July 14, Week 7

Today: Chapter 7, Torque

Homework #7 due Monday, July 21 at 5:00pm

General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ($\overrightarrow{\mathbf{r}}$), only the component of $\overrightarrow{\mathbf{F}}$ which is perpendicular to $\overrightarrow{\mathbf{r}}$ causes torque.

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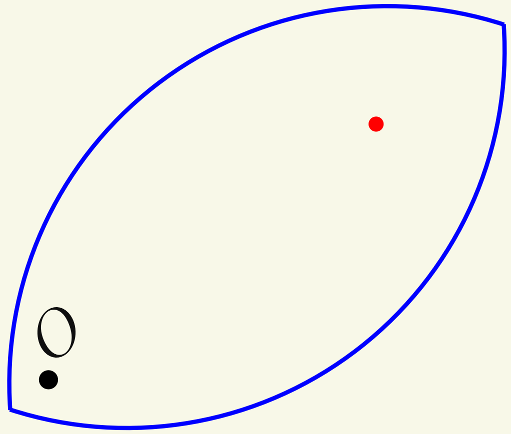

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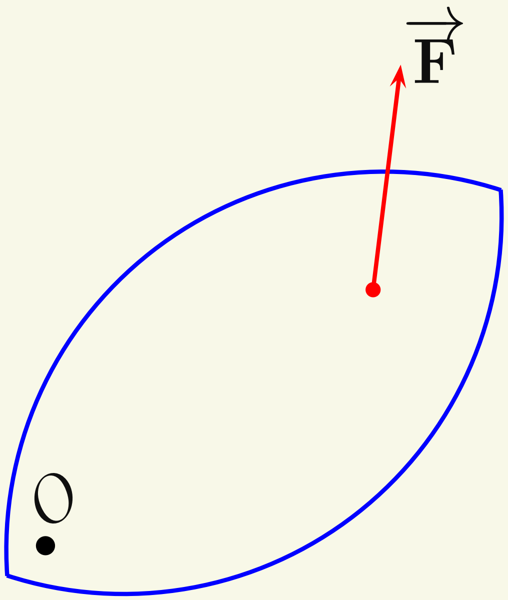

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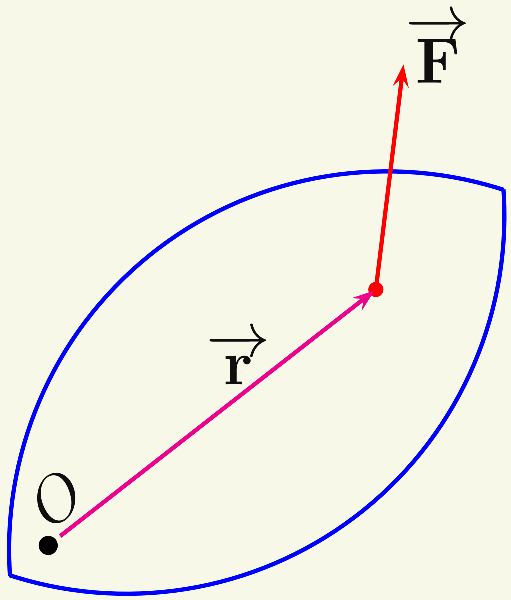

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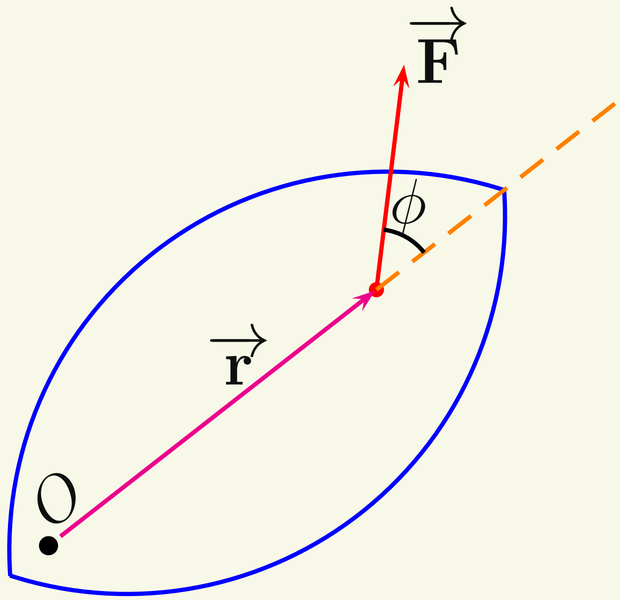

