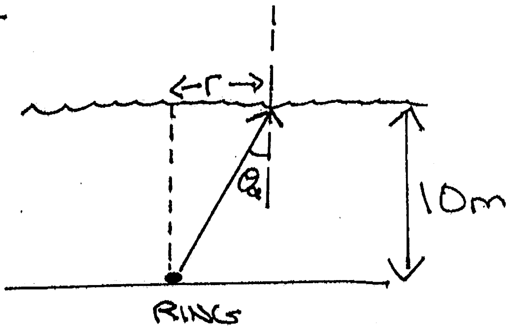
PHYS 2102 : HW#2 33.20, 33.22, 33.34, 33.57, 33.58

33.20



FIND AREA OF LARGEST CIRCLE

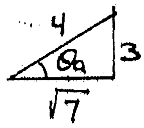
r is radius of circle of ESCAPING LIGHT. WHEN θ_a IS CRITICAL ANGLE, r IS LARGEST.

Area, $A = \pi r^2$

CRITICAL ANGLE FOR WATER TO AIR:

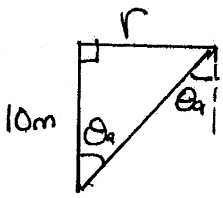
$$n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \tfrac{4}{3}\sin\theta_a = 1(\sin 90^\circ)$$

$$\Rightarrow \sin\theta_a = 3/4 \Rightarrow$$



$\cos\theta_a = \sqrt{7}/4$

$\tan\theta_a = 3/\sqrt{7}$

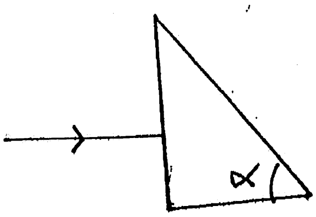


$$\tan\theta_a = \frac{r}{10m} \Rightarrow \frac{3}{\sqrt{7}} = \frac{r}{10m}$$

$$\Rightarrow r = \frac{3(10m)}{\sqrt{7}} = \frac{30}{\sqrt{7}}\,m = 11.339m$$

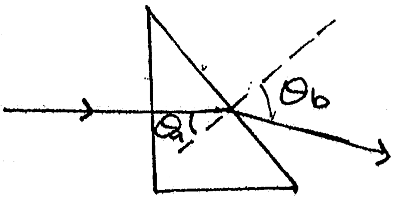
$$A = \pi r^2 = \pi\left(\frac{30}{\sqrt{7}}m\right)^2 \Rightarrow \boxed{A = 403.919m^2 = 404m^2}$$

33.22



PRISM HAS $n = 1.52$. WHAT IS LARGEST α TO HAVE TOTAL INTERNAL REFLECTION FOR AIR AND WATER.

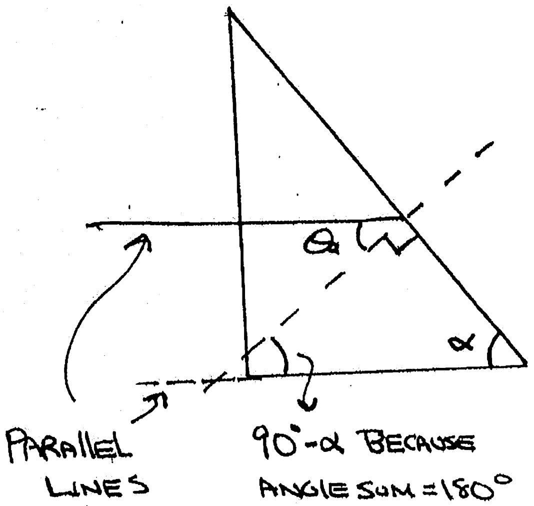
CALL THE OUTSIDE INDEX OF REFRACTION n_b.

