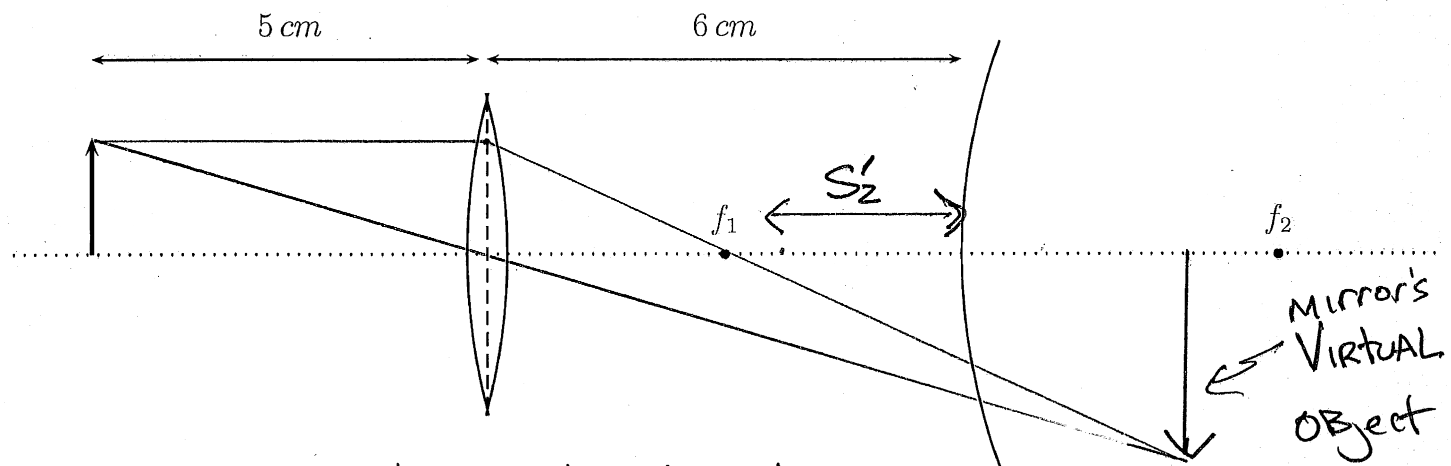
PHYSICS 262
EXAM 3

Please answer any four of the following five questions. Each question is worth five points. Partial credit will be awarded for any **attempted** problem.

1. An object is placed 5 cm in front of an $f_1 = 3\,cm$ converging lens and $f_2 = -4\,cm$ diverging mirror combination. If the lens and the mirror are 6 cm apart, use the thin-lens and mirror equations to find the location of the image created by the mirror. Is this image real or virtual?



LENS: $\frac{1}{S_1} + \frac{1}{S_1'} = \frac{1}{f_1} \Rightarrow \frac{1}{5} + \frac{1}{S_1'} = \frac{1}{3}$

$\Rightarrow \frac{1}{S_1'} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15} \Rightarrow S_1' = \frac{15}{2} = 7.5\,cm$ ← LARGER THAN lens mirror separation

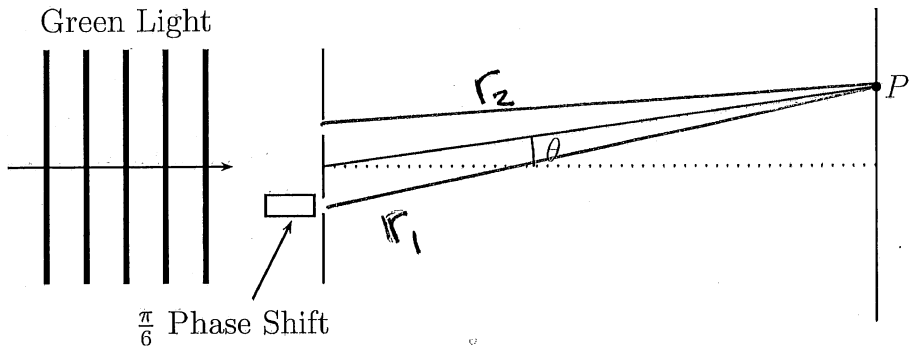
$\Rightarrow$ VIRTUAL OBject FOR Mirror

$\Rightarrow S_2 = 6 - 7.5 = -1.5\,cm$

Mirror: $\frac{1}{S_2} + \frac{1}{S_2'} = \frac{1}{f_2} \Rightarrow \frac{1}{-1.5} + \frac{1}{S_2'} = \frac{1}{-4} \Rightarrow \frac{1}{S_2'} = \frac{-1}{4} + \frac{1}{1.5} = \frac{5}{12}$

$\boxed{S_2' = \frac{12}{5} = 2.4\,cm}$ $S_2' > 0 \Rightarrow$ REAl IMAge $\Rightarrow$ 2.4 cm to left of Mirror OR 3.6 cm to right of lens

2. In a variation on Young's double-slit experiment, a $\pi/6$ "phase shifter" is placed in front of the lower slit. A phase shifter is a device that allows light to pass thorough it, and upon exiting the electric field's phase is increased by, here, $\pi/6\,radians$. Green light ($\lambda = 532\,nm$) is sent through slits that are $d = 8500\,nm$ apart. At what angle θ will the $m = 1$ points of constructive and destructive interference be found? You may assume the screen is far away.



