Phys 262: EM Waves, Chapter 32

Any solution to the wave equation and Maxwell's equations are electromagnetic waves. An example is plane waves

$$\vec{E} = \hat{\imath}\, E_0 \cos(kz - \omega t), \quad \vec{B} = \hat{\jmath}\, B_0 \cos(kz - \omega t). \qquad E = Bc$$

Any wave is characterized by its wavelength, $\boxed{\lambda = \frac{2\pi}{k}}$ and frequency $\boxed{f = \frac{\omega}{2\pi}}$ with $\boxed{\lambda f = c}$

All values of $f > 0$ are possible. (This sets $\lambda = c/f$). Visible light corresponds to a very narrow range of frequencies.
The other frequencies have different names but are still light. The whole range is called the electromagnetic spectrum.

Frequency (Hz)	Name
$10^6 - 10^9$	Radio waves
$10^9 - 10^{12}$	Microwaves
$10^{12} - 10^{14}$	Infrared
$4.29 \times 10^{14} - 7.5 \times 10^{14}$	Visible light
$7.5 \times 10^{14} - 10^{17}$	Ultraviolet
$10^{17} - 10^{20}$	X-rays
Above 10^{20}	Gamma rays

EM WAVES INSIDE OF MATTER

INSIDE A CONDUCTOR $\vec{E}$ MUST BE ZERO $\Rightarrow$ EM WAVES CANNOT PROPAGATE INSIDE A CONDUCTOR

INSIDE AN INSULATOR (ALSO CALLED A DIELECTRIC) ELECTRIC AND MAGNETIC FIELDS ARE REDUCED. FOR AN IDEALIZED MATERIAL CALLED A LINEAR, ISOTROPIC, HOMOGENEOUS (L.i.H.) MEDIUM, WE CAN REPLACE ϵ_0 WITH ϵ AND μ_0 WITH μ.

LINEAR – RESPONSE OF MEDIUM DEPENDS ON E AND B AND NOT E^2, B^2, etc.

ISOTROPIC – SAME RESPONSE FOR ANY DIRECTION OF $\vec{E}$ OR $\vec{B}$

HOMOGENEOUS – RESPONSE IS INDEPENDENT OF POSITION WITHIN MATERIAL

ϵ = PERMITIVITY. WE OFTEN WRITE $\epsilon = \tilde{K}\epsilon_0$. $\tilde{K}$ = DIELECTRIC CONSTANT

μ = PERMEABILITY. $\mu = \tilde{K}_M \mu_0$. $\tilde{K}_M$ = RELATIVE PERMEABILITY.

NOTE: FOR MOST MATERIALS, $\tilde{K}_M = 1$

IN VACUUM: $\nabla^2 \vec{E} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \vec{E}}{\partial t^2}$

IN A L.i.H. DIELECTRIC: $\nabla^2 \vec{E} = \frac{1}{\mu \epsilon} \frac{\partial^2 \vec{E}}{\partial t^2}$

$\Rightarrow$ THE SPEED OF LIGHT IN A L.i.H. DIELECTRIC, $V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\tilde{K}\tilde{K}_M}} \frac{1}{\sqrt{\mu_0\epsilon_0}}$

$$\Rightarrow \boxed{V = \frac{c}{\sqrt{\tilde{K}\tilde{K}_M}}}$$

$\vec{E}$ AND $\vec{B}$ ARE ALWAYS REDUCED IN A DIELECTRIC $\Rightarrow$ $\tilde{K}$ AND $\tilde{K}_M > 1$

$\Rightarrow$ THE SPEED OF LIGHT IN A MATERIAL IS ALWAYS LESS THAN IN A VACUUM, i.e., $\boxed{V < C}$

Index of Refraction

$$\boxed{n = \frac{c}{v}} \Rightarrow \boxed{n = \sqrt{\kappa \kappa_M} \approx \sqrt{\kappa}}$$

Since $\kappa_M \approx 1$ often

Example: The index of refraction for glass is 1.4 (technically for fused quartz). How much longer does it take light to propagate through glass than air?

