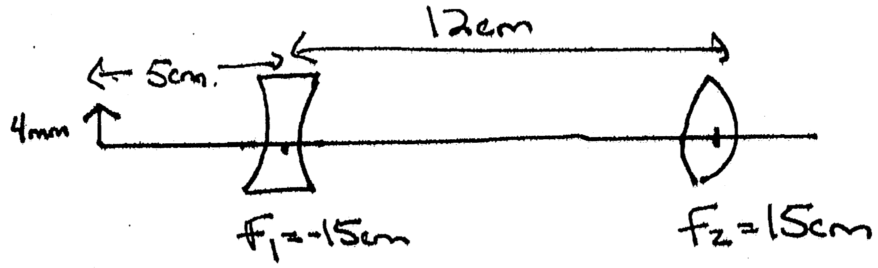
Phys 262 : HW#3 34.89, 34.105, 35.44, 35.47, 35.50, 35.52, 35.35

34.89



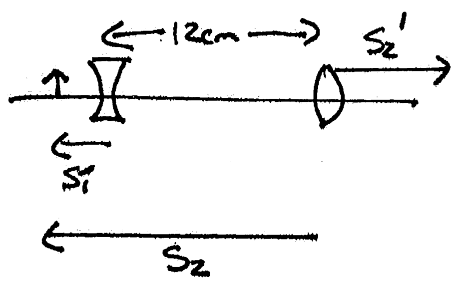
a) WHERE IS IMAGE OF FIRST LENS? $S_1' = ?$. $S_1 = 5cm$

$$\frac{1}{S_1} + \frac{1}{S_1'} = \frac{1}{f_1} \Rightarrow \frac{1}{5cm} + \frac{1}{S_1'} = \frac{1}{-15cm} \Rightarrow \frac{1}{S_1'} = \frac{-1}{15cm} - \frac{1}{5cm} = \frac{-4}{15cm}$$

$$\Rightarrow S_1' = -\frac{15}{4}cm = -3.75cm$$

b) HOW FAR FROM object IS FINAL IMAGE?

1st IMAGE IS 3.75cm to left of lens #1.



$$\Rightarrow S_2 = 3.75cm + 12cm = 15.75cm$$

$$\frac{1}{S_2} + \frac{1}{S_2'} = \frac{1}{f_2} \Rightarrow \frac{1}{15.75cm} + \frac{1}{S_2'} = \frac{1}{15cm}$$

$$\Rightarrow \frac{1}{S_2'} = \frac{1}{15cm} - \frac{1}{15.75cm} \Rightarrow S_2' = 315cm$$

$\Rightarrow$ Final image is $5cm + 12cm + 315cm = \underline{332cm}$ AWAY FROM ORIGINAL object.

c) FINAL IMAGE IS REAL SINCE $S_2' > 0$.

d) Height? $m = m_1 m_2 = \left(\frac{-S_1'}{S_1}\right)\left(\frac{-S_2'}{S_2}\right) = \left(\frac{+3.75}{5}\right)\left(\frac{-315}{15.75}\right) = (.75)(-20) = -15$

$m < 0 \Rightarrow$ \underline{INVERTED}. $\frac{y'}{y} = m \Rightarrow \boxed{y' = -15(4mm) = 60mm}$

34.105 THREE LENSES WITH $f = 40$cm ARE PLACED 52cm APART. OBJECT IS PLACED 80cm TO LEFT OF LENS #1, WHERE IS IMAGE FORMED?