$\vec{E}_2 = \vec{E}_0 \cos(kr_2 - \omega t)$, BECAUSE of phase shifter,

$E_1 = E_0 \cos(kr_1 - \omega t + \pi/6)$. $E_P = E_1 + E_2$

Constructive Intf. $\Rightarrow \cos(kr_2 - \omega t) = \cos(kr_1 - \omega t + \pi/6)$

$\Rightarrow kr_2 - \omega t = kr_1 - \omega t + \pi/6 + 2\pi m \Rightarrow k(r_2 - r_1) = \frac{\pi}{6} + 2\pi m = 2\pi(\frac{1}{12} + m)$

$\Rightarrow r_2 - r_1 = \lambda(\frac{1}{12} + m)$. Screen far away $\Rightarrow r_2 - r_1 \simeq d\sin\theta$

$\Rightarrow \boxed{d\sin\theta = \lambda(m + \frac{1}{12})}$ ← Constructive

Destructive: $\cos(kr_2 - \omega t) = -\cos(kr_1 - \omega t + \pi/6) \Rightarrow kr_2 - \omega t = kr_1 - \omega t + \pi/6 + 2\pi(m + \frac{1}{2})$

$\Rightarrow (r_2 - r_1)k = 2\pi(m + \frac{1}{2} + \frac{1}{12}) \Rightarrow \boxed{d\sin\theta = \lambda(m + \frac{7}{12})}$ ← destructive

(see Next page)

Constructive: $m=1 \Rightarrow d\sin\theta = \lambda\left(\frac{13}{12}\right)$

$\Rightarrow \sin\theta = \frac{\lambda}{d}\left(\frac{13}{12}\right) = \frac{532}{8500}\left(\frac{13}{12}\right) = .0678$

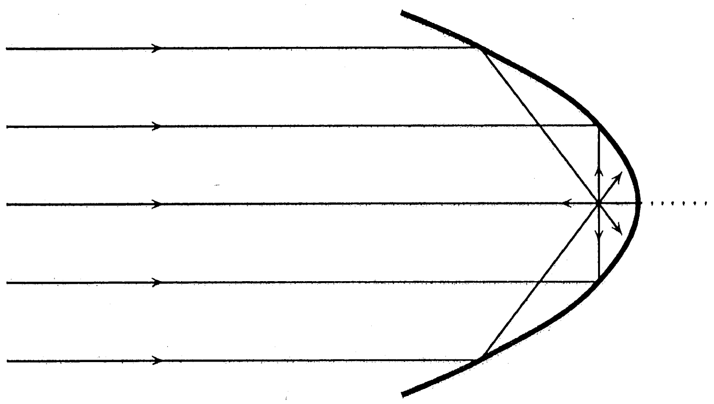
$\Rightarrow \boxed{\theta = \sin^{-1}(.0678) \approx 3.89° \approx 4°}$

Destructive: $m=1 \Rightarrow d\sin\theta = \lambda\left(\frac{19}{12}\right)$

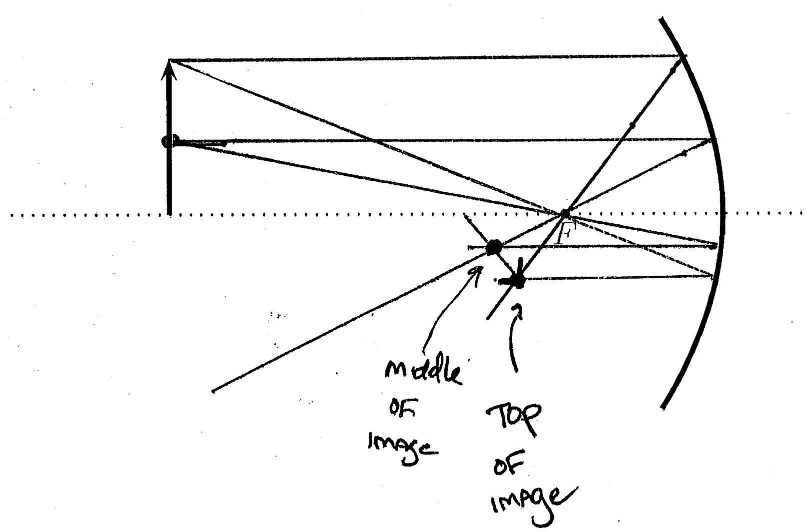
$\Rightarrow \sin\theta = \frac{532}{8500}\left(\frac{19}{12}\right) \approx .099$

$\Rightarrow \boxed{\theta = \sin^{-1}(.099) \approx 5.69°}$

3. Telescopes and car headlights both use parabola shaped mirrors in their design. As shown below, parabolic and spherical mirrors share the behavior of focusing parallel rays through a single point.