For air, $\kappa = 1.00059$ and $\kappa_M = 1 \Rightarrow v \approx c$

In glass, $v = \frac{c}{n} = \frac{c}{1.4}$

v and c are constant $\Rightarrow d = vt$

For glass, $d = vt_g$ For air, $d = t_{AIR}$

$$\Rightarrow \frac{d}{d} = \frac{vt_g}{ct_{AIR}} \Rightarrow 1 = \frac{vt_g}{ct_{AIR}} \Rightarrow \frac{t_g}{t_{AIR}} = \frac{c}{v} = \frac{c}{c/1.4} = 1.4$$

Energy of EM Waves

– Waves of any type are the propagation of energy.

Energy Density – Energy per volume is easy to find for electric and magnetic fields. EM waves have both!

$$u = \underbrace{\frac{1}{2}\epsilon_0 E^2}_{\text{energy stored in } E} + \underbrace{\frac{1}{2}\frac{1}{\mu_0}B^2}_{\text{stored in } B}.$$

For an EM wave $E = Bc \Rightarrow B = E/c \Rightarrow$

$$B = \sqrt{\mu_0 \epsilon_0}\, E$$

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}\left(\sqrt{\mu_0\epsilon_0}\, E\right)^2 = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2 \Rightarrow \boxed{u = \epsilon_0 E^2}$$

Remember $E = E(x,y,z,t) \Rightarrow u = u(x,y,z,t)$. As wave propagates u changes.

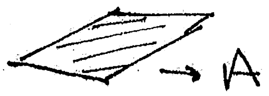
For light, a more useful quantity than $u = \frac{\text{Energy}}{\text{Volume}}$ is $S = \frac{\text{Energy}}{\text{Time} \times (\text{Cross-Sectional Area})}$

$u = \frac{dU}{dV}$, $S = \frac{1}{A}\frac{dU}{dt}$. U = Energy, u = Energy density

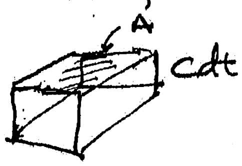
$= \frac{\text{Power}}{\text{Area}}$

Unit: $\frac{\text{Watt}}{m^2}$

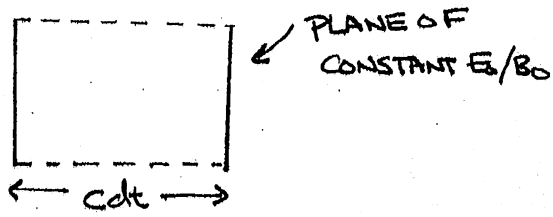
FOR A PLANE WAVE: THE SURFACES OF CONSTANT E_0 AND B_0 MAKE A PLANE. LET'S ASSUME THIS PLANE HAS AN AREA A



IN A TIME dt, THIS PLANE PROPAGATES A DISTANCE cdt



FROM THE SIDE



$$\Rightarrow dV = A\,c\,dt$$

$$\Rightarrow \frac{dU}{dV} = \frac{dU}{A\,c\,dt} \Rightarrow u = \frac{1}{c}\left[\frac{1}{A}\frac{dU}{dt}\right] = \frac{1}{c}S$$

$$\Rightarrow S = uc = \epsilon_0 E^2 c.$$

WHICH CAN BE RE-WRITTEN IN MANY DIFFERENT WAYS.

$$S = \epsilon_0 E \underbrace{(Bc)}_{E=Bc} c = \epsilon_0 c^2 EB = \frac{\epsilon_0}{\underbrace{\epsilon_0 \mu_0}_{c=\frac{1}{\sqrt{\mu_0 \epsilon_0}}}} EB \Rightarrow \boxed{S = \frac{1}{\mu_0} EB}$$

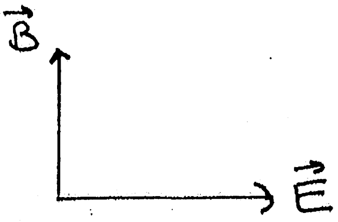
JOHN POYNTING POINTED OUT THAT THE VECTOR $\frac{1}{\mu_0}\vec{E}\times\vec{B}$ GIVES MAGNITUDE AND DIRECTION OF ENERGY FLOW

