PHYS 262, HW# 7, 39.9, 39.15, 39.50, 39.53, 39.60

39.9

a) $e^- \longrightarrow 20eV$, $\lambda = ?$. The electron's rest mass energy is .511 MeV

$\Rightarrow$ 20eV small enough to ignore relativity.

$\lambda = \frac{h}{p}$. $E = K = \frac{p^2}{2M} \Rightarrow 20eV \times 1.6\times10^{-19} J/eV = \frac{p^2}{2(9.11\times10^{-31}kg)}$

$\Rightarrow p = 2.415\times10^{-24} kg\cdot m/s$. (AS A CHECK $p = MV \Rightarrow V = 2.65\times10^{6} m/s = .0088c$)

$\lambda = \frac{6.64\times10^{-34} J\cdot s}{2.415\times10^{-24} kg\cdot m/s} \Rightarrow \boxed{\lambda = 2.75\times10^{-10} m = .275 nm}$

photon

$E = \frac{hc}{\lambda} \Rightarrow 20eV = \frac{(4.15\times10^{-15} eV\cdot s)(3\times10^{8} m/s)}{\lambda}$

$\Rightarrow \boxed{\lambda = 6.225\times10^{-8} m = 62.25 nm}$

b) $\lambda = 250nm$ for electron / photon Find E.

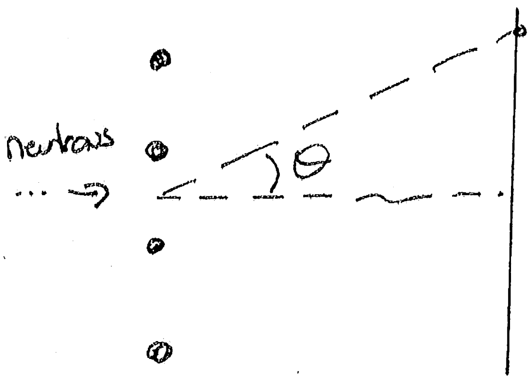
electron : $\lambda = \frac{h}{p} \Rightarrow p = \frac{6.64\times10^{-34} J\cdot s}{250\times10^{-9} m} = 2.656\times10^{-27} kg\cdot m/s$

$E = p^2/2M = \frac{(2.656\times10^{-27} kg\cdot m/s)^2}{2(9.11\times10^{-31} kg)} = 3.87\times10^{-24} J \times \frac{eV}{1.6\times10^{-19} J} \Rightarrow \boxed{E = 2.4\times10^{-5}}$

photon: $E = \frac{hc}{\lambda} = \frac{(4.15\times10^{-15} eV\cdot s)(3\times10^{8} m/s)}{250\times10^{-9} m} \Rightarrow \boxed{E = 4.98 eV}$

c) WANT $\lambda = 250nm$, so electron better choice. Less energy $\Rightarrow$ Less DAMAGE

39.15



MAXIMA OCCUR WHERE

$d \sin\theta = m\lambda$

$d = .091\,nm = .091 \times 10^{-9}\,m$

$\theta = 28.6^\circ$

$m = 1$

$\lambda = ?$

$$\lambda = \frac{d\sin\theta}{m} = \frac{(.091\times10^{-9}m)\sin 28.6^\circ}{(1)} = 4.36\times10^{-11}m$$

FOR MATTER WAVES: $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\,J\cdot s}{(4.36\times10^{-11}m)} = 1.52\times10^{-23}\,kg\cdot m/s$

NEUTRON MASS: $M_N = 1.67\times10^{-27}\,kg$

$$V = \frac{P}{M_N} = \frac{1.52\times10^{-23}\,kg\cdot m/s}{1.67\times10^{-27}\,kg} = 9114\,m/s \rightarrow \text{NON-RELATIVISTIC}$$

$$\Rightarrow K = \frac{p^2}{2M_N} = \frac{(1.52\times10^{-23}\,kg\cdot m/s)^2}{2(1.67\times10^{-27}\,kg)} = 6.92\times10^{-20}\,J \times \frac{eV}{1.6\times10^{-19}\,J}$$

$$\Rightarrow \boxed{K = .432\,eV}$$

39.50 $r = 5\times10^{-15}\,m$ for Nucleus

$\Rightarrow \Delta X = 2(5\times10^{-15}\,m) = 1\times10^{-14}\,m$

$\Delta P_{m.N} = ?$ $\qquad \Delta X \cdot \Delta P_{m.N} = \hbar$

$$\Rightarrow \Delta P_{m.N} = \frac{\hbar}{\Delta X} = \frac{1.06\times10^{-34}\,J\cdot s}{1\times10^{-14}\,m} \Rightarrow \boxed{\Delta P_{m.N} = 1.06\times10^{-20}\,kg\cdot m/s}$$

b) Estimate Kinetic Energy for $p = 1.06\times10^{-20}\,kg\cdot m/s$

$p = M_p V.$ $\qquad M_p = 1.67\times10^{-27}\,kg$ = proton mass.

$$V = \frac{1.06\times10^{-20}\,kg\cdot m/s}{1.67\times10^{-27}\,kg} = 6.35\times10^{6}\,m/s = .02c \rightarrow \text{non-relativistic.}$$

Not sure why book says to use relativity.

