PHYS 262 – Maxwell's Equations, Chapter 32

Optics – Study of Light

James Clerk Maxwell (1831–1879) was the first person to show that light is mutually inducting electric and magnetic fields.

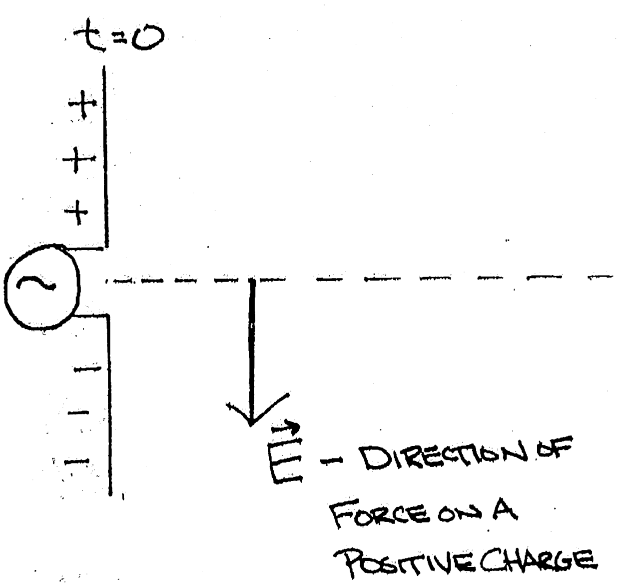
Mutual induction ⇒ Both magnetic and electric induction

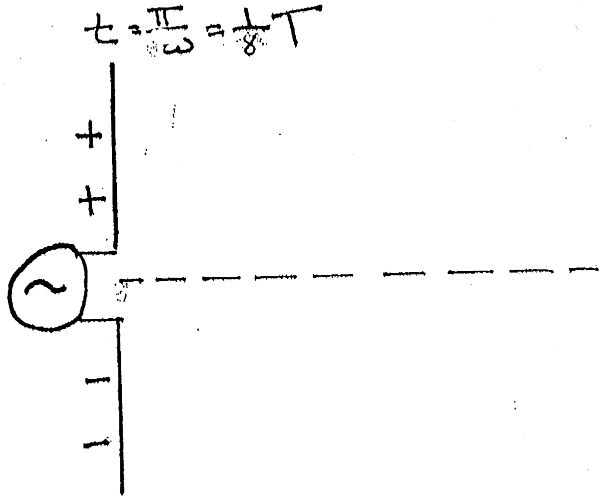
Magnetic Induction – A changing (with time) magnetic field creates an electric field.

Electric Induction – A changing electric field creates a magnetic field.

Electromagnetic Wave – Travelling wave in which electric and magnetic fields work together, i.e., $\vec{E}$ induces $\vec{B}$, which in turn induces $\vec{E}$, which induces $\vec{B}$ . . . , to propagate the wave outwards.

Antenna – Piece of wire connected to an AC generator. The generator puts various amounts of charge on the wire.



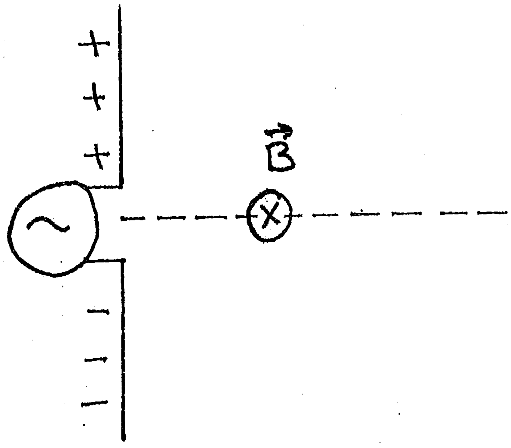


The amount of charge has changed ⇒ A current ⇒ A magnetic field.

