PHYS 262: SPECTRA & BOHR MODEL, CHAPTER 38

ALONG WITH PHOTOELECTRIC EFFECT, ANOTHER SURPRISING EXPERIMENT IN THE EARLY 1900'S WAS THE MEASUREMENT OF ATOMIC SPECTRA.

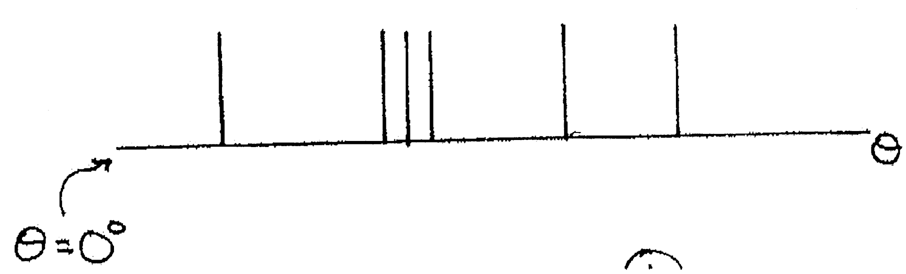
SPECTRUM - LIGHT EMITTED BY AN ATOM.

ATOMS EMIT LIGHT BECAUSE AFTER ABSORBING ENERGY THEIR ELECTRONS ACCELERATE (ACCELERATING CHARGES CREATE EM WAVES - LIKE IN AN ANTENNA). ONE WAY TO GIVE THE ELECTRONS ENERGY IS BY APPLYING A VOLTAGE ACROSS A REGION WHERE THE ATOMS ARE LOCATED - THIS IS HOW NEON SIGNS WORK.

TO MEASURE THE SPECTRA MEANS TO DETERMINE THE DIFFERENT WAVELENGTHS (OR FREQUENCY) IN THE SPECTRUM. AN "EASY" WAY TO DO THIS IS TO USE A DIFFRACTION GRATING. CONSTRUCTIVE INTERFERENCE OCCURS AT $d \sin\theta = m\lambda$, WITH DESTRUCTIVE INTERFERENCE EVERYWHERE ELSE. FOR LIGHT WITH A MIXTURE OF WAVELENGTHS (SUCH AS A SPECTRUM), EACH DIFFERENT WAVELENGTH ENDS UP AT A DIFFERENT ANGLE.

FOR WHITE LIGHT WE OBSERVE THE RAINBOW BECAUSE WHITE LIGHT IS AN EQUAL MIX OF ALL WAVELENGTHS.

FOR THE SPECTRUM OF AN INDIVIDUAL TYPE OF ATOM, SOMETHING MUCH DIFFENT HAPPENS.



WE GET A LINE SPECTRUM

ONLY CERTAIN PARTICULAR WAVELENGTHS ARE BEING EMITTED.

USING $E = \frac{hc}{\lambda}$ $\Rightarrow$ ONLY CERTAIN ENERGIES ARE BEING EMITTED.

THE PHOTONS' ENERGY COMES FROM THE ELECTRONS IN THE ATOM $\Rightarrow$ ONLY SOME ELECTRON ENERGY VALUES ARE POSSIBLE (ALLOWED IS A BETTER TERM).

<u>HYDROGEN</u> <u>SPECTRUM</u> - SIMPLEST ATOM WITH SIMPLEST SPECTRUM.

IN 1885 JOHANN BALMER (SWISS) HAD DETERMINED AN <u>EXPERIMENTAL</u> FORMULA FOR FINDING <u>SOME</u> OF THE WAVELENGTHS IN HYDROGEN'S SPECTRUM.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, \ldots \quad \text{BALMER SERIES}$$

$$R = 1.097 \times 10^7 \, m^{-1} = \text{RYDBERG CONSTANT}$$

TO FIND ALLOWED ENERGIES IN HYDROGEN, WE USE $E = \frac{hc}{\lambda}$

$$\Rightarrow E = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right). \quad E_{PHOTON} = E_{ELECTRON}$$