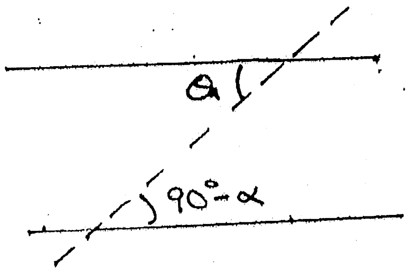


TOTAL INTERNAL REFLECTION $\Rightarrow \theta_b = 90°$

$\Rightarrow n \sin\theta_a = n_b \sin 90° \Rightarrow n \sin\theta_a = n_b$

JUST NEED TO RELATE θ_a to α





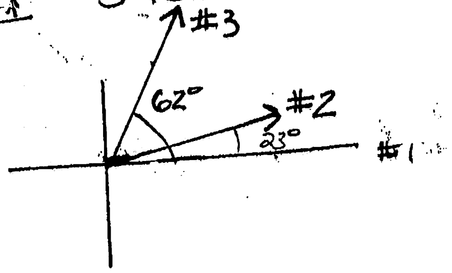
ALTERNATE INTERIOR ANGLES MUST BE EQUAL $\Rightarrow \theta_a = 90° - \alpha$

$\therefore \; n_b = n \sin\theta_a = n \sin(90° - \alpha) = n \cos\alpha \Rightarrow \cos\alpha = \frac{n_b}{n}$

FOR AIR, $n_b = 1 \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°$

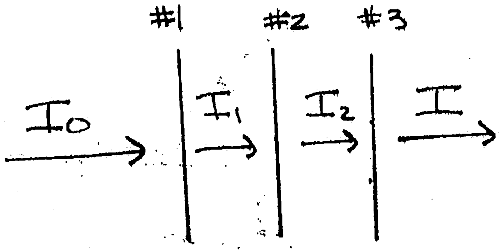
FOR WATER, $n_b = 1.333 \Rightarrow \alpha = \cos^{-1}\left(\frac{\quad}{1.}\right) = 28.7°$

33.34 3 Polarizers with axes 23° and 62° with respect to first.



Unpolarized light is incident on #1 and emerges with $I = 75\ \text{Watt/cm}^2$ after #3. What is intensity if #2 is removed.

So we have:



Output of each polarizer becomes input for the next.

Unpolarized light $\Rightarrow I_1 = \frac{1}{2} I_0$. When light leaves #1, it is polarized along its axis (which we set at 0° above).

Law of Malus $\Rightarrow I_2 = I_1 \cos^2 23°$

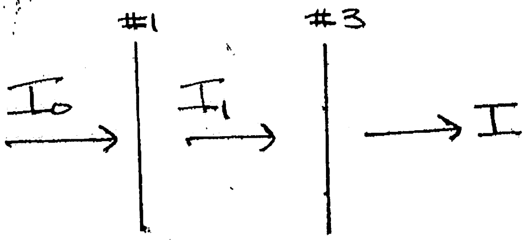
$\Rightarrow I_2 = \frac{1}{2} I_0 \cos^2 23°$

For #3, incoming light is polarized at 23° $\Rightarrow \phi = 62° - 23° = 39°$

$\Rightarrow I = I_2 \cos^2 39° \Rightarrow I = \frac{1}{2} I_0 \cos^2 23° \cos^2 39°$

$\Rightarrow 75\ \text{Watt/cm}^2 = \frac{1}{2} I_0 \cos^2 23° \cos^2 39° \Rightarrow I_0 = 293\ \text{Watt/cm}^2$

Without #2:



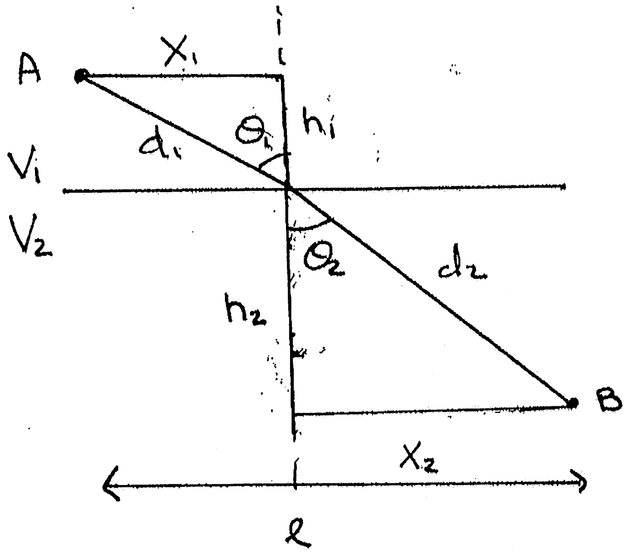
$I_1 = \frac{1}{2} I_0,\ I = I_1 \cos^2 62°$

$\Rightarrow I = \frac{1}{2} I_0 \cos^2 62°$

$\Rightarrow I = \frac{1}{2} (293\ \text{Watt/cm}^2) \cos^2 62°$

$\Rightarrow \boxed{I = 32.3\ \text{Watt/cm}^2}$ Light is dimmer for just #1 and #3!