THE BOOK DOESN'T SPECIFY CONVERGING OR DIVERGING. $f = +40$cm ⇒ CONVERGING

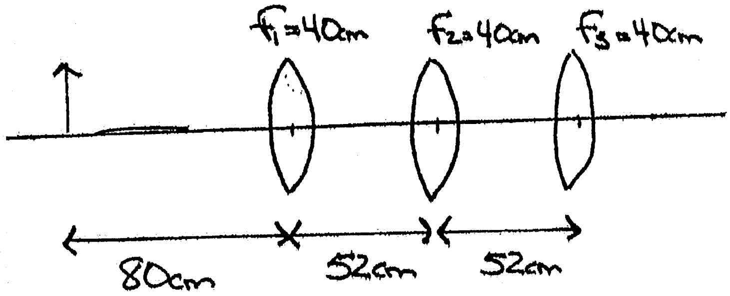


IMAGE OF LENS #1 BECOMES OBJECT FOR LENS #2

IMAGE OF LENS #2 BECOMES OBJECT FOR LENS #3

$S_1 = 80\text{cm},\ f_1 = 40\text{cm}.\quad \frac{1}{S_1} + \frac{1}{S_1'} = \frac{1}{f_1} \Rightarrow \frac{1}{S_1'} = \frac{1}{40\text{cm}} - \frac{1}{80\text{cm}} \Rightarrow S_1' = 80\text{cm}$

↳ FARTHER THAN 52cm ⇒ VIRTUAL OBJECT FOR #2

$S_2 = 52\text{cm} - S_1' = 52\text{cm} - 80\text{cm} = -28\text{cm}$

$\frac{1}{S_2} + \frac{1}{S_2'} = \frac{1}{f_2} \Rightarrow \frac{1}{S_2'} = \frac{1}{40\text{cm}} + \frac{1}{28\text{cm}} \Rightarrow S_2' = 16.5\text{cm}$

↳ LESS THAN 52cm ⇒ REAL OBJECT FOR #3

$S_3 = 52\text{cm} - S_2' = 52\text{cm} - 16.5\text{cm} = 35.5\text{cm}$

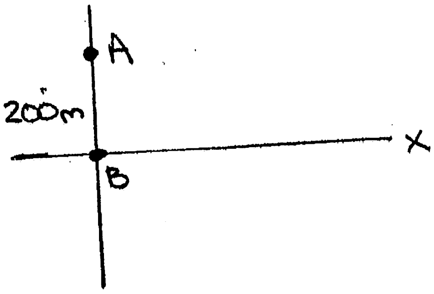
$\frac{1}{S_3} + \frac{1}{S_3'} = \frac{1}{f_3} \Rightarrow \frac{1}{S_3'} = \frac{1}{40\text{cm}} - \frac{1}{35.5\text{cm}} \Rightarrow \boxed{S_3' = -315.6\text{cm}}$

VIRTUAL IMAGE 315.6cm TO LEFT OF LENS #3

⇒ 315.6cm − 52cm − 52cm − 80 = 131.6cm TO LEFT OF ORIGINAL OBJECT

(WITHOUT ROUNDING AT EACH LENS $S_3' = -317.89$ ⇒ 133.89 = 134cm FROM ORIGINAL OBJECT → THIS IS BOOK'S ANSWER)

35.44



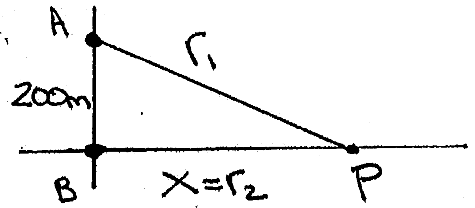
$f = 5.8\,\mathrm{MHz}$

WHERE ALONG X ARE POINTS OF DESTRUCTIVE INTERFERENCE?

DESTRUCTIVE INTERFERENCE OCCURS WHEN PATH DIFFERENCE

$$r_1 - r_2 = (m+\tfrac{1}{2})\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

AT A POINT P:



$$r_1 = \sqrt{X^2 + (200\,\mathrm{m})^2}$$

$$r_2 = X$$

$$\Rightarrow \sqrt{X^2 + (200\,\mathrm{m})^2} - X = (m+\tfrac{1}{2})\lambda \Rightarrow \sqrt{X^2 + (200\,\mathrm{m})^2} = X + (m+\tfrac{1}{2})\lambda$$

$$\Rightarrow X^2 + (200\,\mathrm{m})^2 = (X + (m+\tfrac{1}{2})\lambda)^2 = X^2 + 2X(m+\tfrac{1}{2})\lambda + [(m+\tfrac{1}{2})\lambda]^2$$

$$\Rightarrow \cancel{X^2} + (200\,\mathrm{m})^2 = \cancel{X^2} + 2X(m+\tfrac{1}{2})\lambda + [(m+\tfrac{1}{2})\lambda]^2$$