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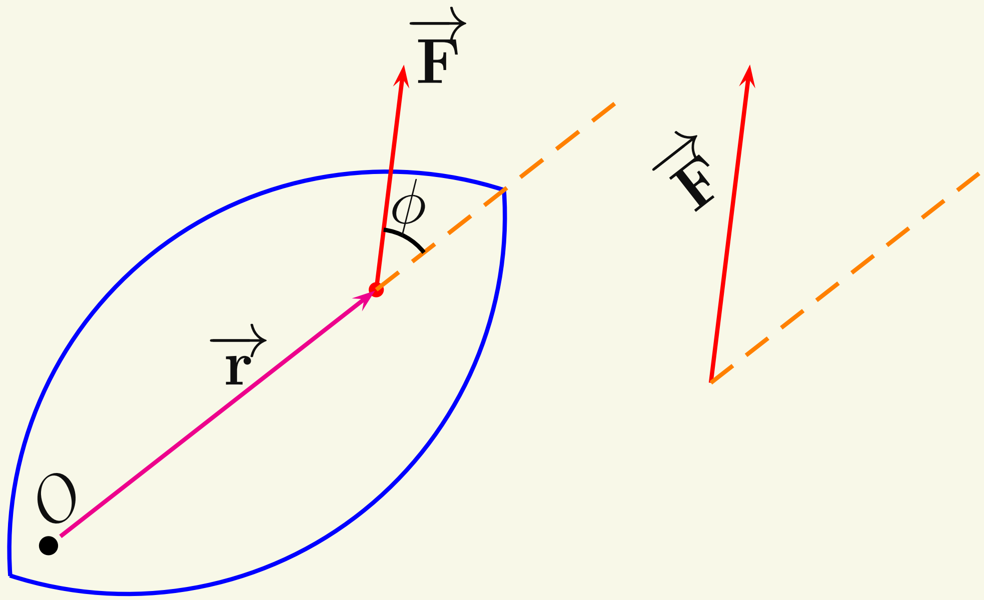

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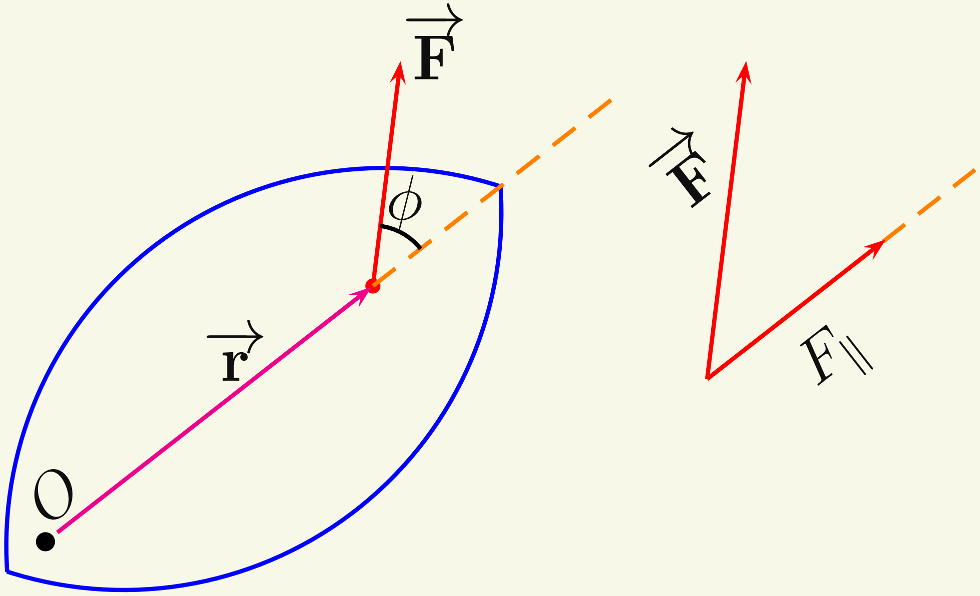

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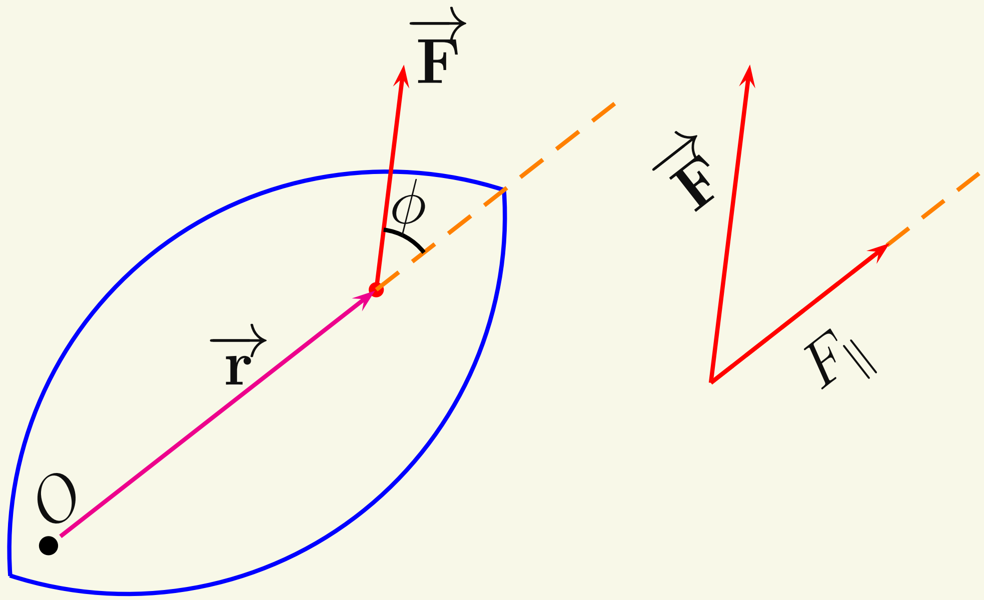

