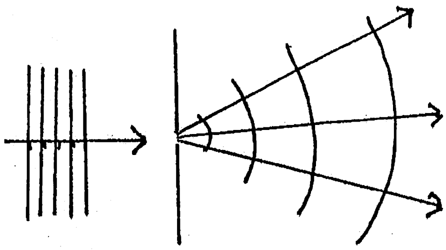
PHYS 262: DIFFRACTION, CHAPTER 36

DIFFRACTION - SPREADING OF ANY WAVE AFTER IT PASSES THROUGH AN OPENING.



SOUND WAVES DIFFRACT AND BEND AROUND OBJECTS. THIS IS WHY WE OFTEN HEAR SOMETHING BEFORE WE CAN SEE IT.

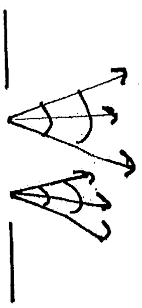
THE SHARPER THE EDGE, THE GREATER THE SPREADING

SINGLE SLIT DIFFRACTION - WITH MONOCHROMATIC LIGHT, WE GET AN INTERFERENCE PATTERN FOR A SINGLE SLIT!

HUYGEN'S PRINCIPLE - EVERY POINT ON A WAVE FRONT ACTS AS A SOURCE WHICH PROPAGATES OUT IN ALL DIRECTIONS.

FOR A SLIT:

VERY POINT
N THE SLIT
TS LIKE
SOURCE

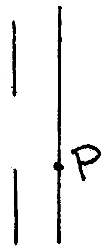


MANY SOURCES OF LIGHT ⇒ INTERFERENCE

THE INTERFERENCE PATTERN CREATED DEPENDS ON HOW FAR AWAY THE SCREEN IS.

FRESNEL DIFFRACTION - AKA NEAR-FIELD DIFFRACTION.

SCREEN IS CLOSE TO SLIT.



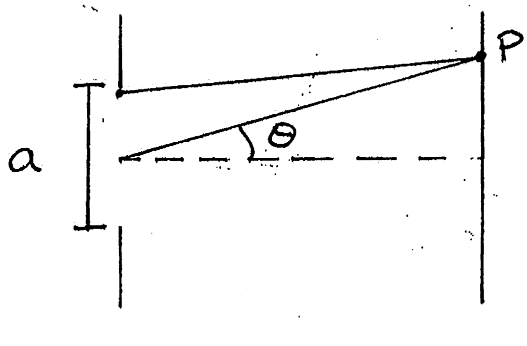
HARD TO DEAL WITH ANALYTICALLY, SO WE'LL NOT DEAL WITH IT!

FRAUNHOFFER DIFFRACTION - FAR-FIELD DIFFRACTION

SCREEN FAR AWAY FROM SLIT

EASIER TO ANALYZE BECAUSE WE CAN MAKE SAME ASSUMPTIONS AS YOUNG'S DOUBLE SLIT.

FOR SINGLE SLIT, LOOK AT TWO PARTICULAR POINTS. ONE JUST BELOW THE TOP AND ONE HALFWAY THROUGH SLIT.



FROM YOUNG'S DOUBLE SLIT, WE KNOW $\Delta r = \left(\frac{a}{2}\right)\sin\theta$

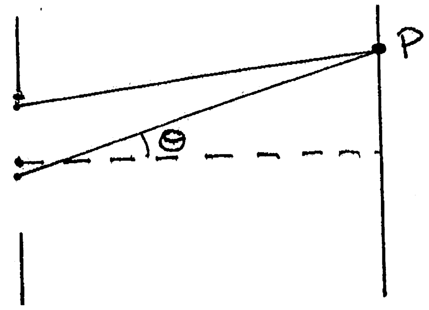
(IT'S A DOUBLE SLIT WITH SEPERATION $d = a/2$)

DESTRUCTIVE INTERFERENCE $\Rightarrow$

$$\Delta r = (m + \tfrac{1}{2})\lambda$$

$$\Rightarrow \frac{a}{2}\sin\theta = (m+\tfrac{1}{2})\lambda \Rightarrow a\sin\theta = \underbrace{(2m+1)\lambda}_{\text{ODD MULTIPLE}}$$

FOR THE NEXT PAIR OF POINTS JUST BELOW ORIGINAL PAIR:

