PHYS 262: HW#1 32.6, 32.13, 32.19, 32.54, AND #1 (BELOW)

#1 SHOW PLANE WAVE $\vec{E}$ AND $\vec{B}$ OBEY WAVE EQUATION AND MAXWELL'S

$\vec{E} = \hat{i} E_0 \cos(Kz - \omega t)$ $\qquad K = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$

$\vec{B} = \hat{j} B_0 \cos(Kz - \omega t)$ $\qquad \lambda f = C$, $E_0 = B_0 C$

COMPONENTS: $E_x = E_0 \cos(Kz - \omega t)$, $E_y = 0$, $E_z = 0$

$B_x = 0$, $B_y = B_0 \cos(Kz - \omega t)$, $B_z = 0$

WAVE EQUATION: $\nabla^2 \vec{E} = \frac{1}{C^2} \frac{\partial^2 \vec{E}}{\partial t^2}$ AND $\nabla^2 \vec{B} = \frac{1}{C^2} \frac{\partial^2 \vec{B}}{\partial t^2}$

$E_y = E_z = 0$ MEANS WAVE EQN FOR $\vec{E}$ IS $\nabla^2 E_x = \frac{1}{C^2} \frac{\partial^2 E_x}{\partial t^2}$

$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}$. $\quad \frac{\partial E_x}{\partial x} = \frac{\partial E_x}{\partial y} = 0$. $\quad \frac{\partial E_x}{\partial z} = -E_0 K \sin(Kz - \omega t)$

$\frac{\partial^2 E_x}{\partial z^2} = -E_0 K^2 \cos(Kz - \omega t)$. $\quad \frac{\partial E_x}{\partial t} = -E_0(-\omega)\sin(Kz - \omega t) = E_0 \omega \sin(Kz$

$\Rightarrow \frac{\partial^2 E_x}{\partial t^2} = E_0 \omega(-\omega)\cos(Kz - \omega t) = -E_0 \omega^2 \cos(Kz - \omega t)$.

$E_0 \omega^2 = E_0 (2\pi)^2 f^2$. $\Rightarrow \frac{1}{C^2} \frac{\partial^2 E_x}{\partial t^2} = -E_0 (2\pi)^2 \frac{f^2}{C^2} \cos(Kz - \omega t)$

$= -E_0 (2\pi)^2 \left(\frac{f}{C}\right)^2 \cos(Kz - \omega t) = -E_0 (2\pi)^2 \left(\frac{1}{\lambda}\right)^2 \cos(Kz - \omega t)$

$= -E_0 \left(\frac{2\pi}{\lambda}\right)^2 \cos(Kz - \omega t) = -E_0 K^2 \cos(Kz - \omega t) = \frac{\partial^2 E_x}{\partial z^2}$

$\Rightarrow \frac{1}{C^2} \frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 E_x}{\partial z^2}$ ✓

FOR $\vec{B}$, WE HAVE $\frac{\partial^2 B_y}{\partial z^2} = \frac{1}{C^2} \frac{\partial^2 B_y}{\partial t^2}$. THE SIMILAR FORM FOR B_y MAKES THIS OBVIOUS.

MAXWELL'S EQUATIONS: $\vec{\nabla}\cdot\vec{E}=0$, $\vec{\nabla}\cdot\vec{B}=0$, $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B}=\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$
$=\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$\vec{\nabla}\cdot\vec{E}=\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}$. $E_y=E_z=0 \Rightarrow \vec{\nabla}\cdot\vec{E}=\frac{\partial}{\partial}\left(E_0\cos(kz-\omega t)\right)=0$

$\vec{\nabla}\cdot\vec{B}=\frac{\partial B_y}{\partial y}=\frac{\partial}{\partial y}\left[B_0\cos(kz-\omega t)\right]=0$

$\vec{\nabla}\times\vec{E}=\hat{\imath}\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right)+\hat{\jmath}\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)+\hat{k}\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)$

$E_y=E_z=0 \Rightarrow \vec{\nabla}\times\vec{E}=\hat{\imath}(0)+\hat{\jmath}\frac{\partial E_x}{\partial z}-\hat{k}\frac{\partial E_x}{\partial y}$