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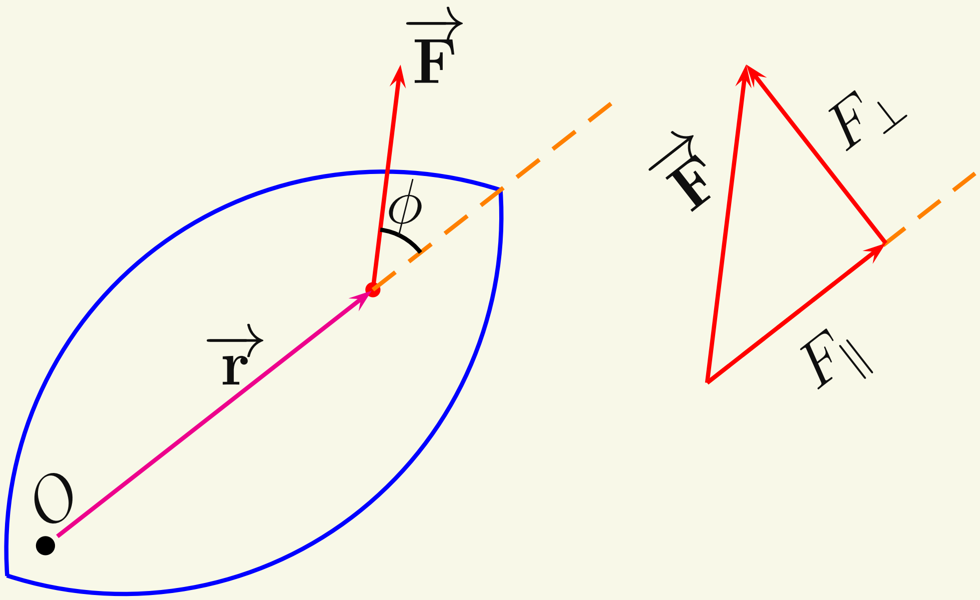

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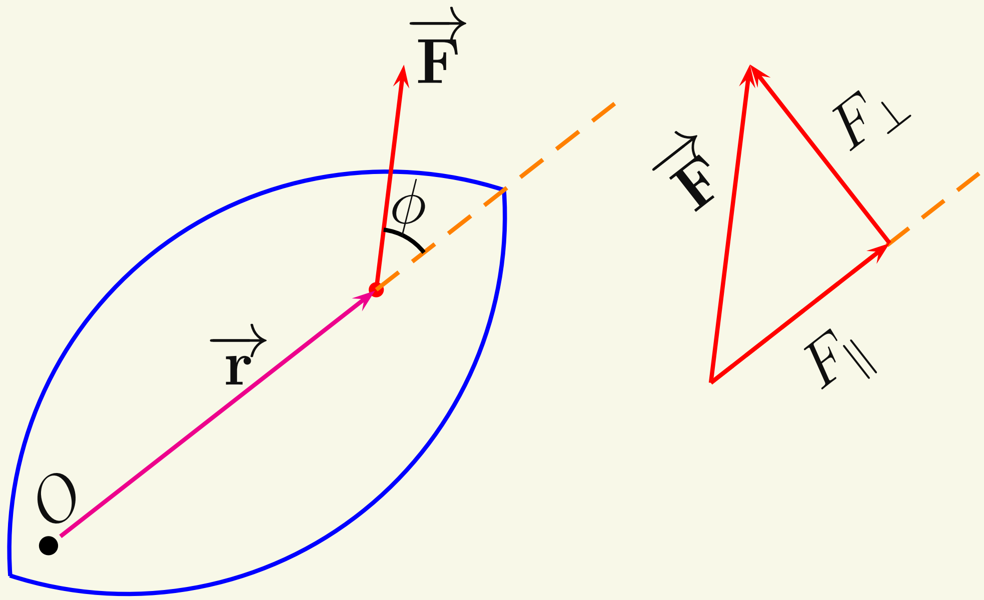