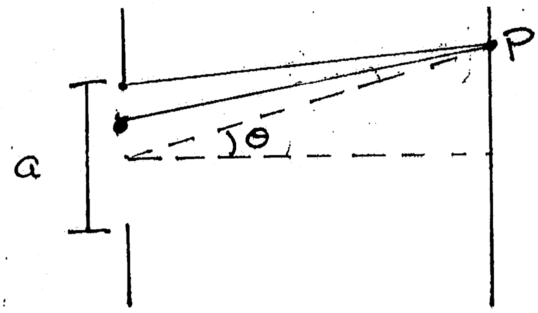


ASSUMING VERY LARGE DISTANCE GIVES SAME $\Delta r = \frac{a}{2}\sin\theta$

$\Rightarrow a\sin\theta = (2m+1)\lambda$ FOR DESTRUCTIVE INTERFERENCE

CONTINUING THIS WAY TELLS US THAT ALL POINTS IN THE SLIT GIVE DESTRUCTIVE INTERFERENCE AT THE POINT P WHERE $a\sin\theta = (2m+1)\lambda$

IF WE SPLIT SLIT INTO FOUR PIECES.



SCREEN VERY FAR AWAY $\Rightarrow$

$$\Delta r = \frac{a}{4}\sin\theta$$

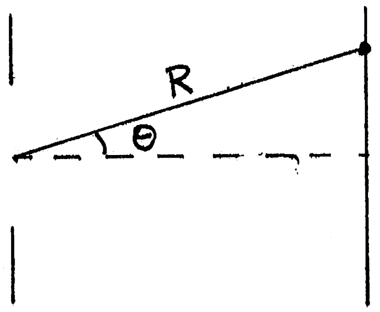
$$\Rightarrow \frac{a}{4}\sin\theta = (m+\tfrac{1}{2})\lambda$$

$\Rightarrow a\sin\theta = (4m+2)\lambda \rightarrow$ EVEN MULTIPLES OF WAVELENGTH BUT NOT ZERO.

SO EITHER EVEN OR ODD MULTIPLES OF λ CREATE DESTRUCTIVE INTERFERENCE:

$$\boxed{a\sin\theta = m\lambda} \qquad m = \pm1, \pm2, \pm3, \ldots \quad m \neq 0$$

INTENSITY - FINDING THE INTENSITY IS A LITTLE TRICKY BECAUSE WE HAVE TO ADD THE CONTRIBUTION OF INFINITELY MANY SOURCES. (AND MAKE A WHOLE BUNCH OF SIMPLIFICATIONS.



FOR PLANE WAVES, $E = E_o \cos(kz - \omega t)$

IT CAN BE SHOWN (SEE LAST PAGE)

$$E_p = E_o \cos(kR - \omega t)\, \frac{2}{k\sin\theta} \sin\left(\frac{ka\sin\theta}{2}\right)$$

IF WE LET $\beta = ka\sin\theta = \frac{2\pi a \sin\theta}{\lambda}$ THEN

$$E_p = E_o \cos(kR - \omega t)\, \frac{2a}{ka\sin\theta} \sin\left(\frac{ka\sin\theta}{2}\right) = E_o a \cos(kR - \omega t)\, \frac{2}{\beta} \sin\left(\beta/2\right)$$

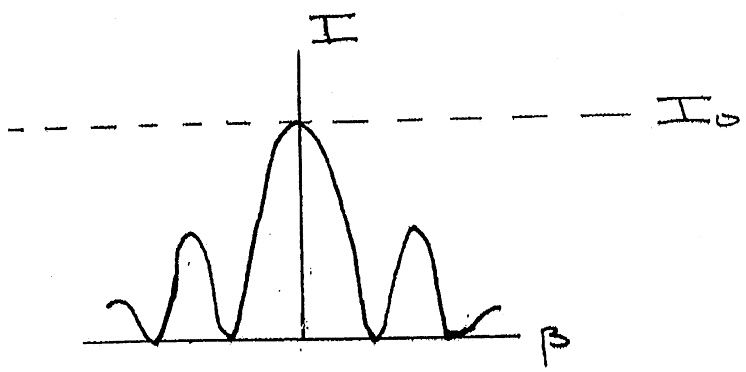
$$= E_o a \cos(kR - \omega t)\, \frac{\sin(\beta/2)}{\beta/2}$$

$$S = \epsilon_o c E_p^2 = \epsilon_o c\, (E_o a)^2 \cos^2(kR - \omega t) \left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$$

$$I = S_{AV} = \underbrace{\epsilon_o c\, (E_o a)^2 (1/2)}_{I_o} \left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$$

$$\Rightarrow \boxed{I = I_o \left(\frac{\sin(\beta/2)}{\beta/2}\right)^2} \qquad \beta = \frac{2\pi a \sin\theta}{\lambda}$$

NOTE: CALCULATOR MUST BE IN RADIANS MODE TO USE THIS EQUATION.