AC generator puts a sinusoidal amount of charge on antenna

$q = Q_0 \cos(\omega t)$, $\underset{\hookrightarrow \text{current}}{i} = \frac{dq}{dt} = -Q_0 \omega \sin(\omega t) = I_0 \sin(\omega t) \Rightarrow q$ and i, $90°$ out of phase $\Rightarrow$ when charges "moving" ↓, current still ↑. So right-hand-rule $\Rightarrow \vec{B} = \otimes$

$t=0$

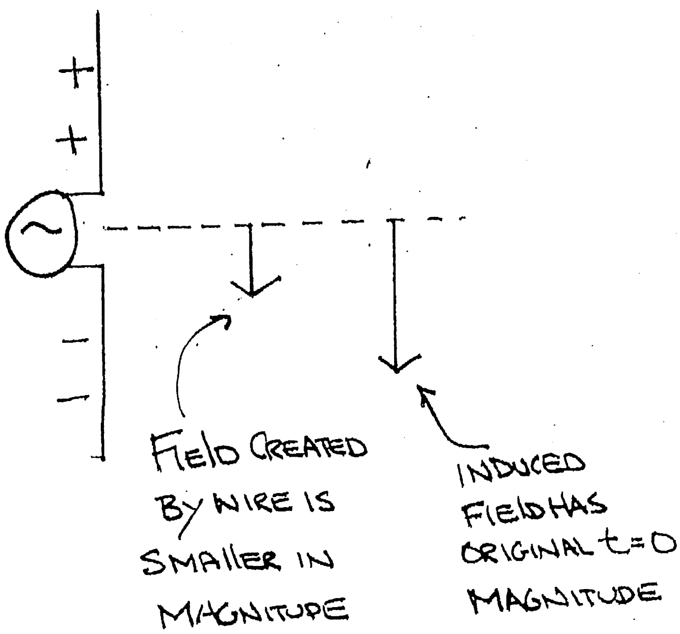


At $t=0$, we have both a $\vec{B}$ and an $\vec{E}$ field.

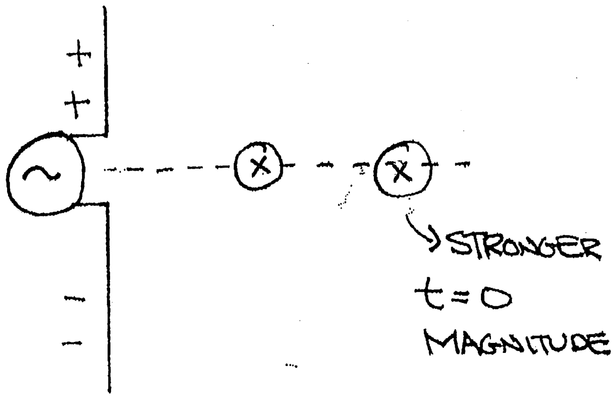
Instantaneously, the amount of charge changes. So $\vec{E}$ also changes (we here have E decreasing) $\Rightarrow$ a magnetic field is induced. <u>This</u> induced magnetic field is farther away from the wire!

At the same time, the current in the wire is also changing $\Rightarrow$ the original $\vec{B}$ changes too $\Rightarrow$ an electric field is induced. This induced electric field is farther from the wire.

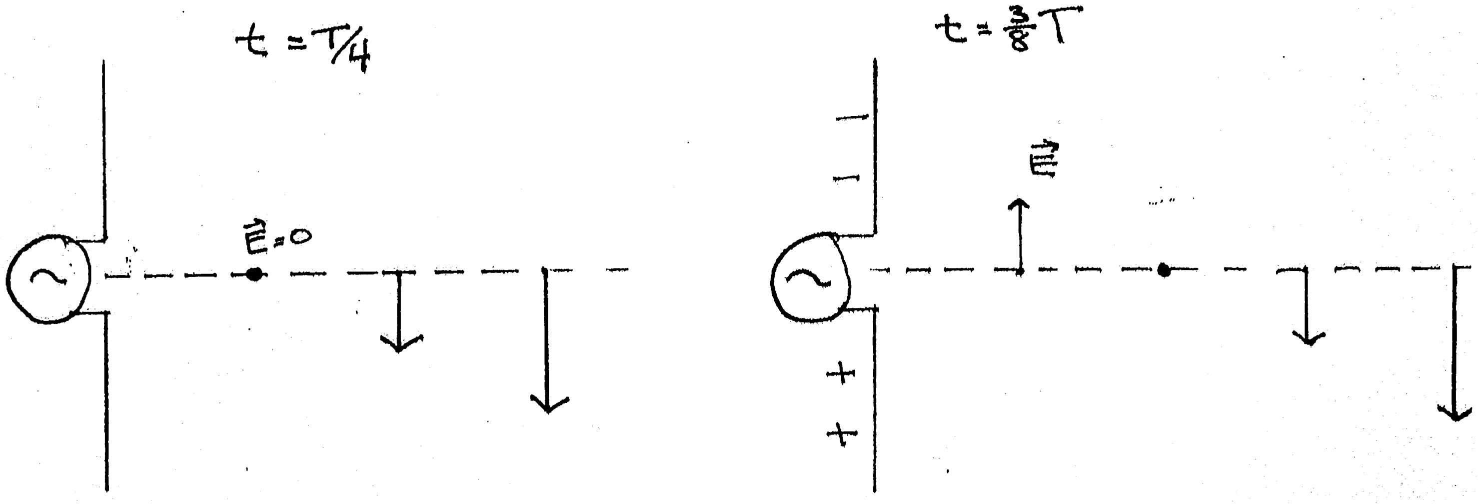
At $t = \frac{1}{8} T$, the electric field looks like:



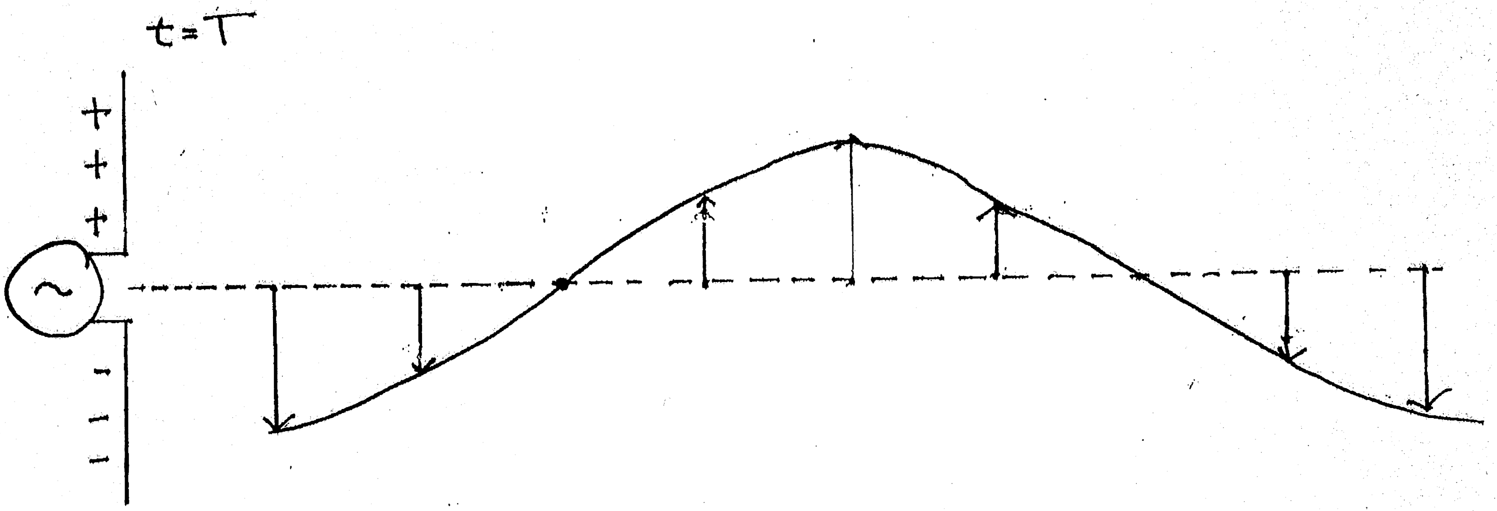
Magnetic field:



At different times, the electric field looks like:

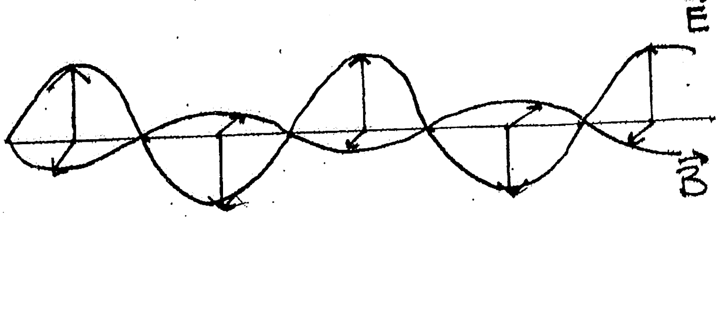


After a complete cycle, the electric field looks like



The magnetic field oscillates into and out of the page!