$$\Rightarrow X = \frac{(200\,\mathrm{m})^2 - [(m+\tfrac{1}{2})\lambda]^2}{2(m+\tfrac{1}{2})\lambda} = \frac{(200\,\mathrm{m})^2 - [(m+\tfrac{1}{2})\lambda]^2}{(2m+1)\lambda}$$

$$\lambda f = c \Rightarrow \lambda = \frac{3\times10^8\,\mathrm{m/s}}{5.8\times10^6\,\mathrm{Hz}} = 51.7\,\mathrm{m} \Rightarrow X = \frac{(200\,\mathrm{m})^2 - ((m+\tfrac{1}{2})51.7\,\mathrm{m})^2}{(2m+1)(51.7\,\mathrm{m})}$$

m	X
0	761m
1	219m
2	90m
3	20m
4	-30m
⋮	⋮

35.41 S_1 AND S_2 ARE OUT OF PHASE BY HALF A CYCLE $\Rightarrow 180° = \pi$ RAD

i) FIND CONDITION FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE.

ASSUME PLANE WAVES TO BEGIN:

$$\vec{E}_1 = \vec{E}_{01}\cos(kz_1 - \omega t), \quad \vec{E}_2 = \vec{E}_{02}\cos(kz_2 - \omega t + \pi)$$

ASSUME SAME POLARIZATION (ALONG X-AXIS) $\Rightarrow$

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 = \hat{\imath}\left(E_{01}\cos(kz_1 - \omega t) + E_{02}\cos(kz_2 - \omega t + \pi)\right)$$

$\Rightarrow$ CONSTRUCTIVE INTERFERENCE WHEN $kz_1 - \omega t = kz_2 - \omega t + \pi + 2\pi m$

$$\Rightarrow kz_1 = kz_2 + \pi(2m+1) \Rightarrow k(z_1 - z_2) = \pi(2m+1)$$

$$\Rightarrow \frac{2\pi}{\lambda}(z_1 - z_2) = 2\pi(m + \tfrac{1}{2}) \Rightarrow z_1 - z_2 = (m + \tfrac{1}{2})\lambda$$

DESTRUCTIVE INTERFERENCE WHEN $kz_1 - \omega t = kz_2 - \omega t + \pi + (2m+1)\pi$

$$\Rightarrow k(z_1 - z_2) = \pi(2m+1+1) = \pi\, 2\underbrace{(m+1)}_{\text{ANOTHER INTEGER}} = 2\pi n \Rightarrow (z_1 - z_2) = n\lambda = m\lambda$$

FOR GENERAL WAVES, WE REPLACE $z_1 - z_2$ WITH $r_1 - r_2$

b) PHASE DIFFERENCE OF $\phi \Rightarrow \vec{E}_p = \hat{\imath}\left(E_{01}\cos(kz_1 - \omega t) + E_{02}\cos(kz_2 - \omega t + \phi)\right)$

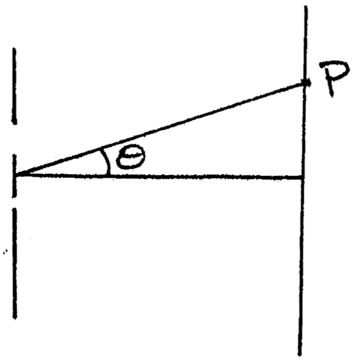
CONSTRUCTIVE INTERFERENCE WHEN $kz_1 - \omega t = kz_2 - \omega t + \phi + 2\pi m$

$$\Rightarrow k(z_1 - z_2) = \phi + 2\pi m = 2\pi\left(\frac{\phi}{2\pi} + m\right) \Rightarrow \frac{2\pi}{\lambda}(z_1 - z_2) = 2\pi\left(\frac{\phi}{2\pi} + m\right)$$

$\Rightarrow z_1 - z_2 = \lambda\left(\frac{\phi}{2\pi} + m\right)$. IN GENERAL, IT'S $\boxed{r_1 - r_2 = \lambda\left(\frac{\phi}{2\pi} + m\right)}$