33.57



POINTS A AND B ARE FIXED ℓ APART AND $h_1 + h_2$ HEIGHT.
$\Rightarrow \ell, h_1, h_2$ CONSTANT.

FIND TIME TO GO FROM A to B.

IN MEDIUM 1, LIGHT TRAVELS WITH CONSTANT V_1 $\Rightarrow d_1 = V_1 t_1 \Rightarrow t_1 = \frac{d_1}{V_1} = \frac{\sqrt{X_1^2 + h_1^2}}{V_1}$

IN MEDIUM 2, $d_2 = V_2 t_2 \Rightarrow t_2 = \frac{d_2}{V_2} = \frac{\sqrt{X_2^2 + h_2^2}}{V_2}$. WRITE X_2 IN TERMS OF CONSTANT ℓ.

$X_2 = \ell - X_1 \Rightarrow t_2 = \frac{\sqrt{(\ell - X_1)^2 + h_2^2}}{V_2}$. THE TOTAL TIME $t = t_1 + t_2$

$\Rightarrow t = \frac{\sqrt{X_1^2 + h_1^2}}{V_1} + \frac{\sqrt{(\ell - X_1)^2 + h_2^2}}{V_2}$. MINIMIZING t GIVES US X_1 WHICH DETERMINES ANGLES.

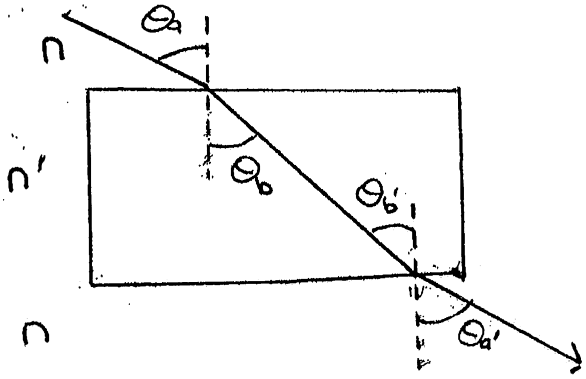
b) $\frac{dt}{dX_1} = \frac{\frac{1}{2}(X_1^2 + h_1^2)^{-1/2}(2X_1)}{V_1} + \frac{\frac{1}{2}((\ell - X_1)^2 + h_2^2)^{-1/2}\, 2(\ell - X_1)(-1)}{V_2} = \frac{X_1}{V_1(X_1^2 + h_1^2)^{1/2}} - \frac{(\ell - X_1)}{V_2((\ell - X_1)^2 + h_2^2)^{1/2}}$

$\frac{dt}{dX_1} = 0 \Rightarrow \frac{X_1}{V_1(X_1^2 + h_1^2)^{1/2}} = \frac{(\ell - X_1)}{V_2((\ell - X_1^2) + h_2^2)^{1/2}}$ $\rightarrow$ I DON'T FANCY SOLVING THIS FOR X_1!

INSTEAD NOTICE : $(X_1^2 + h_1^2)^{1/2} = d_1$, $(\ell - X_1) = X_2$, $((\ell - X_1)^2 + h_2^2)^{1/2} = (X_2^2 + h_2^2)^{1/2} = d_2$

$\Rightarrow \frac{X_1}{V_1 d_1} = \frac{X_2}{V_2 d_2} \Rightarrow \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} \Rightarrow \frac{C\sin\theta_1}{V_1} = \frac{C\sin\theta_2}{V_2} \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$

33.58

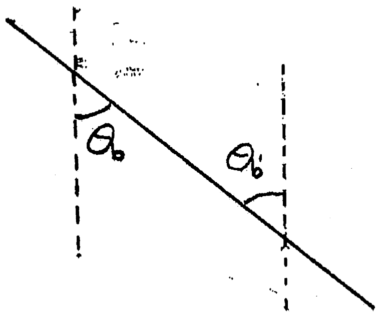


SHOW THAT $\theta_a = \theta_{a'}$

FROM SNELL'S LAW: $n \sin\theta_a = n' \sin\theta_b$ AND $n' \sin\theta_b' = n \sin\theta_a'$

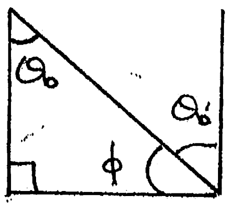
$\Rightarrow$ IF $\theta_b = \theta_b'$ THEN $\theta_a = \theta_a'$.