$$E_{ELECTRON} = -hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = -\frac{hcR}{2^2} - \left(-\frac{hcR}{n^2}\right) = E_2 - E_1$$

$$\Rightarrow \boxed{E_n = -\frac{hcR}{n^2}} \qquad hcR = (4.136 \times 10^{-15}\, eV \cdot s)(3 \times 10^8\, m/s)(1.097 \times 10^7 /m) = 13.6\, eV$$

$$\boxed{E_n = -\frac{13.6\, eV}{n^2}}$$

OTHER PEOPLE SOON FOUND SIMILAR EXPRESSIONS FOR THE OTHER WAVELENGTHS

LYMAN SERIES: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2, 3, 4, \ldots$ ← UV

BALMER SERIES: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, $n = 3, 4, 5, \ldots$ ← VISIBLE MOSTLY

PASCHEN SERIES: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6, \ldots$

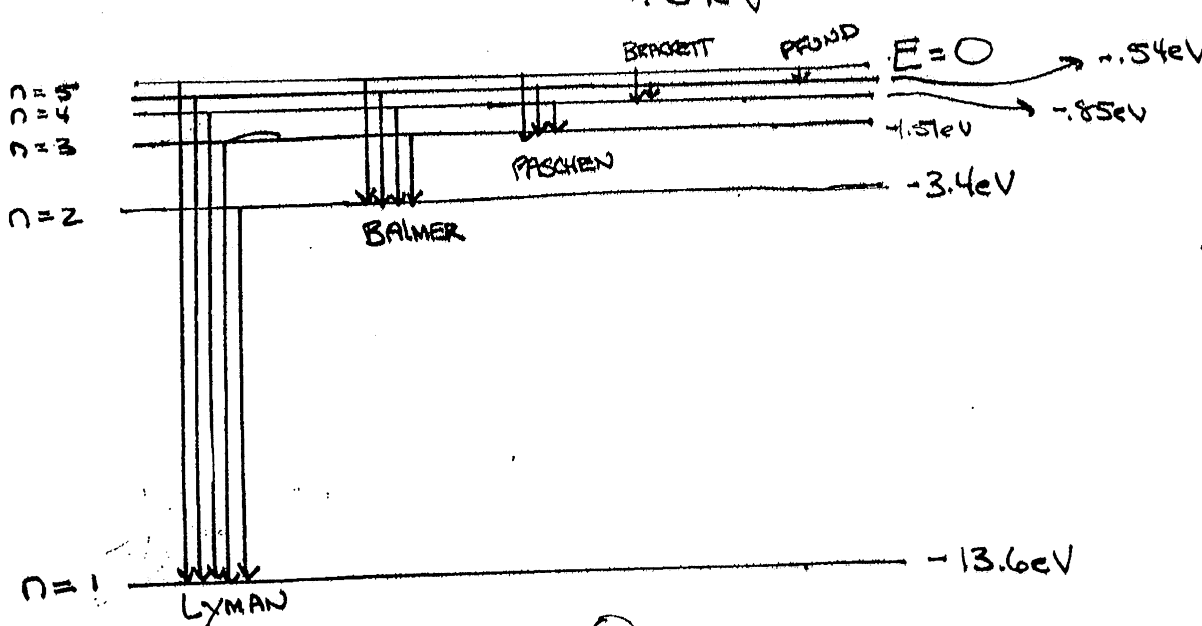
BRACKETT SERIES: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, $n = 5, 6, 7, \ldots$

PFUND SERIES: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$ $n = 6, 7, 8, \ldots$

(PASCHEN, BRACKETT, PFUND:) INFRARED

PHOTON ENERGY IS DETERMINED BY ELECTRON'S "TRANSITION" FROM AN EXCITED STATE TO A FINAL STATE.

