Exploring Primordial Black Holes as Probes for Early Cosmological History

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Abstract

Primordial Black Holes (PBHs) are objects that could have formed in the early universe as a result of density fluctuations that collapsed under their own gravity. Their mass can be (much) smaller than the mass of stellar black holes by many orders of magnitude and is greatly influenced by the cosmological epoch in which they formed. This thesis investigates the formation of PBHs and their growth due to accretion when the universe was a tiny fraction of a second old. By analyzing equations that describe these phenomena, we aim to connect the early cosmological history to the resulting mass spectrum of PBHs.

1 Introduction

PBHs behave very differently from those black holes most common in our universe now. PBHs form when regions of the early universe experience density fluctuations large enough to overcome pressure and grow gravitationally [1]. These fluctuations, described by the density contrast $\delta = \delta\rho/\rho$, where ρ is the average energy density of the universe [2]. Various theoretical mechanisms have been proposed to explain how PBHs could form shortly after the Big Bang. An interesting formation mechanism involves cosmological phase transitions, during which the universe underwent a significant change in its equation of state, briefly lowering the pressure support in the plasma. This change could have heightened the likelihood of gravitational collapse in overdense regions.

The main mechanism that we consider here involves primordial density fluctuations. These fluctuations can be traced back to quantum fluctuations during cosmic inflation that were stretched to cosmological scales. Regions with higher than the average density could decouple from the cosmic expansion and undergo a gravitational collapse if the density contrast δ exceeds a critical threshold δ_c, most commonly around $\delta_c \sim 0.1$ [2]. In a radiation-dominated (RD) era, radiation pressure prevents the collapse of density fluctuations at sub-horizon scales. As a result, only fluctuations whose physical size is on the scale of the cosmic horizon could collapse under their own gravity (if their amplitude is large enough). However, during a matter-dominated epoch, the absence of pressure allows fluctuations at scales much smaller than the horizon size to collapse.

Accretion also plays an important role in the evolution of PBHs following their initial formation. PBHs can grow through Bondi-Hoyle accretion where matter falls spherically inward. On the other hand, in the Eddington-limited accretion a rotating disk forms around the PBH, thereby limiting its growth rate. The conditions for accretion depend on many factors, such as the density of the surrounding medium, the epoch in which the black hole is accreting, and the mass of the PBH [3]. These effects have a great impact on the mass spectrum of PBHs formed in different cosmological epochs.

This work focuses on understanding the mass spectrum of PBHs that formed during an early matter-dominated (EMD) era when the universe was a fraction of a second old By deriving and analyzing equations that govern the evolution of the PBHs, we examine how the collapse of density fluctuations followed by accretion from the surrounding influenced PBH formation and growth. Of particular interest to us is the $10^{17} - 10^{22}$ g mass range where PBHs can constitute the entirety of dark matter (DM) in the universe [4].

2 Background

In the "standard cosmological history", The early universe underwent different eras that were defined by the dominant form of energy during the era. In the "standard cosmological history", the universe entered a RD phase shortly after the end of inflation, meaning its energy density was primarily in the form of relativistic particles with an equation of state $w = 1/3$. However, important classes of particle physics models of the early universe predict one (or more) periods of EMD, where the energy density was temporarily dominated by non-relativistic matter, such as coherent oscillations of massive scalar fields or long lived heavy particles [5]. These components behave like pressureless matter ($w = 0$) and eventually decay to establish a RD universe. Such a scenario is fully consistent with observations as long as the EMD epoch ends within the first second after Big Bang. The best observational probes of the early universe, namely the cosmic microwave background (CMB) and big bang nulceosyntheis (BBN), inform us about the state of the universe at about 400,000 years and one second after Big Bang respectively. However, currently, we do not have any observational