DESTRUCTIVE $\Rightarrow k(z_1 - z_2) = \phi + (2m+1)\pi \Rightarrow \boxed{r_1 - r_2 = \lambda\left(\frac{\phi}{2\pi} + m + \tfrac{1}{2}\right)}$

35.50



$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

ASSUME SAME POLARIZATION (Along x)
AND we're told $E_2 = 2E_1$

$$\Rightarrow \vec{E}_p = \hat{\imath}\, E_0 \big(\cos(kz_1 - \omega t) + 2\cos(kz_2 - \omega t)\big)$$

$$\vec{E}_p = \hat{\imath}\, E_0 \big(\cos(kz_1 - \omega t) + \cos(kz_2 - \omega t) + \cos(kz_2 - \omega t)\big)$$

USE $\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$$\Rightarrow \vec{E}_p = \hat{\imath}\, E_0 \left(2\cos\left(\frac{k(z_1+z_2)}{2} - \omega t\right)\cos\left(\frac{k(z_1-z_2)}{2}\right) + \cos(kz_2 - \omega t)\right)$$

$$= \hat{\imath}\, E_0 \left(2\cos\left(\frac{k(z_1+z_2)}{2} - \omega t\right)\cos \phi/2 + \cos(kz_2 - \omega t)\right)$$

$$S = \epsilon_0 c E_p^2 = \epsilon_0 c E_0^2 \Big(4\cos^2\left(\frac{k(z_1+z_2)}{2} - \omega t\right)\cos^2\phi/2 + \cos^2(kz_2 - \omega t)$$
$$+ 4\cos\left(\frac{k(z_1+z_2)}{2} - \omega t\right)\cos(kz_2 - \omega t)\cos\phi/2\Big)$$

Now use $\cos(a-b) = \cos a\cos b + \sin a \sin b$

$$\cos\left(\frac{k(z_1+z_2)}{2} - \omega t\right)\cos(kz_2 - \omega t) = \left[\cos\left(\frac{k(z_1+z_2)}{2}\right)\cos\omega t + \sin\left(\frac{k(z_1+z_2)}{2}\right)\sin\omega t\right]$$
$$\times \left[\cos kz_2 \cos\omega t + \sin kz_2 \sin\omega t\right]$$

$$= \cos\left(\frac{k(z_1+z_2)}{2}\right)\cos kz_2 \cos^2\omega t + \sin\left(\frac{k(z_1+z_2)}{2}\right)\sin kz_2 \sin^2\omega t +$$
$$\left(\cos\frac{k(z_1+z_2)}{2}\sin kz_2 + \sin\frac{k(z_1+z_2)}{2}\cos kz_2\right)\cos\omega t \sin\omega t$$

$I = S_{AV} \rightarrow$ A TIME AVERAGE

THE AVERAGE OF $\cos^2(a - \omega t) = \sin^2(b - \omega t) = 1/2$ FOR ANY a & b

THE AVERAGE OF $\cos\omega t \sin\omega t = 0$

$$\Rightarrow I = \epsilon_0 c E_0^2 \left(4\left(\tfrac{1}{2}\right)\cos^2\phi/2 + \tfrac{1}{2} + 4\left(\cos\left(\frac{k(z_1+z_2)}{2}\right)\cos kz_2\left(\tfrac{1}{2}\right) + \sin\left(\frac{k(z_1+z_2)}{2}\right)\right.\right.$$

$$\left.\left.\times \sin kz_2\left(\tfrac{1}{2}\right)\right)\cos\phi/2\right)$$

$$= \epsilon_0 c E_0^2 \left(2\cos^2\phi/2 + \tfrac{1}{2} + 2\left(\cos\left(\frac{k(z_1+z_2)}{2}\right)\cos kz_2 + \sin\left(\frac{k(z_1+z_2)}{2}\right)\sin kz_2\right) \times \cos\phi/2\right)$$

$$\cos\left(\frac{k(z_1+z_2)}{2}\right)\cos kz_2 + \sin\left(\frac{k(z_1+z_2)}{2}\right)\sin kz_2 = \cos\left(\frac{k(z_1+z_2)}{2} - kz_2\right)$$

$$= \cos\left(\frac{kz_1}{2} - \frac{kz_2}{2}\right) = \cos\left(\frac{k(z_1-z_2)}{2}\right) = \cos\phi/2$$