THE MINIMA OCCUR WHERE $\sin \beta/2 = 0 \Rightarrow \beta/2 = m\pi \Rightarrow \beta = 2m\pi$

$$\Rightarrow \frac{2\pi a \sin\theta}{\lambda} = 2\pi m \Rightarrow a\sin\theta = m\lambda \quad \text{(AS REQUIRED)}$$

THE GREATEST MAXIMA OCCURS AT $\beta = 0 \Rightarrow \frac{2\pi a \sin\theta}{\lambda} = 0 \Rightarrow \theta = 0^\circ$

THE LOCAL MAXIMA OCCUR WHERE $\frac{dI}{d\beta} = 0$

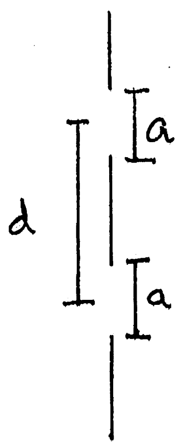
$$\frac{dI}{d\beta} = 2I_0\left(\frac{\sin\beta/2}{\beta/2}\right)\left(\frac{\frac{1}{2}\cos\beta/2}{\beta/2} - \frac{1/2\sin\beta/2}{(\beta/2)^2}\right) = I_0\frac{\sin\beta/2}{(\beta/2)^3}\left(\beta/2\cos\beta/2 - \sin\beta/2\right)$$

$$\Rightarrow \boxed{\beta/2\cos\beta/2 - \sin\beta/2 = 0} \quad \text{TRANSCENDENTAL EQUATION}$$

THE SOLUTIONS GIVE MAXIMA OF APPROXIMATELY:

$$I = .0472 I_0,\ .0165 I_0,\ .0083 I_0,\ \ldots$$

MULTIPLE SLITS



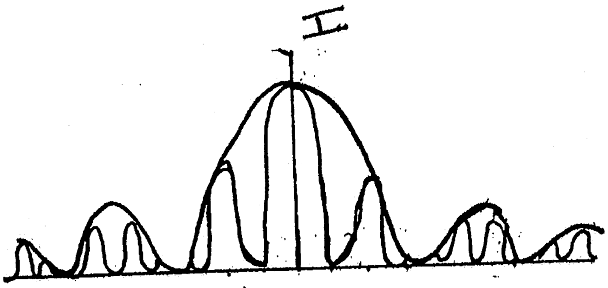
NOTICE THAT THIS IS NOT ~~YOUNG'S~~ DOUBLE SLIT EXP. BECAUSE THE SLITS ARE LARGE.

THE INTENSITY IS THE COMBINATION OF YOUNG'S AND DIFFRACTION'S INTENSITY.

$$\boxed{I = I_0 \cos^2 \phi/2 \left(\frac{\sin \beta/2}{\beta/2}\right)^2}$$

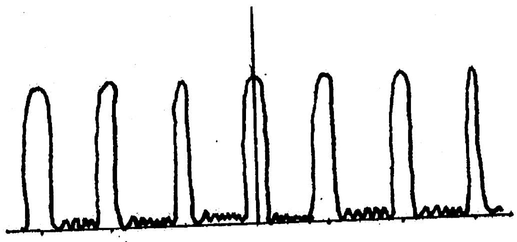
$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$



GET FASTER OSCILLATING FUNCTION INSIDE A SINGLE SLIT "ENVELOPE."

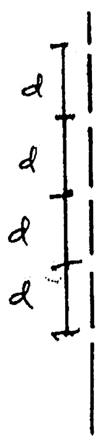
FOR MANY SLITS THE INTENSITY FUNCTION LOOKS LIKE



A DIFFRACTION GRATING HAS SO MANY SLITS THAT WE GET INTENSIT ONLY AT EQUALLY SPACED POINTS.

IF THE DISTANCE BETWEEN SLITS IS d. THE NON-ZERO POINTS OCCUR AT

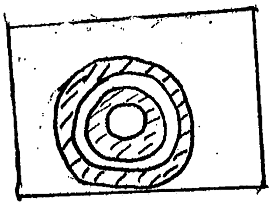
$$\boxed{d \sin\theta = m\lambda}$$



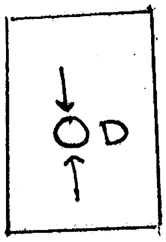
NOTE: MANY DIFFRACTION GRATINGS ARE GIVEN IN TERMS OF LINES/m (SLITS/m). $d = $ m/SLIT