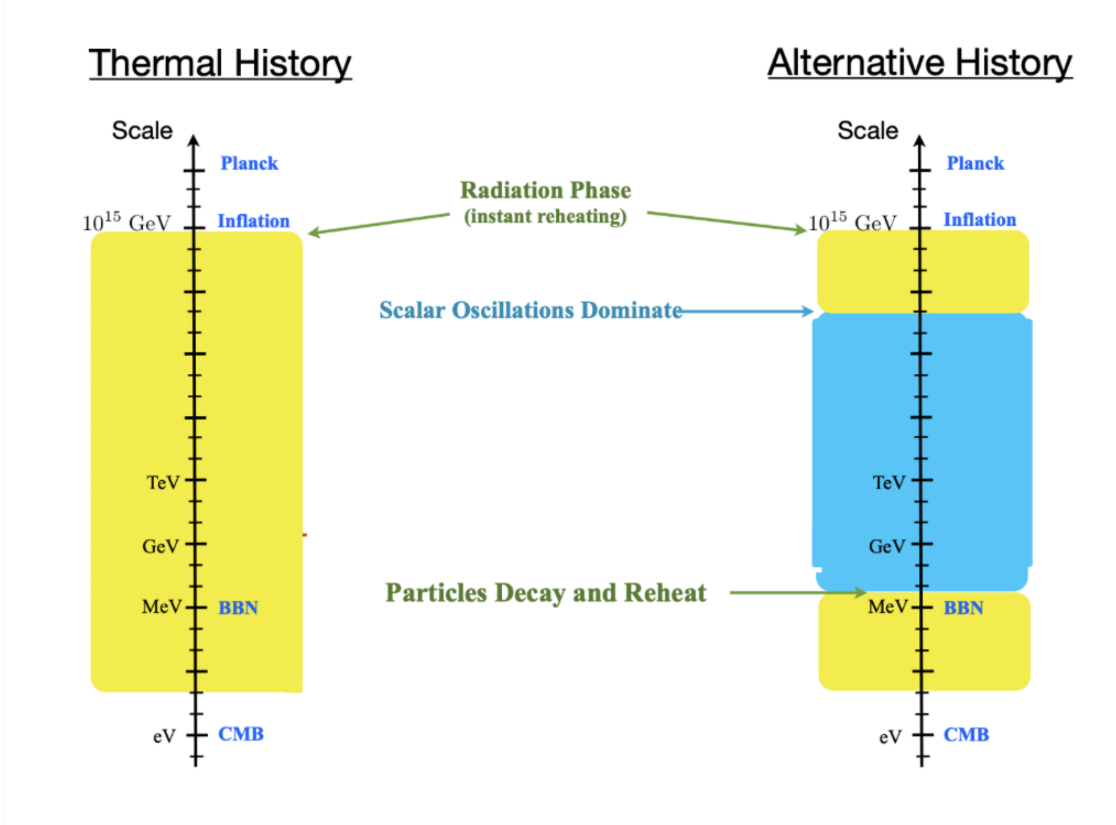

Figure 1: Comparison of the standard thermal history of the universe (left) with an alternative scenario (right) including an EMD phase. (Adapted from [5])

probes of the universe prior to one second.

Fig. 1 compares the standard thermal history of the universe with an alternative scenario involving EMD. Oscillations of a scalar field (for example, modulus fields from string theory) that behave like matter can dominate the energy density of the universe and then decay and reheat the universe back into RD before the onset of BBN.

The absence of pressure during EMD has important implications for the formation and evolution of PBHs. First, it leads to the growth of subhorizon density fluctuations in time. As a result, fluctuations whose initial amplitude is very small δ can reach the threshold δ_c during EMD and collapse to form PBHs. Second, for the same reason, PBHs can efficiently accrete from their surrounding upon formation, and hence their initial mass can grow by a significant factor. The Bondi-Hoyle accretion is especially efficient and could increase PBH masses by a few orders of magnitude [6]. After reheating, when the universe transitions back to RD, accretion is must less effective [3]. The maximum mass a PBH can reach is therefore largely determined by how long it accretes during the EMD phase.

Unlike astrophysical black holes, the mass of PBHs that are around today spans a very