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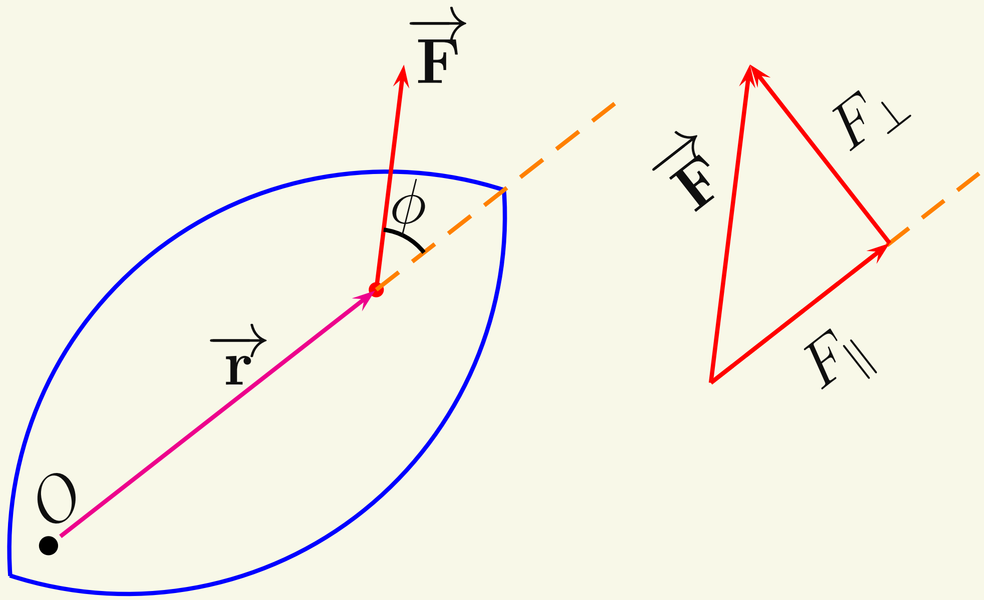

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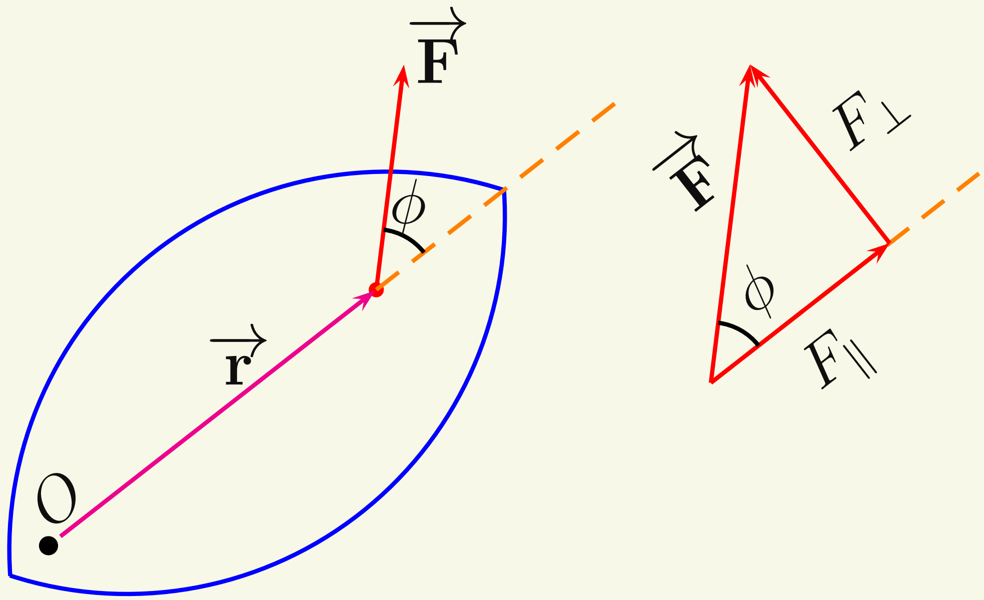

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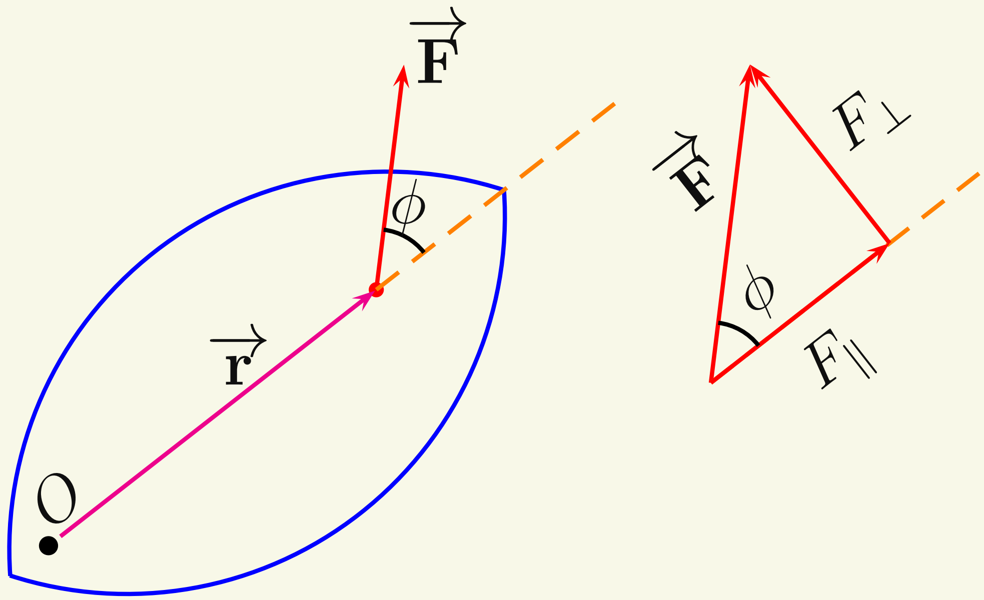