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$

POYNTING VECTOR

<u>Example</u>: FIND THE POYNTING VECTOR FOR A PLANE WAVE WHICH PROPAGATES IN THE +Z DIRECTION.

$$\vec{E} = \hat{\imath}\, E_0 \cos(kz - \omega t), \quad \vec{B} = \hat{\jmath}\, B_0 \cos(kz - \omega t)$$



FROM RHR, $\vec{E}\times\vec{B}$ POINTS OUT OF PAGE $\Rightarrow +\hat{z}$

$$\Rightarrow \vec{S} = \hat{k}\left(\frac{E_0 B_0}{\mu_0}\right)\cos^2(kz-\omega t)$$

NOTICE S IS NOT CONSTANT.

RELATED TO $\vec{S}$ IS THE INTENSITY (I).

$$\boxed{I = S_{AV}}$$

FOR A PLANE WAVE : THE AVERAGE VALUE OF $\cos^2(kz-\omega t) = \frac{1}{2}$

$$\Rightarrow \boxed{I = \left(\frac{1}{2\mu_0}\right)E_0 B_0}$$

WHICH CAN BE RE-WRITTEN MANY WAYS USING $E_0 = B_0 c$ AND $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

RADIATION PRESSURE – LIGHT CARRIES "MOMENTUM" i.e., THE NET RESULT OF THE ENERGY DEPOSITED BY LIGHT IS TO CAUSE MOTION (AS IF IT STRIKES SOMETHING WITH A MOMENTUM).

FOR A PARTICLE : $E = \frac{1}{2M}p^2 \Rightarrow \frac{dE}{dp} = \frac{1}{M}(p) = \frac{1}{M}(MV) = V \Rightarrow dE = V\,dp$

FOR LIGHT $dE = dU$, $V = c \Rightarrow dU = c\,dp \Rightarrow \frac{dU}{dV} = c\frac{dp}{dV}$ (V = VOLUME)

$$\Rightarrow \frac{dp}{dV} = \frac{u}{c} = \frac{S/c}{c} = S/c^2$$

LIKE LAST TIME $dV = Ac\,dt \Rightarrow \frac{dp}{Ac\,dt} = S/c^2 \Rightarrow \frac{dp}{dt} = \frac{SA}{c}$

$\frac{dp}{dt} = F$ (FORCE) FROM NEWTON'S 2ND LAW

$$\Rightarrow F = \frac{SA}{c} \Rightarrow F/A = S/c \,. \quad F/A = P \text{ (PRESSURE)}$$

THE AVERAGE PRESSURE IS CALLED THE RADIATION PRESSURE, P_{RAD}.

WHEN LIGHT BOUNCES OFF THE SURFACE, THE FORCE IS DOUBLED.

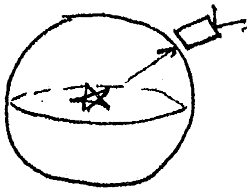
$$P_{RAD} = 2P_{AV} = 2\left(S/c\right)_{AV} = 2/c\, S_{AV} \Rightarrow \boxed{P_{RAD} = \frac{2I}{c}}$$

IF LIGHT IS TOTALLY ABSORBED : $\boxed{P_{RAD} = I/c}$

EXAMPLE: 32.15 p. 12.43

A SPACE PROBE $r = 2\times10^{10}$m AWAY FROM A STAR MEASURES $I = 5\times10^{3}$ W/m^2.
WHAT IS THE TOTAL POWER OF THIS STAR?

THE STAR RADIATES IN ALL DIRECTIONS, THE LIGHT TRAVELS IN ALL DIRECTIONS WITH THE SAME SPEED $\Rightarrow$ ENERGY IS SPREAD OUT OVER A SPHERE OF RADIUS 2×10^{10}m.



$I = S_{AV}$. $\quad S = \frac{\text{POWER}}{A} \Rightarrow \text{POWER} = S \times A$

$(\text{POWER})_{AV} = IA$. $\quad$ SPHERE $\Rightarrow A = 4\pi r^2$

$$\Rightarrow (\text{POWER})_{AV} = (5\times10^{3}\,\text{W/m}^2)(4\pi)(2\times10^{10}\,\text{m})^2 = 2.5\times10^{25}\,\text{WATT}$$