Let's compare:

Non-rel: $K = \frac{p^2}{2M} = \frac{(1.06\times10^{-20}\,kg\cdot m/s)^2}{2(1.67\times10^{-27}\,kg)} = 3.36\times10^{-14}\,J \times \frac{eV}{1.6\times10^{-19}\,J} \approx 2.1\times10^{5}\,eV$

$\Rightarrow \boxed{K = .21\,MeV}$

relativity: $K = E - M_0c^2$; $E^2 = p^2c^2 + m_0^2c^4 \Rightarrow$

$K = \sqrt{p^2c^2 + m_0^2c^4} - M_0c^2 = 3.36\times10^{-14}\,J$ (wow!)

c) For proton to stay in nucleus, potential energy must be more negative than kinetic $\Rightarrow \boxed{U = -3.36\times10^{-14}\,J = -.21\,MeV}$

Much bigger than $-13.6\,eV$

39.53

$$\pi^0 \to \gamma \; \gamma$$

LIFETIME, $8.4\times10^{-17}s$

So $\Delta t = 8.4\times10^{-17}s$

$\Delta E\cdot\Delta t \geq \hbar \Rightarrow \Delta E\cdot\Delta t = \hbar$ for MINIMUM UNCERTAINTY.

$\Delta E\,(8.4\times10^{-17}s) = 6.56\times10^{-16}eV\cdot s$

$\Rightarrow \Delta E = 7.8eV$

The rest MASS of AN electron IS About $.511\ MeV/c^2$

$\Rightarrow E_0 = .511\ MeV = 5.11\times10^{5}eV$

$\Rightarrow 7.8eV = \dfrac{7.8}{5.11\times10^{5}}E_0 = 1.52\times10^{-5}E_0 = \Delta M c^2$

π^0 has $M \approx 264\ E_0/c^2 \Rightarrow \dfrac{\Delta M}{M} = \dfrac{1.52\times10^{-5}E_0}{264E_0} = 5.78\times10^{-8}$

So probably too small to worry about.

To get ΔM, we have to CONVERT to J.

$\Delta E = \Delta M c^2 \Rightarrow 7.8eV \times \dfrac{1.6\times10^{-19}J}{eV} = \Delta M\,(3\times10^{8}m/s)^2$

$$\boxed{\Rightarrow \Delta M = 1.386\times10^{-35}kg = 1.4\times10^{-35}kg}$$

39.60 $E = \frac{p^2}{2M} + \frac{1}{2}KX^2$

WITH $pX \approx h$ (THOUGH $\hbar$ WOULD BE BETTER) $\Rightarrow pX \approx \hbar$

$\Rightarrow p = \frac{\hbar}{X} \Rightarrow E = \frac{1}{2M}\left(\frac{\hbar}{X}\right)^2 + \frac{1}{2}KX^2 = \frac{\hbar^2}{2M}\frac{1}{X^2} + \frac{K}{2}X^2$

MINIMUM ENERGY IS found BY SETTING DERIVATIVE to ZERO.

$\frac{dE}{dX} = \frac{\hbar^2}{2m}\left(\frac{-2}{X^3}\right) + \frac{K}{2}(2X) = 0 \Rightarrow \frac{-\hbar^2}{M}\frac{1}{X^3} + KX = 0$

$\Rightarrow \frac{\hbar^2}{M}\frac{1}{X^3} = KX \Rightarrow X^4 = \frac{\hbar^2}{MK} \Rightarrow X^2 = \frac{\hbar}{\sqrt{MK}}$

$E_{MIN} = E\left(X^2 = \frac{\hbar}{\sqrt{MK}}\right) = \frac{\hbar^2}{2M}\left(\frac{1}{\hbar/\sqrt{MK}}\right) + \frac{K}{2}\left(\frac{\hbar}{\sqrt{MK}}\right) = \frac{\hbar}{2M}\sqrt{MK} + \frac{\hbar}{2}\sqrt{\frac{K}{M}}$

$= \frac{\hbar}{2}\sqrt{\frac{K}{M}} + \frac{\hbar}{2}\sqrt{\frac{K}{M}} \Rightarrow \boxed{E_{MIN} = \hbar\sqrt{\frac{K}{M}}}$ → Nothing CAN ever truly be at Rest!

USING $\omega = \sqrt{\frac{K}{M}} \Rightarrow E_{MIN} = \hbar\omega$

$\frac{\frac{p^2}{2M}}{\frac{1}{2}KX^2} = \frac{\frac{\hbar}{2}\sqrt{\frac{K}{M}}}{\frac{\hbar}{2}\sqrt{\frac{K}{M}}} \Rightarrow \boxed{\frac{K}{U} = 1}$