BECAUSE THE TOP AND BOTTOM SURFACES ARE PARALLEL, SO ARE THE TWO NORMAL LINES.



θ_b AND θ_b' ARE ALTERNATE INTERIOR ANGLES $\Rightarrow \boxed{\theta_b = \theta_b'}$.

IF YOU DON'T REMEMBER (AIA), WE CAN ALWAYS DO THIS:



$\theta_b + \phi + 90° = 180°$ (SUM OF ANGLES)

$\Rightarrow \phi = 90° - \theta_b$

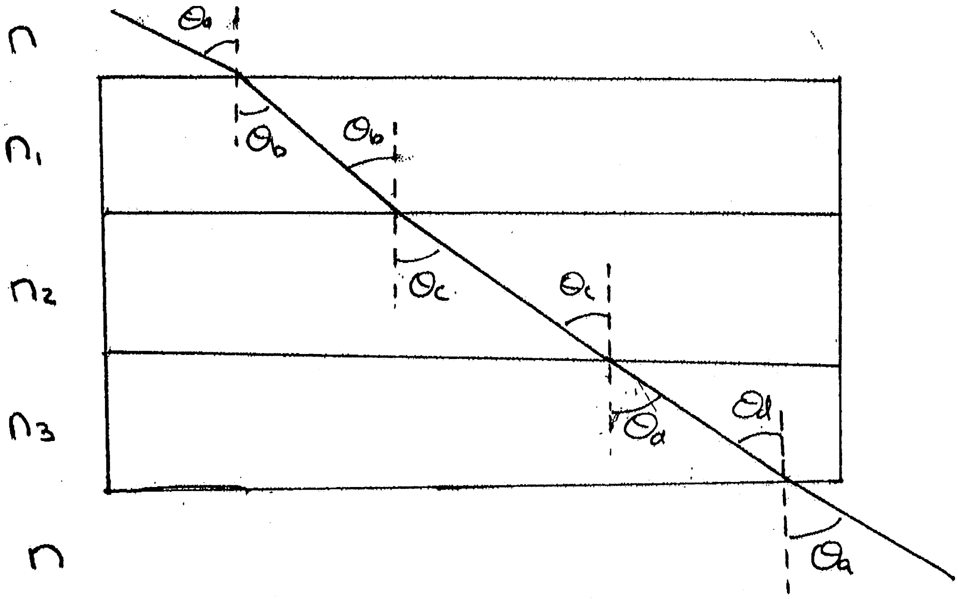
$\theta_b' + \phi = 90°$ (DEFINITION OF NORMAL)

$\Rightarrow \theta_b' = 90° - \phi = 90° - 90° + \theta_b = \theta_b$

FOR ANY NUMBER OF PARALLEL PLATES, THE SAME GEOMETRY WILL HOLD. THE TOP (θ_b) ANGLE MUST BE EQUAL TO THE BOTTOM (θ_b'). FOR PLATES WITH $n_1, n_2, n_3, \ldots$

$n \sin\theta_a = n_1 \sin\theta_b$, $n_1 \sin\theta_b = n_2 \sin\theta_c$, $n_2 \sin\theta_c = n_3 \sin\theta_d$, $n_3 \sin\theta_d = n \sin\theta_a'$

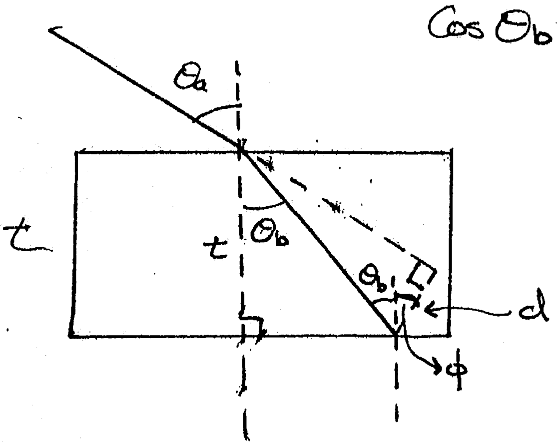
$\Rightarrow n \sin\theta_a = n \sin\theta_a' \Rightarrow \theta_a = \theta_a'$ (SEE PICTURE ON NEXT PAGE.)