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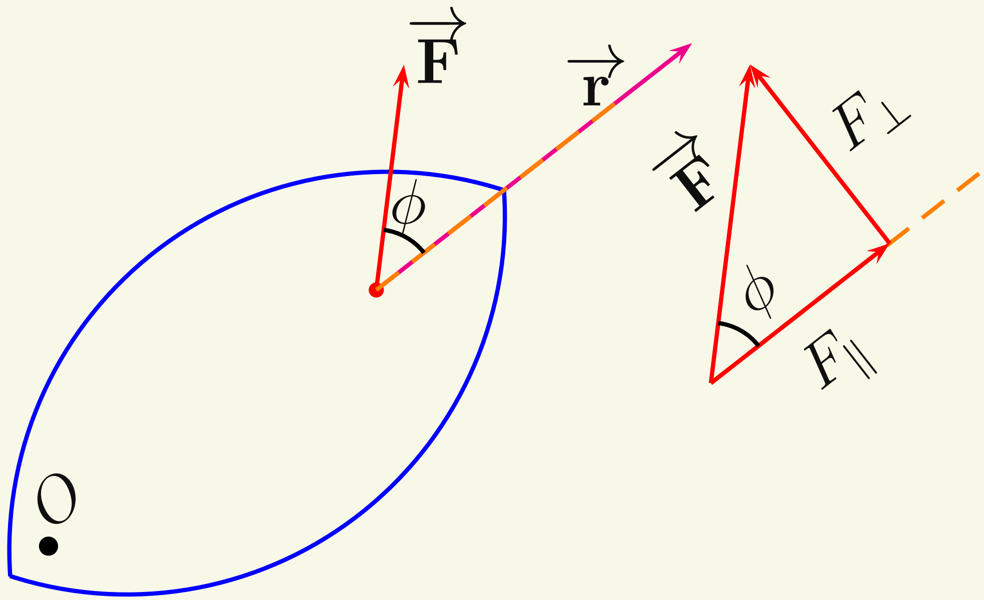

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ϕ is angle between $\overrightarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{F}}$

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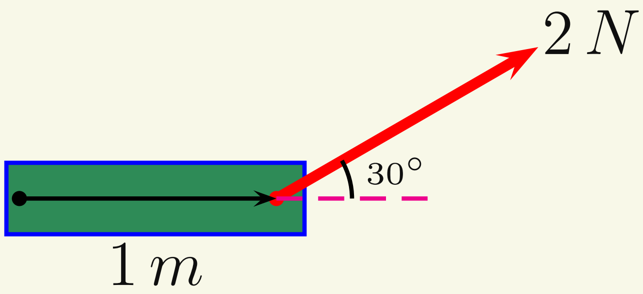

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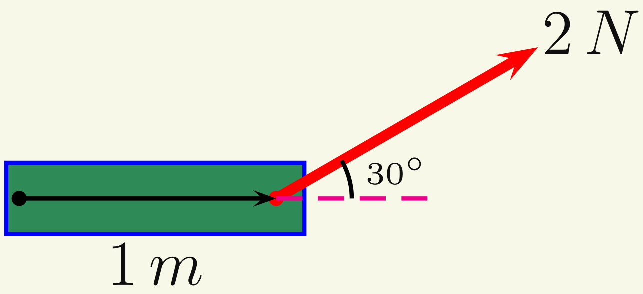

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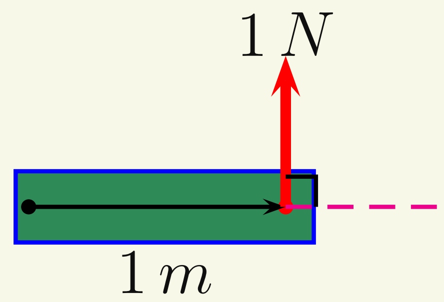

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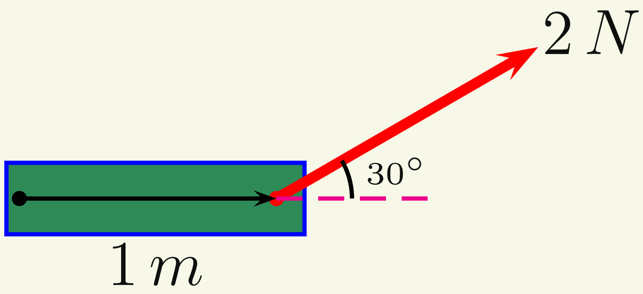