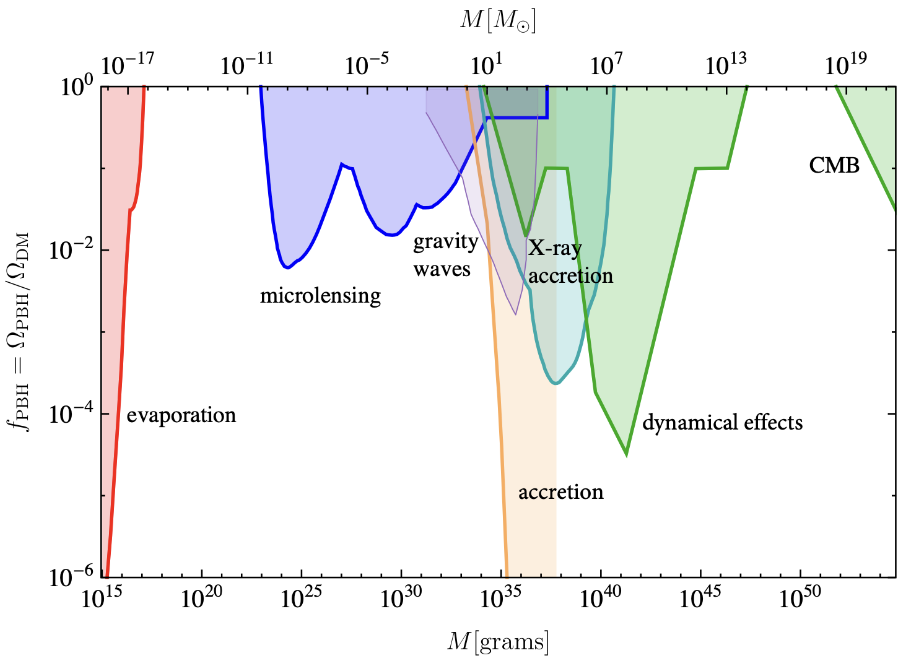

Figure 2: Observational constraints on the fraction of dark matter in the form of PBHs across a wide range of masses. The shaded regions are excluded by various probes (e.g., microlensing, CMB, gamma-ray background), but a window between 10^{17} and 10^{22} g remains unconstrained. (Adapted from [7])

broad range from $10^{-18}M_\odot$ to masses much larger than $M_\odot$ (see Fig. 2) [7]. Here, $M_\odot \approx 2 \times 10^{33}$ g denotes the solar mass). PBHs with masses below 10^{15} g have evaporated via Hawking radiation before the present time. For $M_{\rm PBH} \simeq 10^{15}-10^{17}$ g, the abundance of PBHs is tightly constrained by the emission of gamma-rays from the Hawking radiation [7]. For masses above 10^{22} g, the PBH abundance is subject to various astrophysical constraints [7]. There exists an interesting mass window $10^{17} - 10^{22}$ g within which the PBHs can account for the entirety of DM in the universe [4].

3 Theoretical Framework

3.1 Collapse Conditions and Thresholds

The formation of a PBH begins when a region of the early universe becomes dense enough for gravity to overcome pressure and cause collapse [2]. The likelihood of this happening depends on the amplitude of the density contrast, defined as $\delta = \frac{\delta\rho}{\rho}$, which is a measure of the overdensity in a region to the average energy density of the universe at the time [2]. If δ exceeds a critical threshold, denoted δ_c, the region will collapse [2].

Figure 3 illustrates how this difference in thresholds affect PBH formation. Because Gaussian distribution of the amplitude of primordial fluctuations drops off steeply, a small shift in δ_c can lead to orders of magnitude differences in the probability of collapse.

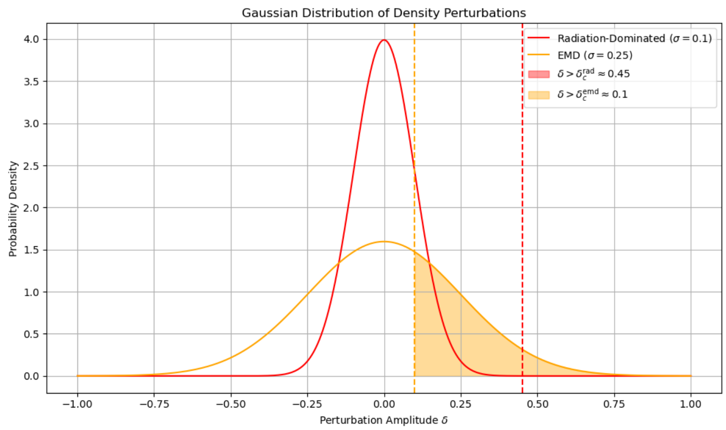

Figure 3: Gaussian distributions of density perturbations for two cosmological scenarios. In radiation domination (red), the variance is smaller and the threshold for collapse is higher ($\delta_c \approx 0.45$). In an early matter-dominated phase (orange), the variance is larger and the threshold is lower ($\delta_c \approx 0.1$), resulting in a significantly larger fraction of perturbations collapsing into PBHs (shaded areas) (Figure created by the author.

The density contrast is commonly expressed in terms of the primordial curvature perturbation, $\zeta(k)$, which remains constant on superhorizon, or larger than the cosmological horizon scale [3]. The relation between $\delta(k,t)$ and $\zeta(k)$ for modes that enter the horizon is given by:

$$\delta(k,t) = \frac{4}{9}\left(\frac{k}{aH}\right)^2 \zeta(k), \tag{1}$$

where k is the comoving wavenumber, a is the scale factor, and H is the Hubble parameter. This equation states that the amplitude of the density fluctuations at horizon entry is directly related to the initial curvature perturbation spectrum set during inflation [3]. Because of this connection, the abundance of PBHs formed can be traced back to features in the primordial power spectrum [9].