Together they make light!



TO SHOW ALL THIS IS TRUE MATHEMATICALLY (AS MAXWELL DID), WE USE MAXWELL'S EQUATIONS.

MAXWELL'S EQUATIONS ARE A SUMMARY OF EVERYTHING WE KNOW ABOUT ELECTROMAGNETISM.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \qquad \text{GAUSS'S LAW}$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \qquad \text{GAUSS'S LAW FOR MAGNETISM}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_c + \epsilon_0 \frac{d\Phi_E}{dt}\right) \qquad \text{AMPERE'S LAW}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{FARADAY'S LAW}$$

Φ = FLUX. $\Phi_E = \int \vec{E}\cdot d\vec{A}$, $\Phi_B = \int \vec{B}\cdot d\vec{A}$ → INTEGRAL OVER PART OF A SURFACE

$\oint \vec{E}\cdot d\vec{A}$ → NET (OR TOTAL) FLUX. $\oint$ → INTEGRAL OVER AN ENTIRE CLOSED SURFACE (ONE THAT COMES BACK TO WHERE IT STARTED).

GAUSS'S LAW – THE NET ELECTRIC FLUX THROUGH ANY CLOSED SURFACE MUST EQUAL THE TOTAL CHARGE ENCLOSED IN THAT SURFACE DIVIDED BY ϵ_0.

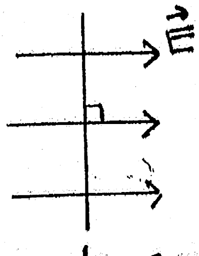
ϵ_0 = PERMITIVITY OF FREE SPACE = $8.85\times10^{-12}\ C^2/Nm^2$

FLUX MEASURES THE NUMBER OF FIELD LINES PASSING THROUGH A SURFACE.

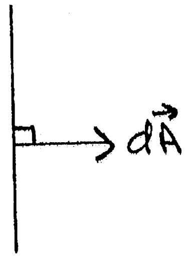
$d\vec{A}$ – SURFACE AREA ELEMENT. THE DIRECTION OF $d\vec{A}$ IS NORMAL (90°) TO THE SURFACE.

EXAMPLE CALCULATE THE FLUX THROUGH A RECTANGULAR SURFACE OF AREA $2.5\,m^2$ CREATED BY A UNIFORM ELECTRIC FIELD OF MAGNITUDE 250N/C FOR THE FOLLOWING ARRANGEMENT.

$\vec{E}$ STRIKES THE SURFACE PERPENDICULARLY:



SIDE VIEW OF SURFACE



$\Rightarrow$ ANGLE BETWEEN $\vec{E}$ AND $d\vec{A}$ IS $0°$

$$\Phi_E = \int \vec{E}\cdot d\vec{A} = \int E\,dA \cos 0° = \int E\,dA = E\int dA = EA \Rightarrow \Phi_E = (250\,N/C)(2.5\,m^2) = 625\,Nm^2/C$$

UNIFORM

IF WE SET $d\vec{A}$ TO BE IN THE Z-DIRECTION, THEN $dA = dx\,dy$, i.e., $d\vec{A} = dx\,dy\,\hat{k}$

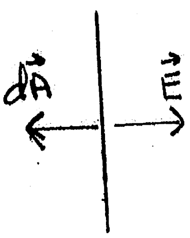
IN GENERAL, $d\vec{A} = dy\,dz\,\hat{i} + dx\,dz\,\hat{j} + dx\,dy\,\hat{k}$

- FIND THE NET FLUX THROUGH THE SURFACE.