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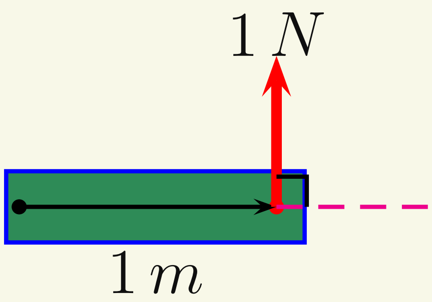

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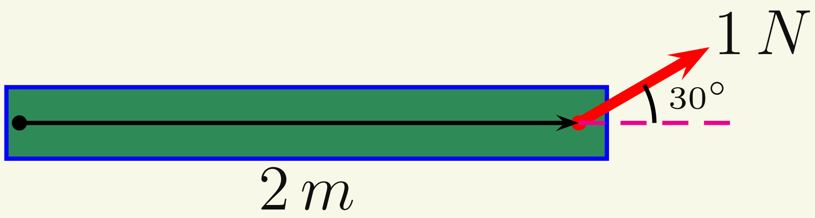

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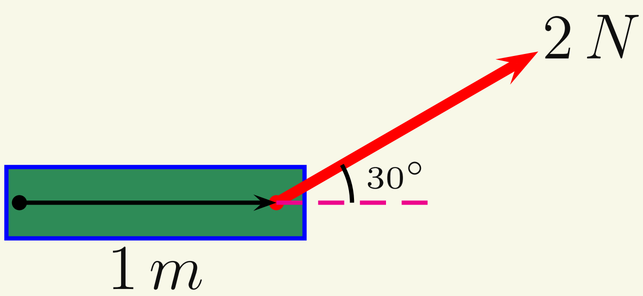

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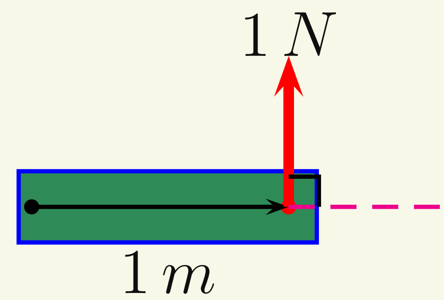

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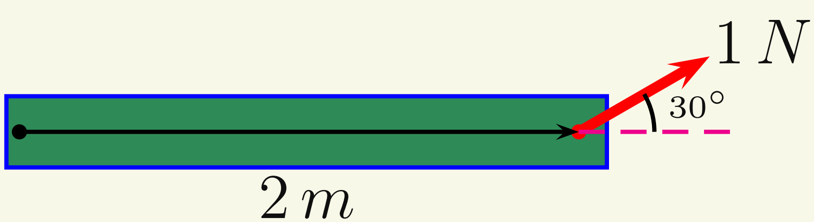

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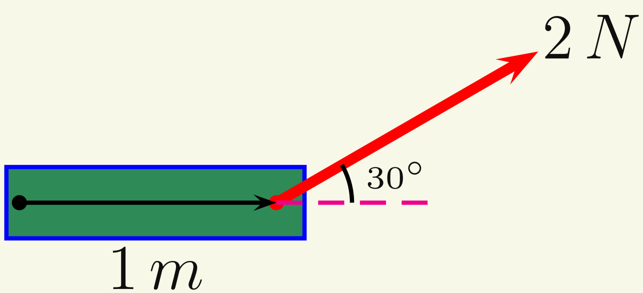

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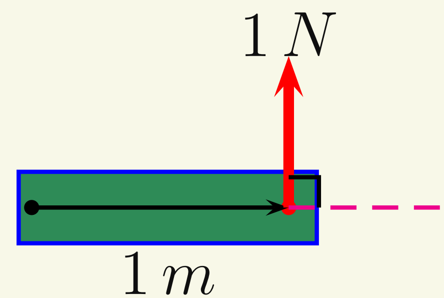

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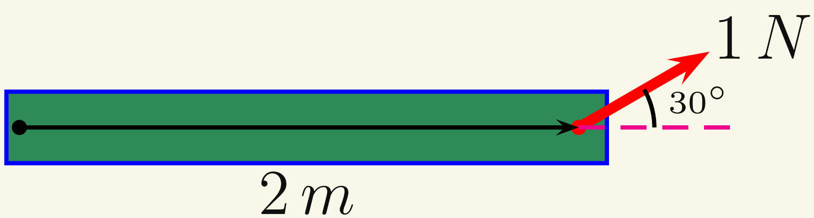

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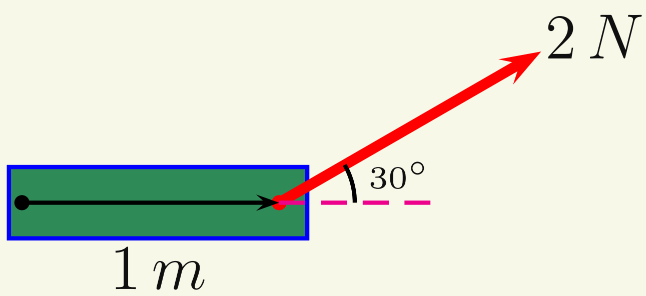