In a RD era, large amplitude fluctuations with $\delta_c \sim 0.45$ can overcome the pressure if their physical size is on the scale of the horizon [2]. On the other hand, during an EMD, pressure is negligible, and hence subhorizon fluctuations can collapse if the corresponding value of δ exceeds $\delta_c \sim 0.1$ [7]. More importantly, due to the absence of pressure, subhorizon fluctuations freely grow during EMD:

$$\delta(t) = \delta_i \left(\frac{t}{t_i}\right)^{2/3}, \tag{2}$$

where t_i is the initial time corresponding to horizon entry and $\delta_i = \delta(t_i)$. Thus, a fluctuation with a small initial value δ_i can collapse and form PBHs as long as it has enough time to grow and reach the critical value δ_c in the EMD epoch. This dramatically increases the probability of PBH formation during EMD [2].

3.2 PBH Mass Scaling

The mass of a PBH at the time of its formation is closely linked to the mass enclosed within the cosmological horizon. This is because the horizon sets the maximum region over which causal processes can operate and thus defines the largest possible collapsing region at a given time. The horizon mass at a time t is given by:

$$M_H(t) = \frac{4\pi}{3}\rho(t)R_H^3, \tag{3}$$

where $\rho(t)$ is the energy density of the universe and R_H is the horizon radius that is related to Hubble parameter $H(t)$ according to $R_H \simeq H^{-1}$. For a spatially flat universe dominated by a single component (e.g., matter or radiation), the horizon mass can be simplified using Friedmann's equation:

$$H^2(t) = \frac{8\pi G}{3}\rho(t), \tag{4}$$

resulting in:

$$M_H(t) = \frac{M_{\rm P}^2}{H(t)}, \tag{5}$$

where $M_{\rm P} \equiv (\hbar c/8\pi G)^{-1/2}$ is the reduced Planck mass. Since $H(t) \propto t^{-1}$, the horizon mass scales with time as:

$$M_H(t) \propto t. \tag{6}$$

The mass of a PBH formed at time t is then expected to be a fraction of the horizon mass at that time [4]:

$$M_{\rm PBH}(t) = \gamma M_H(t), \tag{7}$$

where $\gamma \sim 0.2$–0.5 is a model-dependent efficiency factor related to the equation of state and the shape of the collapsing perturbation. This relation implies that PBHs formed earlier in the universe (at smaller t) are lighter, while those formed closer to the end of an early matter-dominated phase or during radiation domination are heavier. The scaling between PBH mass and formation time is a crucial input for predicting the PBH mass spectrum and its dependence on the thermal history of the early universe.

3.3 PBH Mass Fraction

To estimate the fraction of the energy density that collapses into PBHs, the Press-Schechter formalism can be used, which is a statistical approach originally developed for halo formation in structure formation studied, but it is also applicable to PBHs. This formalism assumes that the initial density fluctuations δ) follow a Gaussian distribution with zero mean and variance $\sigma^2(M)$, where M refers to the smoothing sale (or mass scale) of the perturbation. The probability density function is:

$$P(\delta) = \frac{1}{\sqrt{2\pi}\,\sigma(M)} \exp\left(-\frac{\delta^2}{2\sigma^2(M)}\right). \tag{8}$$

The fraction $\beta(M)$ of regions collapsing into PBHs of mass M is given by the integral of this probability above the critical collapse threshold δ_c:

$$\beta(M) = \int_{\delta_c}^{\infty} P(\delta)\,d\delta = \int_{\delta_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma(M)} \exp\left(-\frac{\delta^2}{2\sigma^2(M)}\right) d\delta. \tag{9}$$

This can be expressed in terms of the complementary error function:

$$\beta(M) = \frac{1}{2}\,\mathrm{err}\left(\frac{\delta_c}{\sqrt{2}\,\sigma(M)}\right). \tag{10}$$

In the regime where $\delta_c \gg \sigma(M)$, which is typical for PBH formation, we can use the asymptotic form of the complementary error function to approximate:

$$\beta(M) \approx \frac{\sigma(M)}{\delta_c\sqrt{2\pi}} \exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right). \tag{11}$$

This shows that the abundance of PBHs formed in a RD phase is extremely sensitive to the amplitude of fluctuations encoded in $\sigma(M)$. Even small changes in σ can lead to exponential differences in PBH formation fraction.

3.4 Accretion

Following their initial formation, PBHs do not necessarily remain static in mass. In an EMD epoch, the lack of radiation pressure allows PBHs to accrete surrounding matter efficiently [6]. This accretion processes plays an important role in shaping the final mass distribution of PBHs, and consequently determining their viability as DM candidates or sources of observational signals. Two distinct accretion mechanisms are relevant in this context: Bondi-Hoyle and Eddington-limited accretion.