$E_x=E_0\cos(kz-\omega t) \Rightarrow \vec{\nabla}\times\vec{E}=\hat{\jmath}\left(-E_0 k\sin(kz-\omega t)\right)-\hat{k}(0)$

$\Rightarrow \vec{\nabla}\times\vec{E}=\hat{\jmath}\left(-E_0 k\right)\sin(kz-\omega t)$

$\frac{\partial\vec{B}}{\partial t}=\hat{\imath}\frac{\partial B_x}{\partial t}+\hat{\jmath}\frac{\partial B_y}{\partial t}+\hat{k}\frac{\partial B_z}{\partial t}$. $B_x=B_z=0$, $B_y=B_0\cos(kz-\omega t)$

$\Rightarrow \frac{\partial\vec{B}}{\partial t}=\hat{\jmath}\left(-B_0(-\omega)\sin(kz-\omega t)\right)=\hat{\jmath}\left(B_0\omega\right)\sin(kz-\omega t)$

FOR $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$, WE NEED $E_0 k=B_0\omega \Rightarrow E_0=B_0\left(\frac{\omega}{k}\right)=B_0\left(\frac{2\pi f}{2\pi/\lambda}\right)$

$\Rightarrow E_0=B_0(f\lambda)=B_0 c$. ✓

$\vec{\nabla}\times\vec{B}=\hat{\imath}\left(\frac{-\partial B_y}{\partial z}\right)+\hat{k}\left(\frac{\partial B_y}{\partial x}\right)$ SINCE $B_x=B_z=0$. $B_y=B_0\cos(kz-\omega t)$

$\Rightarrow \vec{\nabla}\times\vec{B}=\hat{\imath}(-1)\left(-B_0 k\sin(kz-\omega t)\right)=\hat{\imath}\,B_0 k\sin(kz-\omega t)$

$\frac{\partial\vec{E}}{\partial t}=\hat{\imath}\frac{\partial}{\partial t}\left(E_0\cos(kz-\omega t)\right)=\hat{\imath}\left(-E_0(-\omega)\sin(kz-\omega t)\right)=\hat{\imath}\,E_0\omega\sin(kz-\omega t)$

$\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}=\hat{\imath}\,E_0\frac{\omega}{c^2}\sin(kz-\omega t) \Rightarrow$ WE NEED $E_0\frac{\omega}{c^2}=B_0 k$

$\Rightarrow E_0=B_0\left(\frac{k}{\omega}\right)c^2=B_0\left(\frac{2\pi/\lambda}{2\pi f}\right)c^2=B_0\left(\frac{1}{\lambda f}\right)c^2=B_0\left(\frac{1}{c}\right)c^2 \Rightarrow E_0=B_0 c$.

32.6 EM WAVE WITH $\lambda = 435\text{nm}$ IS PROPAGATING IN $-Z$ DIRECTION.

$E_0 = 2.7 \times 10^{-3}$ V/m AND IS IN $+X$ DIRECTION.

a $f = ?$ $\quad \lambda f = c \Rightarrow (435 \times 10^{-9}\text{m}) f = 3 \times 10^{8} \text{m/s} \Rightarrow \boxed{f = 6.9 \times 10^{14} \text{Hz}}$

$\omega = 2\pi f \Rightarrow \boxed{\omega = 4.33 \times 10^{15} \text{RAD/s}}$

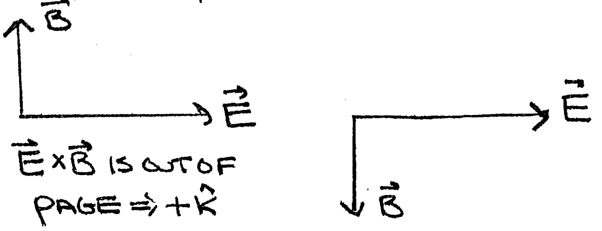
b $B_0 = ?$ $\quad E_0 = B_0 c \Rightarrow \boxed{B_0 = \dfrac{2.7 \times 10^{-3} \text{V/m}}{3 \times 10^{8} \text{m/s}} = 9 \times 10^{-12} \text{T}}$