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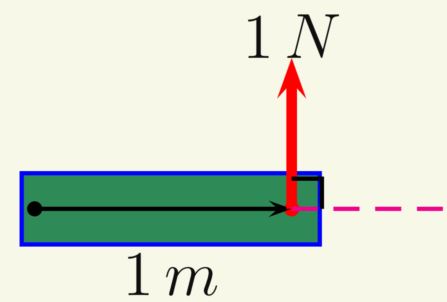

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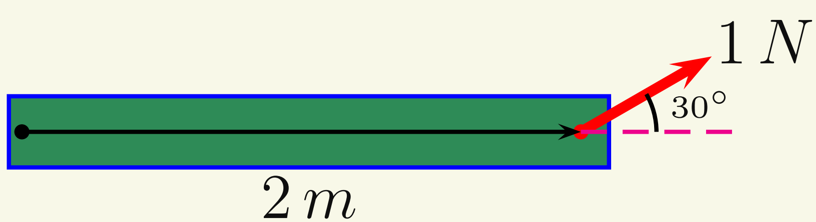

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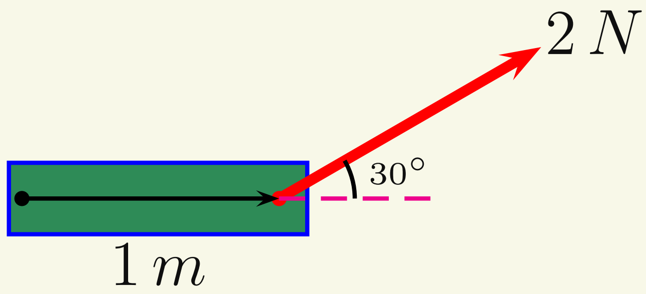

$\tau_1 = (1\,m)(2\,N)\sin 30^\circ = 1\,N\cdot m$

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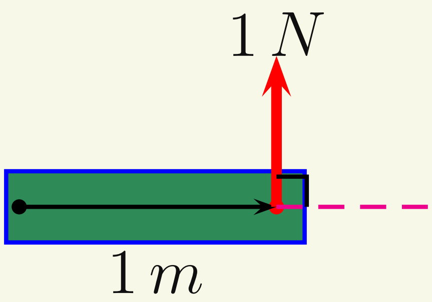

$\tau_2 = (1\,m)(1\,N) = 1\,N\cdot m$

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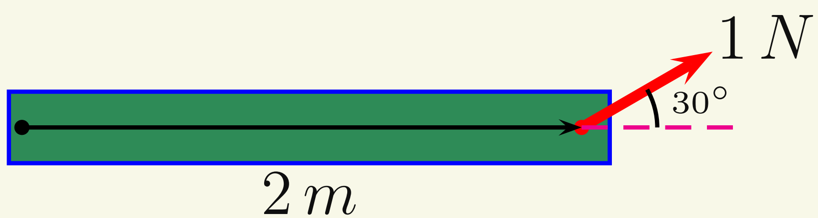

$\tau_3 = (2\,m)(1\,N)\sin 30^\circ = 1\,N\cdot m$

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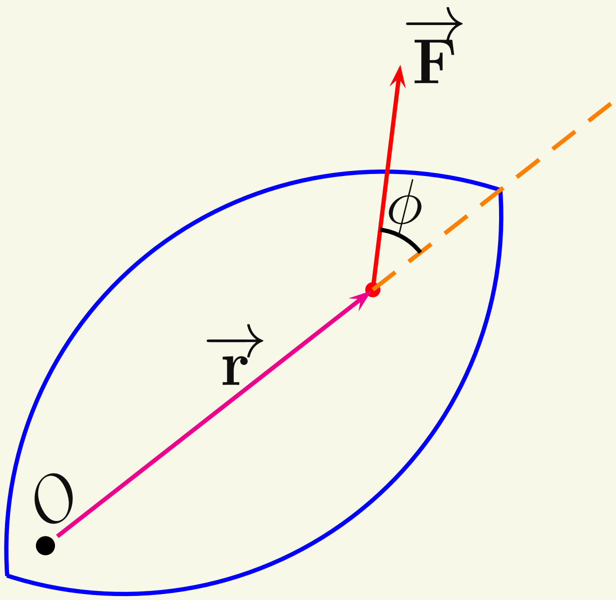