Bondi-Hoyle accretion is a spherically symmetric process in which matter is gravitationally drawn into the PBH from the surrounding medium. In the absence of significant pressure gradients, the infalling material flows inward rather efficiently, allowing for rapid and relatively unrestricted growth. The general form of the Bondi-Hoyle accretion rate is given by [6]:

$$\dot{M}_{\rm Bondi} = 4\pi r^2 \rho(r,t) v(r,t) \tag{12}$$

where r is the radius at which the accretion flow is evaluated, $\rho(r,t)$ is the ambient density and $v(r,t)$ is the inflow velocity.

As shown in the appendix, Bondi-Hoyle accretion duing in EMD results in [6]:

$$M_{\rm B-H}(t) \propto t^{2/3}. \tag{13}$$

This power-law growth indicates that PBHs can gain several orders of magnitude in mass during the EMD phase.

Due to the conservation of angular momentum, infalling matter gains angular velocity as it gets closer to the black hole. If the angular velocity of infalling particles reaches the Keplerian value before crossing the black hole horizon, a rotating disk can form. This signals termination of Bondi-Hoyle accretion after which the PBH starts accreting from the disk. The formation of a disk further restricts the rate at which PBHs can grow. This reinforces the importance of early Bondi-Hoyle accretion during EMD in determining the final PBH mass.

The Eddington luminosity, which is the maximum luminosity a black hole can sustain while still accreting, is given by

$$L_{\rm Edd} = \frac{4\pi G M m_p c}{\sigma_T}, \tag{14}$$

where m_p is the proton mass and σ_T is the Thomson scattering cross-section. If the accreting black hole emits radiation near this limit, the outward pressure from the photons effectively stops any additional infall, capping the accretion rate. The Eddington-limited accretion rate is the maximum rate at which a black hole can accrete matter while radiating at its Eddington luminosity. It is given by:

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\epsilon c^2} = \frac{4\pi G M m_p}{\epsilon c \sigma_T}, \tag{15}$$

where ϵ is the radiative efficiency. In the post-reheating universe, this limit becomes like a bottleneck for further PBH growth.

This reiterates that the final mass of a PBH is determined primarily by how long it is able to grow through Bondi-Hoyle accretion before transitioning to Eddington-limited regime [7].

4 Results

To estimate the maximum mass a PBH can attain during the EMD phase, we start from the Bondi-Hoyle growth relation in Eq. (15) [6]:

$$M(t) = M_i \left(\frac{t}{t_i}\right)^{2/3}. \tag{16}$$

If Bondi-Hoyle accretion ceases at $t = t_{\rm max}$, the final mass will be:

$$M_{\rm max} = M_i \left(\frac{t_{\rm max}}{t_i}\right)^{2/3}. \tag{17}$$

As shown in the appendix:

$$t_{\rm max} \approx H_i^{-1} \delta_c^{6/5} \sigma_H^{-9/5}, \tag{18}$$

where, σ_H is the variance of primordial density fluctuations. This leads to citeDeLuca:2022:

$$M_{\rm max} \approx 0.3 \delta_c^{9/5} \sigma_H^{-6/5} M_i. \tag{19}$$

We take $\sigma_H \simeq 10^{-5}$ below, which implies no enhancement of density fluctuations over the CMB value at small scales relevant for PBH formation.

We consider a situation where the onset and end of the EMD period correspond to t_O and t_R respectively. Then, for PBHs that were formed at the onset of EMD, $t_i = t_O$. Also, from Eqs. (5,7), we have :

$$M_i \simeq \frac{M_{\rm P}^2}{H_O^2}. \tag{20}$$

The final mass of PBHs M_f depends on the relation between t_R and $t_{\rm max}$. If $t_{\rm max} \lesssim t_R$, we will have $M_f = M_{\rm max}$. On the other hand, $M_f = M_i (t_R/t_O)^{2/3}$.if $t_{\rm max} > t_{\rm R}$.