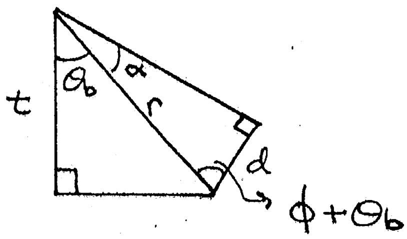
C) PROVE $d = t\,\dfrac{\sin(\theta_a - \theta_b)}{\cos\theta_b}$.

t = THICKNESS

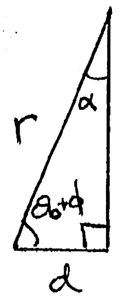
d = PERP. DISTANCE BETWEEN INGOING AND OUTGOING RAYS.



THE ATTACHED Δ'S WILL HELP!



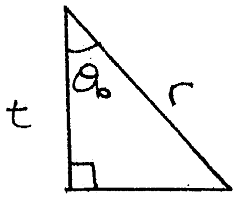
LABEL NEW ANGLES. CALL THEIR COMMON SIDE r.



$\cos(\theta_b+\phi) = \dfrac{d}{r} \Rightarrow d = r\cos(\theta_b+\phi)$. WE ALSO WILL NEED THAT

$\alpha + (\theta_b+\phi) + 90^\circ = 180^\circ$

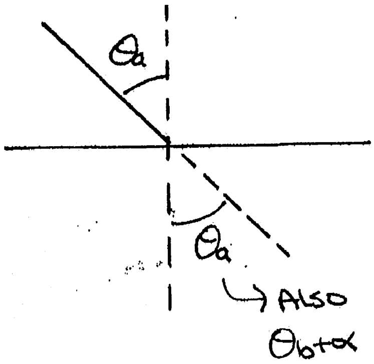
$\Rightarrow \alpha + \theta_b + \phi = 90^\circ$



$\cos\theta_b = \dfrac{t}{r} \Rightarrow r = \dfrac{t}{\cos\theta_b} \Rightarrow d = \dfrac{t}{\cos\theta_b}\cos(\theta_b+\phi)$

$$d = t\,\frac{\cos(\theta_b + \phi)}{\cos\theta_b}.$$

TO GET RID OF ϕ, WE REMEMBER THAT OPPOSITE ANGLES ARE EQUAL.



LOOKING BACK AT OUR ATTACHED Δ'S, WE SEE THAT $\theta_a = \theta_b + \alpha$. $\Rightarrow \alpha = \theta_a - \theta_b$

NOW USE $\alpha + \theta_b + \phi = 90°$

$\Rightarrow \theta_a - \theta_b + \theta_b + \phi = 90°$

$\Rightarrow \theta_a + \phi = 90° \Rightarrow \phi = 90° - \theta_a$

$$\Rightarrow d = t\,\frac{\cos(\theta_b - \theta_a + 90°)}{\cos\theta_b}.$$

$\cos(\beta + 90°) = -\sin\beta$

$$\Rightarrow d = t\left(\frac{-\sin(\theta_b - \theta_a)}{\cos\theta_b}\right) \Rightarrow d = t\,\frac{\sin(\theta_a - \theta_b)}{\cos\theta_b}.$$

$(\sin(-\beta) = -\sin\beta)$

QED!!!

1) $\theta_a = 66°$, $t = 2.4\text{cm}$, $n' = 1.8$, $n = 1$, $d = ?$

FIND θ_b USING SNELL'S LAW: $n \sin\theta_a = n' \sin\theta_b \Rightarrow 1 \sin 66° = 1.8 \sin\theta_b$

$\Rightarrow \theta_b = 30.5°$

$$\therefore d = (2.4\text{cm})\,\frac{\sin(66° - 30.5°)}{\cos(30.5°)} \Rightarrow \boxed{d = 1.62\text{cm}}$$