WE NEED TO INCLUDE THE LEFT-HAND SIDE AS WELL AS THE RIGHT-HAND SIDE.

$$\oint \vec{E}\cdot d\vec{A} = \int_{\text{RIGHT}} \vec{E}\,d\vec{A} + \int_{\text{LEFT}} \vec{E}\,d\vec{A} = 625\,Nm^2/C + \int_{\text{LEFT}} \vec{E}\,d\vec{A}$$

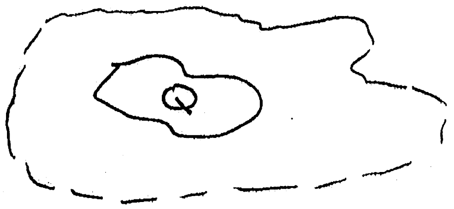
FOR THE LEFT-HAND $d\vec{A}$ IS IN THE $-\hat{k}$ DIRECTION:



$$\int_{\text{LEFT}} \vec{E}\cdot d\vec{A} = \int E\,dA \cos 180° = -\int E\,dA = -EA = -625\,Nm^2/C$$

$$\oint \vec{E}\cdot d\vec{A} = 0$$

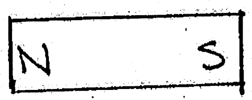
GAUSS'S LAW



$\leftarrow \oint \vec{E} \cdot d\vec{A}$ MUST BE Q/ϵ_0

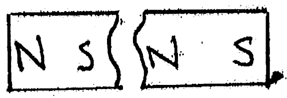
GAUSS'S LAW FOR MAGNETISM : $\oint \vec{B} \cdot d\vec{A} = 0 \Rightarrow$ THE NET MAGNETIC FLUX THROUGH A CLOSED SURFACE MUST BE ZERO.

THERE ARE NO MAGNETIC MONOPOLES!



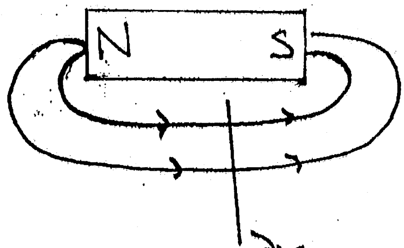
N = NORTH POLE
S = SOUTH POLE

IF I BREAK IT!



YOU NEVER FIND A NORTH W/OUT A SOUTH

MAGNETIC FIELD LINES!



SURFACE HAS EQUAL # OF LINES ENTERING AND LEAVING SO $\oint \vec{B} d\vec{A} = 0$

NOTE: $\Phi_B = \int \vec{B} \cdot d\vec{A}$ IS NOT NECESSARILY ZERO BECAUSE IT IS THE FLUX THROUGH PART OF A SURFACE.

FARADAY'S LAW : $\frac{d\Phi_B}{dt} = -\oint \vec{E} \cdot d\vec{\ell} \rightarrow$ A CHANGING MAGNETIC FLUX INDUCES AN ELECTRIC FIELD

$d\vec{\ell}$ = LINE ELEMENT $\Rightarrow$ WE INTEGRATE OVER A LINE.

$$d\vec{\ell} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$$

FOR A LOOP OF WIRE : $\oint \vec{E} \cdot d\vec{l} = \mathcal{E}_{IND}$ (THE INDUCED EMF)

$$\vec{F} = q_0 \vec{E} \Rightarrow \vec{E} = \vec{F}/q_0 \Rightarrow \frac{\oint \vec{F} \cdot d\vec{l}}{q_0} = \frac{\text{Work}}{q_0} = \text{VOLTAGE}$$

<u>AMPERE'S LAW</u> $\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_c + \epsilon_0 \frac{d\Phi_E}{dt} \right)$

μ_0 = PERM~~ITTIVITY~~EABILITY OF FREE SPACE $= 4\pi \times 10^{-7}\ Ns^2/C^2$

i_c = CURRENT ENCLOSED. $i_c = \frac{dq}{dt}$

AMPERE'S LAW IS USUALLY WRITTEN AS $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

WHEN $\frac{d\Phi_E}{dt} = 0$

AMPERE'S LAW TELLS US THAT MAGNETIC FIELDS ARE CREATED BY CURRENT OR CHANGING ELECTRIC FLUX.