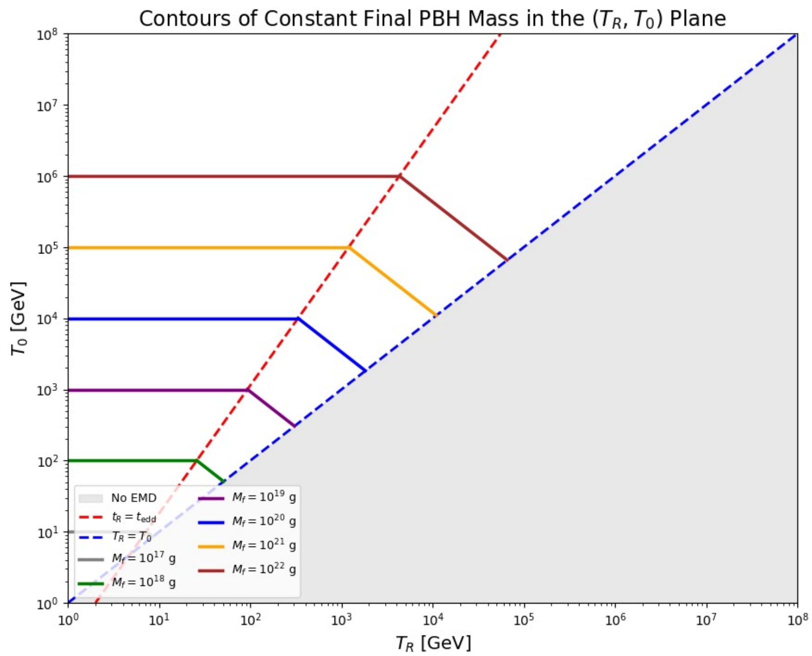

Figure 4: Final PBH mass contours as a function of initial temperature T0 and reheating temperature TR. The shaded region indicates parameter space where efficient growth is suppressed. (figure created by author)

In the RD phase before and and after EMD, we have:

$$t = \frac{1}{2}H \quad , \quad H = \left(\frac{\pi^2}{90} g_*(T)\right)^{1/2} \frac{T^2}{M_{\rm P}}, \tag{21}$$

where $g_*(T)$ is the number of relativistic degrees of freedom at temperature T. Thus:

$$M_f \simeq \delta_c^{9/5}\, \sigma_H^{-6/5}\, \frac{M_{\rm P}^3}{T_O^2} \qquad t_{\rm max} \lesssim t_{\rm R}\,, \tag{22}$$

$$M_f \simeq 3\left(\frac{T_O}{T_R}\right)^{4/3} \frac{M_{\rm P}^3}{T_O^2} \qquad t_{\rm max} > t_R\,, \tag{23}$$

where we have taken $g_*(T_O) = g_*(T_R) = 106.75$ as is the case in the standard model of particle physics.

Fig. 4 depicts contours of constant M_f values in the T_0–T_R plane [7]. Regions with lower T_R and higher T_0 support more efficient PBH growth, while high T_R sharply limits the accretion window. The dashed red line marks the transition boundary where the EMD phase ends earlier than $t_{\rm max}$ (i.e., $t_{\rm R} < t_{\rm max}$). In this case M_f only grows partially and does not reach the maximum value of $M_{\rm max}$. The shaded region below the dashed blue line is excluded because it does not give rise to EMD (that is, $t_{\rm R} < t_{\rm O}$).

The values of M_f chosen in Fig. 4 correspond to the $10^{17} - 10^{22}$ g window. PBHs in this mass window are not ruled out by current observational constraints, including those from microlensing, the CMB, and gamma-ray background [7]. This leads to a very interesting possibility that PBHs can constitute 100% of DM in the universe within this mass range. What is specially compelling about this scenario is that it does not rely on particle DM candidates that would require new physics beyond the standard model of particle physics.

Our derived mass contours show that if PBH forms early enough and the reheating temperature is low, it can grow to reach the viable DM window. The exponential dependence of the PBH formation fraction $\beta(M)$ on the amplitude of density fluctuations $\sigma(M)$ implies that even small enhancements in the primordial power spectrum can lead to a high abundance of PBHs. This sensitivity is important because it means that relatively modest features in the inflationary potential or reheating dynamics could result in a universe where PBHs make up for all or most of the dark matter.

4.1 Discussion and Conclusion

PBHs formed in the early universe could leave observable imprints through scalar-induced gravitational waves (GWs), most commonly if the primordial curvature power spectrum is enhanced at small scales [9]. These GWs are sensitive to the amplitude and shape of the inflationary spectrum and can serve as an indirect probe of PBH production. During an EMD phase, the generation of these scalar-induced gravitational waves is further amplified, especially near the transition from matter to radiation domination. Depending on the reheating temperature and the PBH mass scale, these signals may fall within the sensitivity range of upcoming gravitational wave detectors like LISA, DECIGO, or the Einstein Telescope [9]. In addition, PBHs with masses between 10^{15} and 10^{16} g are being evaporated at the present time through Hawking radiation. This process could contribute to the diffuse gamma-ray background and may leave distinct spectral features [7]. While constraining this low-mass end of the PBH spectrum, it also offers a potential detection route for evaporating black holes. Altogether, GWs and gamma-ray signals provide valuable observational tools for testing the scenarios discussed in this thesis and for constraining the early universe conditions that could lead to PBH formation.