SERIES	FINAL STATE	ENERGY ($E_n = \frac{-13.6\,eV}{n^2}$)
LYMAN	$n = 1$	$-13.6\,eV$
BALMER	2	$-3.4\,eV$
PASCHEN	3	$-1.51\,eV$
BRACKETT	4	$-.85\,eV$
PFUND	5	$-.54\,eV$



NOT SHOWN IS $n = 6, 7, 8, \ldots$

EXAMPLE WHAT TYPE OF LIGHT IS EMITTED FROM AN ELECTRON WHICH TRANSITIONS FROM $n_1 = 7$ to $n_2 = 2$.

$$E_1 = \frac{-13.6\,eV}{7^2} = -.278\,eV \quad , \quad E_2 = -\frac{13.6\,eV}{2^2} = -3.4\,eV$$

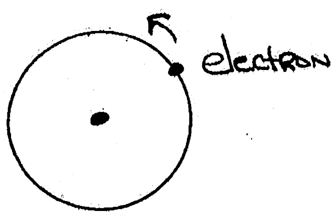
$$\Rightarrow \Delta E = -3.4\,eV - (-.278\,eV) = -3.122\,eV$$

electron LOSES ENERGY WHICH photon gAINS $\Rightarrow E = 3.122\,eV$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(4.136\times10^{-15}\,eV\cdot s)(3\times10^{8}\,m/s)}{3.122\,eV} = 3.97\times10^{-7}\,m = 397\,nm$$

$\approx 400\,nm$ = Violet Light

BOHR MODEL — (NIELS BOHR — DANISH)



ELECTRON CIRCLES NUCLEUS LIKE PLANET CIRCLING SUN.

PROBLEM WITH THIS MODEL: ELECTRON HAS CENTRIPETAL ACCELERATION. ACCELERATING CHARGES EMIT EM WAVES, THEREFORE, THE ELECTRON SHOULD LOSE ENERGY AND QUICKLY SPIRAL INTO NUCLEUS.

TO AVOID THIS, BOHR ASSUMED THE ELECTRON'S ANGULAR MOMENTUM IS QUANTIZED.

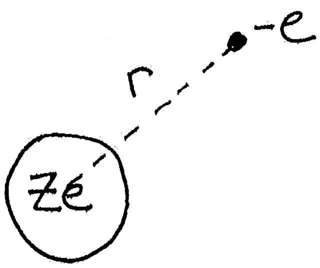
QUANTIZED — HAVING CERTAIN VALUES ONLY.

ANGULAR MOMENTUM FOR PARTICLE IN CIRCULAR MOTION OF RADIUS r: $L = mvr$.

BOHR SET $mvr = n\frac{h}{2\pi}$ $\quad n = 1, 2, 3, 4, \ldots$ (L IS A MULTIPLE OF PLANCK'S CONSTANT DIVIDED BY 2π)

WE USE NOTATION $\boxed{\hbar = \frac{h}{2\pi} = 1.05\times10^{-34}\,Js = 6.583\times10^{-16}\,eV\cdot s}$

$\hbar$ = "h BAR"



Z = # OF PROTONS. ALLOWS US TO GENERALIZE TO ANY 1 ELECTRON ATOM.

COULOMB'S LAW GIVES FORCE: $F = \frac{1}{4\pi\epsilon_0}\frac{Ze \cdot e}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$

$\downarrow$

$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$

THIS ATTRACTIVE FORCE MUST BE PROVIDING THE CENTRIPETAL ACCELERATION $\left(\frac{v^2}{r}\right)$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = \frac{Mv^2}{r} \Rightarrow \frac{Ze^2}{4\pi\epsilon_0} = \frac{Mv^2}{r}(r^2) = Mvr(v)$$

$$\Rightarrow \frac{Ze^2}{4\pi\epsilon_0} = n\hbar v \Rightarrow v = \frac{1}{4\pi\epsilon_0\hbar}\frac{Ze^2}{n} \rightarrow \text{ONLY CERTAIN VELOCITIES ARE ALLOWED!}$$