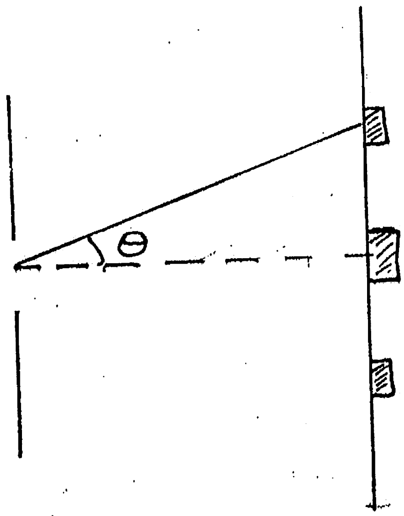
$$\Rightarrow \frac{\text{LINES}}{m} = \frac{1}{d}$$

CIRCULAR SLITS - GET CIRCULAR DIFFRACTION Pattern



CENTER BRIGHT SPOT IS CALLED THE AIRY DISK

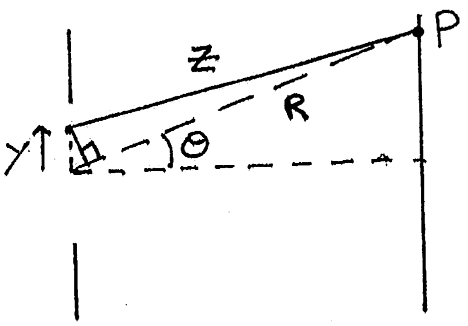




BRIGHT RINGS: $\sin\theta = \frac{1.63\lambda}{D}, \frac{2.68\lambda}{D}, \frac{3.70\lambda}{D}, \ldots$

DARK RINGS: $\sin\theta = \frac{1.22\lambda}{D}, \frac{2.23\lambda}{D}, \frac{3.24\lambda}{D}, \ldots$

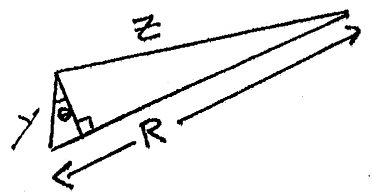
APPENDIX: THE ELECTRIC FIELD E_p OF SINGLE SLIT.



THE ADDITION TO THE FIELD AT POINT P FROM THE INFINITESIMAL SOURCE A DISTANCE y ABOVE THE CENTER OF THE SLIT IS

$$dE_p = E_0 \cos(kz - \omega t)$$

LET'S EXPAND THINGS:



$$z^2 = (R - y\sin\theta)^2 + (y\cos\theta)^2$$
$$= R^2 - 2yR\sin\theta + y^2\sin^2\theta + y^2\cos^2\theta$$
$$= R^2 - 2yR\sin\theta + y^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{1}$$
$$= R^2\left(1 - \frac{2y\sin\theta}{R} + \frac{y^2}{R^2}\right)$$

IN THE FAR FIELD, $R \gg y \Rightarrow y^2/R^2 \approx 0 \Rightarrow z^2 = R^2\left(1 - \frac{2y\sin\theta}{R}\right)$

$\Rightarrow z = R\left(1 - \frac{2y\sin\theta}{R}\right)^{1/2}$. TAKE SERIES EXPANSION: $(1+x)^{1/2} = 1 + \frac{1}{2}x + \ldots$

$\Rightarrow z = R\left(1 + \frac{1}{2}\left(\frac{-2y\sin\theta}{R}\right) + \ldots\right) = R - \frac{Ry\sin\theta}{R} + \ldots = R - y\sin\theta$

(WE DROP HIGHER TERMS BECAUSE $y \ll R$)

$$\Rightarrow dE_p = E_0\cos(kR - ky\sin\theta - \omega t) = E_0\cos(kR - \omega t - ky\sin\theta)$$

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\Rightarrow dE_p = E_0\left[\cos(kR - \omega t)\cos(ky\sin\theta) + \sin(kR - \omega t)\sin(ky\sin\theta)\right]$$

TO FIND E_p, WE INTEGRATE OVER ALL SOURCES, i.e., $-a/2 \le y \le a/2$

$$E_p = \int_{-a/2}^{a/2} dE_p = E_0\left[\cos(kR - \omega t)\frac{1}{k\sin\theta}\left(\sin\left(\frac{k\sin\theta\, a}{2}\right) - \sin\left(-\frac{k\sin\theta\, a}{2}\right)\right) + \sin(kR - \omega t)\frac{1}{k\sin\theta}\left(\cos\left(\frac{k\sin\theta\, a}{2}\right) - \cos\left(-\frac{k\sin\theta\, a}{2}\right)\right)\right]$$

$$= E_0\cos(kR - \omega t)\frac{1}{k\sin\theta}\, 2\sin\left(\frac{ka\sin\theta}{2}\right) + 0 \qquad (\cos(-\theta) = \cos\theta,\ \sin(-\theta) = -\sin\theta)$$