Finally, some simplifications were made in this study that should be kept in mind when interpreting the results. First, we assumed an idealized EMD phase with a sharp transition to radiation domination. In reality, the end of EMD may be more gradual, and reheating could be inhomogeneous or prolonged. These effects would change the effective growth window for PBHs and might slightly shift the final mass contours. Also, we did not model the detailed shape of the curvature power spectrum. The specific inflationary model responsible for those features was not addressed, and the variance $\sigma(M)$ was treated parametrically. Despite these simplifications, the analytical approach provides a useful framework for estimating PBH growth and abundance trends. Future work including more precise numerical simulations or modified accretion scenarios can build on these foundations.

In summary, the results presented in this thesis offer important insights into how PBH could have formed and evolved during an EMD phase in the early universe. We explored the formation and growth of PBHs during an EMD era focusing on the role of accretion. By starting from first principles derivations and applying physically motivated accretion models, we have shown how PBHs can grow significantly, reaching masses that not only fall within observationally viable windows, but also make them compelling candidates for DM. One of the most important outcomes is the link between thermal history (specifically an epoch of EMD) and the final PBH mass. This reinforces the idea that understanding reheating isn't just important for inflationary models, it's also important for any PBH-related DM scenario.

5 Acknowledgments

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6 References

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[10] Craig, L. (2025). Figure generated by the author using Python 3.9, NumPy, SciPy, and Matplotlib.

[11] Craig, L. (2025). Contour plot of final PBH mass generated by the author using Python 3.9, NumPy, and Matplotlib.

Appendix: Derivations of PBH Growth and Accretion Timescales

In this appendix, we provide detailed derivations of key relations used for PBH mass growth during an EMD phase.

Bondi-Hoyle Accretion Growth During EMD

During an EMD era, PBHs grow through spherical Bondi-Hoyle accretion. The accretion rate is [6]:

$$\dot{M}_{\text{Bondi}} \propto r_{\text{ta}}^2 \rho(t) t, \tag{24}$$

where:

- $r_{\text{ta}}(t)$ is the turnaround radius,
- $\rho(t)$ is the background density,
- t is cosmic time.

The self-similar collapse solution during EMD gives the following scalings:

$$r_{\text{ta}}(t) \propto t^{1/3}, \quad \rho(t) \propto t^{-2}. \tag{25}$$

Substituting these into the accretion rate, we find:

$$\dot{M}_{\text{Bondi}} \propto (t^{1/3})^2 \times t^{-2} \times t, \tag{26}$$

$$\propto t^{2/3} \times t^{-2} \times t, \tag{27}$$

$$\propto t^{-1/3}. \tag{28}$$

Thus, the Bondi-Hoyle accretion rate decreases with time as $\dot{M} \propto t^{-1/3}$.

Integrating this mass growth over time:

$$M(t) = M_i \left(\frac{t}{t_i}\right)^{2/3}, \tag{29}$$

where M_i is the mass at the initial time t_i. This shows that PBH mass grows as $t^{2/3}$ during the Bondi-Hoyle phase.

Maximum PBH Mass

Bondi-Hoyle accretion can continue until angular velocity effects become important, leading to disk formation. The maximum accretion time $t_{\max}$ is given by [8]:

$$t_{\max} \approx H_i^{-1} \delta_c^{6/5} \sigma_H^{-9/5}, \tag{30}$$

where H_i is the Hubble parameter at formation, δ_c is the collapse threshold, and σ_H is the variance of density perturbations at horizon crossing. Substituting $t_{\max}$ into the mass growth formula, we find:

$$M_{\max} = M_i \left(\frac{t_{\max}}{t_i} \right)^{2/3}. \tag{31}$$

Since $t_i \sim H_i^{-1}$, we have:

$$\frac{t_{\max}}{t_i} \simeq \delta_c^{6/5} \sigma_H^{-9/5}. \tag{32}$$

Thus:

$$M_{\max} \simeq M_i \left(\delta_c^{6/5} \sigma_H^{-9/5} \right)^{2/3}, \tag{33}$$

$$= M_i \delta_c^{4/5} \sigma_H^{-6/5}. \tag{34}$$

Now, recalling that the initial PBH mass M_i itself scales with the collapse threshold δ_c at formation, we can absorb this into a prefactor [8], resulting in:

$$M_{\max} \approx 0.3\, \delta_c^{9/5} \sigma_H^{-6/5} M_H, \tag{35}$$

where M_H is the horizon mass at formation. We see that the maximum PBH mass depends on both the amplitude of primordial fluctuations and the collapse threshold.