UNITS: $\dfrac{\text{V/m}}{\text{m/s}} = \dfrac{\text{V}}{\text{m}^2/\text{s}} = \dfrac{(\text{J/C})}{\text{m}^2/\text{s}} = \dfrac{\text{J}}{\text{m}^2(\text{C/s})} = \dfrac{\text{J}}{\text{m}^2 \cdot \text{A}} = \dfrac{\text{N} \cdot \text{m}}{\text{m}^2 \cdot \text{A}} = \dfrac{\text{N}}{\text{m} \cdot \text{A}} = \text{T}$

I HAD TO LOOK ON p.1023 TO FIND WHAT A TESLA IS. DANG E&M UNITS!

c WRITE $\vec{E}$ AND $\vec{B}$ EQUATIONS FOR A PLANE WAVE (WHICH YOUR BOOK CALLS A SINUSOIDAL PLANE WAVE)

$\vec{E} = \hat{\imath} E_0 \cos(kz - \omega t)$ $\vec{B} = \hat{\jmath} B_0 \cos(kz - \omega t)$ PROPAGATES IN $+Z$ DIRECTION, WHICH IS GIVEN BY $\vec{E} \times \vec{B}$'S DIRECTION. $\Rightarrow$ LET $\vec{B}$ BE IN $-\hat{\jmath}$ DIRECTION.



$\vec{E} \times \vec{B}$ IS OUT OF PAGE $\Rightarrow +\hat{k}$

$\vec{E} \times \vec{B}$ IS INTO PAGE $\Rightarrow -\hat{k}$

$\vec{E} = \hat{\imath} E_0 \cos[k(-z) - \omega t]$

$\vec{B} = -\hat{\jmath} B_0 \cos[k(-z) - \omega t]$

HAVE TO FLIP Z to $-Z$ IN COSINE

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{435 \times 10^{-9}\text{m}} \Rightarrow \boxed{k = 1.44 \times 10^{7} \text{RAD/m}}$

$\cos(-kz - \omega t) = \cos[-(kz + \omega t)] = \cos(kz + \omega t) \leftarrow$ EVEN

$\Rightarrow \vec{E} = \hat{\imath} E_0 \cos(kz + \omega t), \quad \vec{B} = -\hat{\jmath} B_0 \cos(kz + \omega t)$

32.13 EM WAVE WITH $f = 5.7\times10^{14}\,Hz$, $V = 2.17\times10^{8}\,m/s$

a $\lambda = ?$ $\quad \lambda f = \quad \Rightarrow \lambda = \dfrac{2.17\times10^{8}\,m/s}{5.7\times10^{14}\,Hz} \Rightarrow \boxed{\lambda = 3.81\times10^{-7}\,m = 381\,nm}$

b FIND λ FOR SAME WAVE IN AIR. FOR AIR $V = C$

$$\Rightarrow \lambda = \frac{3\times10^{8}\,m/s}{5.7\times10^{14}\,Hz} \Rightarrow \boxed{\lambda = 5.26\times10^{-7}\,m = 526\,nm}$$

c $n = ?$ $\quad n = C/V \Rightarrow n = \dfrac{3\times10^{8}\,m/s}{2.17\times10^{8}\,m/s} \Rightarrow \boxed{n = 1.38}$

d FIND K ASSUMING $K_M = 1$ $\quad n = \sqrt{K K_M} \Rightarrow K = \dfrac{n^2}{K_M} = \dfrac{(1.38)^2}{1} \Rightarrow \boxed{K = 1.91}$

32.19 FOR ANTENNA 2.5 Km AWAY, YOU FIND $E_0 = .09\,V/m$, $f = 244\,MHz$.

a $I = ?$ $\quad I = \dfrac{1}{2}\dfrac{1}{\mu_0 c} E_0^2 = \dfrac{1}{2}\dfrac{(.09\,V/m)^2}{(4\pi\times10^{-7}\frac{Ns^2}{C^2})(3\times10^{8}\,m/s)} \Rightarrow \boxed{I = 1.07\times10^{-5}\,\frac{WATT}{m^2}}$

UNIT: $\dfrac{V^2C^2}{m^3Ns} = \dfrac{(J/C)^2C^2}{(Nm)m^2s} = \dfrac{J^2}{Jm^2s} = \dfrac{J}{m^2s} = \dfrac{J/s}{m^2} = \dfrac{WATT}{m^2}$ $\rightarrow$ DANG EM UNITS!