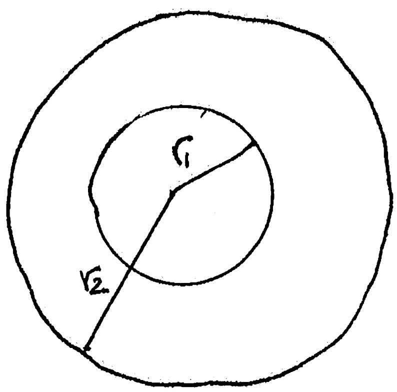
$$\Rightarrow \boxed{v_n = \frac{1}{4\pi\epsilon_0\hbar}\frac{Ze^2}{n}} \quad \text{for } Z=1, \quad v_n = \underline{\frac{2.18\times10^6\,\text{m/s}}{n}}$$

$$Mvr = n\hbar \Rightarrow r = \frac{n\hbar}{M}\left(\frac{1}{v}\right) = \frac{n\hbar}{M}\left(\frac{4\pi\epsilon_0\hbar n}{Ze^2}\right) = \frac{4\pi\epsilon_0\hbar^2}{M}\frac{n^2}{Ze^2} \rightarrow \text{ONLY CERTAIN RADII ARE ALLOWED.}$$

$$\Rightarrow \boxed{r_n = \frac{4\pi\epsilon_0\hbar^2}{M}\frac{n^2}{Ze^2}} \quad \text{for } Z=1, \quad \underline{r_n = a_0 n^2} \quad a_0 = 5.29\times10^{-11}\,\text{m} = .5\,\text{Å}$$

$\text{Å} = 10^{-10}\,\text{m}$

a_0 = BOHR RADIUS.



ELECTRON CAN CIRCLE AT r_1 OR r_2 BUT NOT ANYWHERE INBETWEEN.

DURING TRANSITIONS, ELECTRONS MAKE QUANTUM JUMPS FROM ONE TO ANOTHER.

TO FIND ENERGY, WE REMEMBER THAT THE TOTAL ENERGY IS KINETIC PLUS POTENTIAL ENERGY.

FOR TWO POINT CHARGES $U = +\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \Rightarrow U = \frac{1}{4\pi\epsilon_0}\frac{Ze(-e)}{r} = \frac{-1}{4\pi\epsilon_0}\frac{Ze^2}{r}$

$$K = \frac{1}{2}MV^2 \Rightarrow E = \frac{1}{2}MV^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}$$

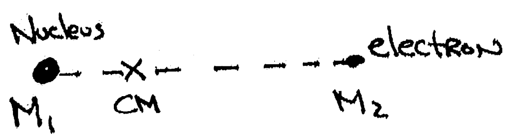
USE $V = V_n$ AND $r = r_n$

$$\Rightarrow E = \frac{1}{2}M\left(\frac{1}{4\pi\epsilon_0\hbar}\right)^2\left(\frac{Ze^2}{n}\right)^2 - \frac{1}{4\pi\epsilon_0}Ze^2\left(\frac{M}{4\pi\epsilon_0\hbar^2}\right)\frac{Ze^2}{n^2} = \frac{-1}{2(4\pi\epsilon_0\hbar)^2}M\left(\frac{Ze^2}{n}\right)^2$$

$$\boxed{E_n = \frac{-M}{2(4\pi\epsilon_0\hbar)^2}\left(\frac{Ze^2}{n}\right)^2 = \frac{-E_0}{n^2}}$$

FOR $Z=1$, $E_0 = 13.6\,eV$ → VERY PERSUASIVE!

WE CAN MAKE IT EVEN BETTER! — BOTH THE NUCLEUS AND ELECTRON MOVE, THEY ROTATE ABOUT THE CENTER OF MASS.



$$\vec{r}_{CM} = \frac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_1+M_2}$$

IT TURNS OUT THAT TO INCLUDE THE NUCLEUS MOTION, ALL WE HAVE TO DO IS REPLACE THE ELECTRON MASS WITH THE REDUCED MASS, μ.

$$\boxed{\mu = \frac{M_1 M_2}{M_1+M_2}}$$

FOR HYDROGEN $\mu = .99946\,M$ (M = electron MASS)

FOR OTHER HYDROGEN-LIKE ATOMS, REDUCED MASS IS ESSENTIAL TO DETERMINE correct ENERGIES, RADII, etc.