Python Code for Gaussian Distribution Plot

The following code was used to generate the Gaussian distribution figure comparing early matter domination and radiation domination conditions:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# Constants and parameters
mu = 0  # Mean perturbation amplitude
```

```
sigma_rad = 0.1  # Small variance for radiation-dominated era
sigma_emd = 0.25  # Larger variance for early matter-dominated era

# Collapse thresholds
delta_c_rad = 0.45
delta_c_emd = 0.1

# x-values and PDF values
x = np.linspace(-1, 1, 1000)
y_rad = norm.pdf(x, mu, sigma_rad)
y_emd = norm.pdf(x, mu, sigma_emd)

# Shaded areas
x_fill_rad = np.linspace(delta_c_rad, 1, 500)
x_fill_emd = np.linspace(delta_c_emd, 1, 500)
y_fill_rad = norm.pdf(x_fill_rad, mu, sigma_rad)
y_fill_emd = norm.pdf(x_fill_emd, mu, sigma_emd)

# Plot
plt.figure(figsize=(10, 6))
plt.plot(x, y_rad, 'r-', label='Radiation-Dominated ($\\sigma=0.1$)')
plt.plot(x, y_emd, 'orange', label='EMD ($\\sigma=0.25$)')

# Fill the region above thresholds
plt.fill_between(x_fill_rad, y_fill_rad, color='red', alpha=0.4, label=r'$\delta > \delt
plt.fill_between(x_fill_emd, y_fill_emd, color='orange', alpha=0.4, label=r'$\delta > \d

# Vertical lines for collapse thresholds
plt.axvline(delta_c_rad, color='red', linestyle='--')
plt.axvline(delta_c_emd, color='orange', linestyle='--')

# Labels and styling
plt.title('Gaussian Distribution of Density Perturbations')
plt.xlabel('Perturbation Amplitude $\\delta$')
plt.ylabel('Probability Density')
plt.legend()
```

```
plt.grid(True)
plt.tight_layout()
plt.show()
```

Python Code for PBH Mass Contour Plot

The following code was used to generate the contours of constant final PBH mass in the (T_R, T_0) plane:

```
import numpy as np
import matplotlib.pyplot as plt

# Mass labels and colors
mass_labels = [1e17, 1e18, 1e19, 1e20, 1e21, 1e22]
colors = ['gray', 'green', 'purple', 'blue', 'orange', 'brown']

# Turning points
T0_turns = np.logspace(1, 6, len(mass_labels))
TR_turns = 2 * T0_turns**(5/9)

# Temperature grid
T0 = np.logspace(0, 8, 500)

# Setup plot
fig, ax = plt.subplots(figsize=(10, 8))

# No EMD region
ax.fill_between(T0, 1, T0, color='lightgray', alpha=0.5, label='No EMD')

# Dashed reference lines
TR_edd = 2 * T0**(5/9)
ax.plot(TR_edd, T0, '--', color='red', lw=2, label=r'$t_R = t_{\rm edd}$')
ax.plot(T0, T0, '--', color='blue', lw=2, label=r'$T_R = T_0$')

# Draw Mf contours
for Mf, col, T0_turn, TR_turn in zip(mass_labels, colors, T0_turns, TR_turns):
    TR_neg = np.logspace(np.log10(TR_turn), np.log10(T0_turn), 1000)
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    T0_neg = (T0_turn * TR_turn) / TR_neg
    mask = T0_neg >= TR_neg
    ax.plot(TR_neg[mask], T0_neg[mask], color=col, lw=2.5)

    TR_flat = np.logspace(0, np.log10(TR_turn), 300)
    T0_flat = np.full_like(TR_flat, T0_turn)
    ax.plot(TR_flat, T0_flat, color=col, lw=2.5, label=rf'$M_f = 10^{{{int(np.log10(Mf))

# Final formatting
ax.set_xscale('log')
ax.set_yscale('log')
ax.set_xlim(1, 1e8)
ax.set_ylim(1, 1e8)
ax.set_xlabel(r'$T_R\;[\mathrm{GeV}]$', fontsize=14)
ax.set_ylabel(r'$T_0\;[\mathrm{GeV}]$', fontsize=14)
ax.set_title(r'Contours of Constant Final PBH Mass in the $(T_R, T_0)$ Plane', fontsize=
ax.grid(False)
ax.legend(fontsize=9, loc='lower left', ncol=2)

plt.tight_layout()
plt.show()
```