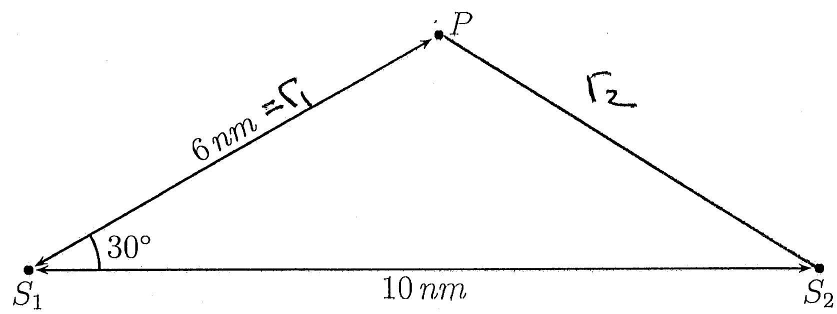


To see what image a parabolic mirror creates, sketch the P-ray and F-ray for the top and midpoint of the image in the diagram below. Comment on your results. *Note:* To make the drawing easier, the mirror below has a much lower curvature than the one above and may appear spherical to you, but I promise, its equation is parabolic. ($x = -\frac{1}{8}y^2$ to be exact.)



So parabolic mirror "skew" their images.
If objects is small, the skewing will be small & you'll get an image like that of a spherical mirror.
But it won't obey the mirror equation.

4. Two sources of monochromatic, coherent light, S_1 and S_2, are placed $10\,nm$ apart as shown below. If destructive interference is occurring at the point labeled P, find a possible wavelength of the light emitted by the two sources.



Destructive INTF $\Rightarrow r_1 - r_2 = (m+\tfrac{1}{2})\lambda$

I'll use Law of Cosines! $r_2^2 = r_1^2 + 10^2 - 2(r_1)(10)\cos 30^\circ$

$\Rightarrow r_2^2 = (6)^2 + (10)^2 - 2(6)(10)\cos 30^\circ = 32.077$

$\Rightarrow r_2 = 5.664\,nm$

$\Rightarrow r_1 - r_2 = 6\,nm - 5.664\,nm = .336\,nm$

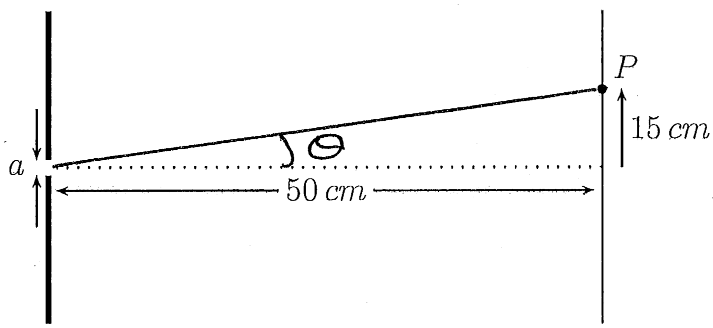
$\Rightarrow .336\,nm = (m+\tfrac{1}{2})\lambda \Rightarrow \lambda = \dfrac{.336\,nm}{m+\tfrac{1}{2}}$

So here are the possibilities:

m	λ (nm)
0	.672
1	.224
2	.1344
3	.096

etc.

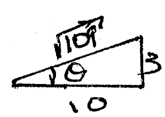
5. Red laser light, wavelength 635 nm, is sent through a single slit of width $a = 800\,nm$. If the screen is 50 cm away, what is the intensity (as a fraction of the maximum intensity I_0) at the point P which is 15 cm above the center line?



Single Slit, $I = I_0 \left[\frac{\sin \beta/2}{\beta/2}\right]^2$ $\qquad \beta = \frac{2\pi a \sin\theta}{\lambda}$

$\tan\theta = \frac{15}{50} = \frac{3}{10}$ So either $\theta = \tan^{-1}(\frac{3}{10}) = 16.7° \Rightarrow \sin\theta = .287$

OR



$\Rightarrow \sin\theta = \frac{3}{\sqrt{109}} \Rightarrow \beta = 2\pi \frac{(800nm)(3/\sqrt{109})}{(635nm)} \doteq 2.275 rad$

$\Rightarrow \beta/2 = 1.1373 rad$

$\Rightarrow I = I_0 \left[\frac{\sin(1.1373 rad)}{1.1373}\right]^2 = I_0 \left[\frac{0.9075}{1.1373}\right]^2 = I_0 (.6367)$

$\Rightarrow \boxed{\frac{I}{I_0} = 0.6367}$