$$\left[\cos(a)\cos(b) + \sin(a)\sin(b) = \cos(a-b)\right]$$

$$\Rightarrow I = \epsilon_0 c E_0^2\left(2\cos^2\phi/2 + \tfrac{1}{2} + 2\cos\phi/2\cos\phi/2\right)$$

$$= \epsilon_0 c E_0^2\left(4\cos^2\phi/2 + 1/2\right)$$

$$I = I_0 \text{ when } \phi = 0 \Rightarrow I_0 = \epsilon_0 c E_0^2 (4 + 1/2) = \epsilon_0 c E_0^2 (9/2)$$

$$\Rightarrow \epsilon_0 c E_0^2 = \tfrac{2}{9} I_0$$

$$\Rightarrow I = \tfrac{2}{9} I_0 \left(4\cos^2\phi/2 + 1/2\right) = I_0\left(\tfrac{8}{9}\cos^2\phi/2 + \tfrac{1}{9}\right)$$

ONE FINAL TRIG IDENTITY: $\cos^2 a = \tfrac{1}{2}(1 + \cos 2a)$

$$\Rightarrow I = I_0\left(\tfrac{8}{9}\cdot\tfrac{1}{2}(1+\cos\phi) + \tfrac{1}{9}\right) = I_0\left(\tfrac{4}{9} + \tfrac{4}{9}\cos\phi + \tfrac{1}{9}\right)$$

$$\Rightarrow I = I_0\left(\tfrac{5}{9} + \tfrac{4}{9}\cos\phi\right).$$ QED!

35.52 YOUNG'S DOUBLE SLIT $\Rightarrow d\sin\theta = m\lambda$ FOR CONSTRUCTIVE

$d\sin\theta = (m+\tfrac{1}{2})\lambda$ FOR DESTRUCTIVE

FOR RED LIGHT ($\lambda = 700nm$) WE GET THE $m=3$ CONSTRUCTIVE INTERFERENCE AT THE SAME ANGLE AT WHICH ANOTHER WAVELENGTH, λ_2 IS HAVING DESTRUCTIVE INTERFERENCE

$\Rightarrow d\sin\theta = 3(700nm)$ AND $d\sin\theta = (m+\tfrac{1}{2})\lambda_2$

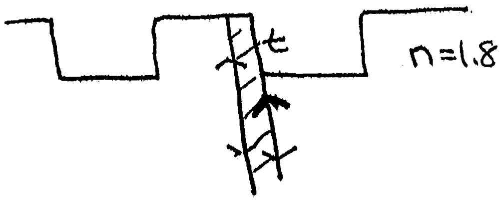
$\Rightarrow 3(700nm) = (m+\tfrac{1}{2})\lambda_2$

$$\Rightarrow \boxed{\lambda_2 = \frac{3(700nm)}{(m+\tfrac{1}{2})}}$$

IF THIS WAS λ_2'S

m	λ_2	
0	4200nm	INFRARED
1	1400nm	
2	840nm	
3	600nm	VISIBLE
4	466nm	
5	381nm	

35.35



NOTICE THAT LIGHT IS INSIDE THE PLASTIC. REFLECTING OFF SLOWER SURFACE BENEATH FASTER AIR $\Rightarrow$ NO PHASE SHIFT

$\Rightarrow$ DESTRUCTIVE INTERFERENCE WHEN $2t = (m+\frac{1}{2})\lambda$

LET $m=0 \Rightarrow 2t = \frac{1}{2}\lambda \Rightarrow t = \frac{\lambda}{4}$

$\lambda_{AIR} = 790nm$. REMEMBER HOW WAVELENGTH CHANGES WITHIN A MATERIAL $\Rightarrow n_1\lambda_1 = n_2\lambda_2$

$$\Rightarrow \lambda_{PLASTIC} = \frac{n_{AIR}\lambda_{AIR}}{n_{plastic}} = \frac{1(790nm)}{1.8}$$

$$\Rightarrow t = \frac{790nm}{4(1.8)} \Rightarrow \boxed{t = 110nm = 1.1\times10^{-7}m}$$