b $B_0 = ?$ $\quad B_0 = \dfrac{E_0}{C} = \dfrac{.09\,V/m}{3\times10^{8}\,m/s} \Rightarrow \boxed{3\times10^{-10}\,T = B_0}$

c POWER = ? $\quad I = \left(\dfrac{POWER}{AREA}\right)_{AVG} \Rightarrow POWER = IA$

IF ANTENNA RADIATES IN ALL DIRECTIONS, ITS ENERGY WILL "SPREAD OUT" OVER THE SURFACE OF A SPHERE $\Rightarrow A = 4\pi r^2$. $r = 2.5\,Km = 2.5\times10^{3}\,m$

$$\Rightarrow POWER = (1.07\times10^{-5}\,WATT/m^2)(4\pi)(2.5\times10^{3}\,m)^2 \Rightarrow \boxed{POWER = 840\,WATT}$$

d SEE ABOVE WHY IN ALL DIRECTIONS.

32.54 SOLAR SAIL. SUN'S POWER $= 3.9 \times 10^{26}$ WATT.

a SAIL SHOULD BE REFLECTIVE TO DOUBLE RADIATION PRESSURE.

b WHAT AREA SAIL DO WE NEED TO PROPEL 10,000 Kg SHIP?

$P_{RAD} = \frac{2I}{c}$. PRESSURE $=$ F/A SO PRESSURE ON SAIL IS F/A_{SAIL}

$$\Rightarrow \frac{F}{A_{SAIL}} = \frac{2I}{c} \Rightarrow A_{SAIL} = \frac{Fc}{2I}$$

TO HAVE SHIP MOVE, IT MUST OVERCOME ITS GRAVITATIONAL ATTRACTION TO THE SUN (AND THE PLANETS TOO, BUT THAT'S AN UN-NEEDED COMPLICATION!)

$\Rightarrow F = \frac{GM_1M_2}{r^2}$ $M_1 =$ SHIP'S MASS $= 10,000$ Kg
$M_2 =$ SUN'S MASS $= 1.99 \times 10^{30}$ Kg

$\Rightarrow A_{SAIL} = \frac{GM_1M_2}{r^2}\left(\frac{c}{2I}\right)$. $I =$ SUNLIGHT'S INTENSITY. $I = \frac{\text{POWER}}{\text{AREA}}$

WHERE THIS AREA IS THE SURFACE OVER WHICH THE SUN'S POWER IS SPREAD OUT. THE SUN RADIATES IN ALL DIRECTIONS $\Rightarrow$ A SPHERE

$\Rightarrow A = 4\pi r^2 \Rightarrow I = \frac{\text{POWER}}{4\pi r^2}$.

$\Rightarrow A_{SAIL} = \frac{GM_1M_2}{\cancel{r^2}}\left(\frac{c}{2\,\text{Power}}\right)(4\pi \cancel{r^2})$ THE r's CANCEL! $\rightarrow$ THIS IS WHAT MAKES A SOLAR SEAL FEASIBLE.

$A_{SAIL} = \frac{2\pi GM_1M_2c}{\text{POWER}}$. $G = 6.67 \times 10^{-11}\, Nm^2/Kg^2$

$$\Rightarrow A_{SAIL} = \frac{2\pi\,(6.67\times10^{-11}\,Nm^2/Kg^2)(10{,}000\,Kg)(1.99\times10^{30}\,Kg)(3\times10^8\,m/s)}{3.9\times10^{26}\,WATT} = 6.42\times10^6\,m^2$$

$$\text{UNIT: } \frac{Nm^3/s}{WATT} = \frac{(Nm)(m^2)}{s\cdot(J/s)} = \frac{Jm^2}{J} = m^2$$

$$A_{SAIL} = 6.42\times10^6\,m^2 = 6.42\times10^6\,m^2 \times \frac{Km}{(1000m)^2}$$

$$\Rightarrow \boxed{A_{SAIL} = 6.42\,Km^2}$$

WHICH IS ABOUT 1600 ACRES. YIKES! THAT'S BIG.