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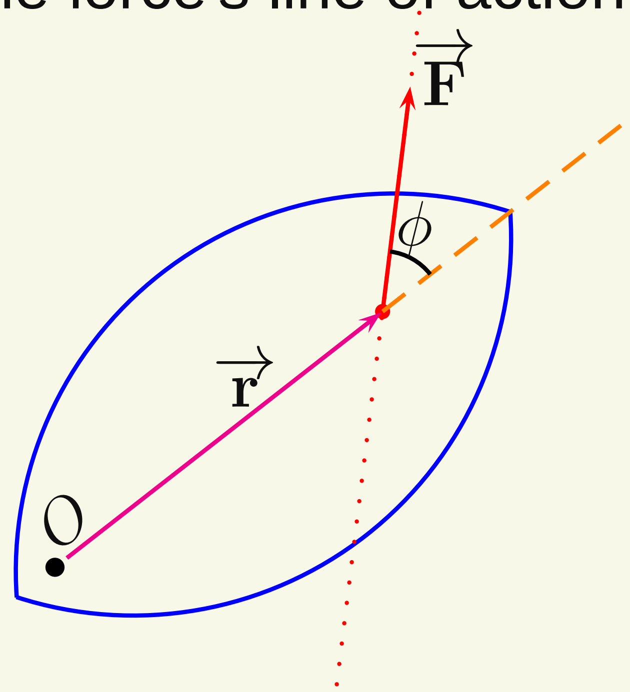

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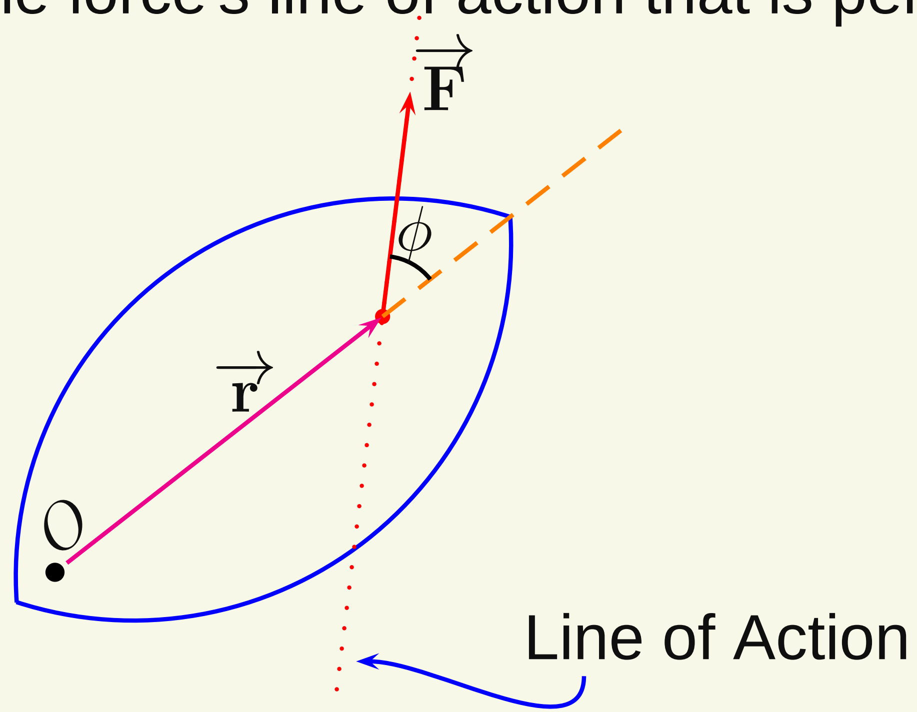

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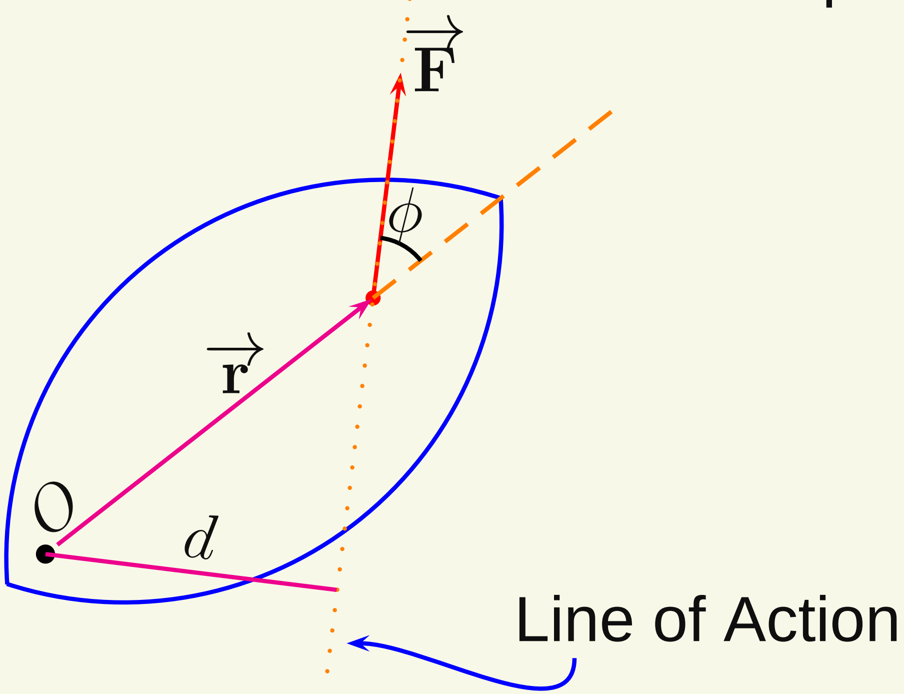

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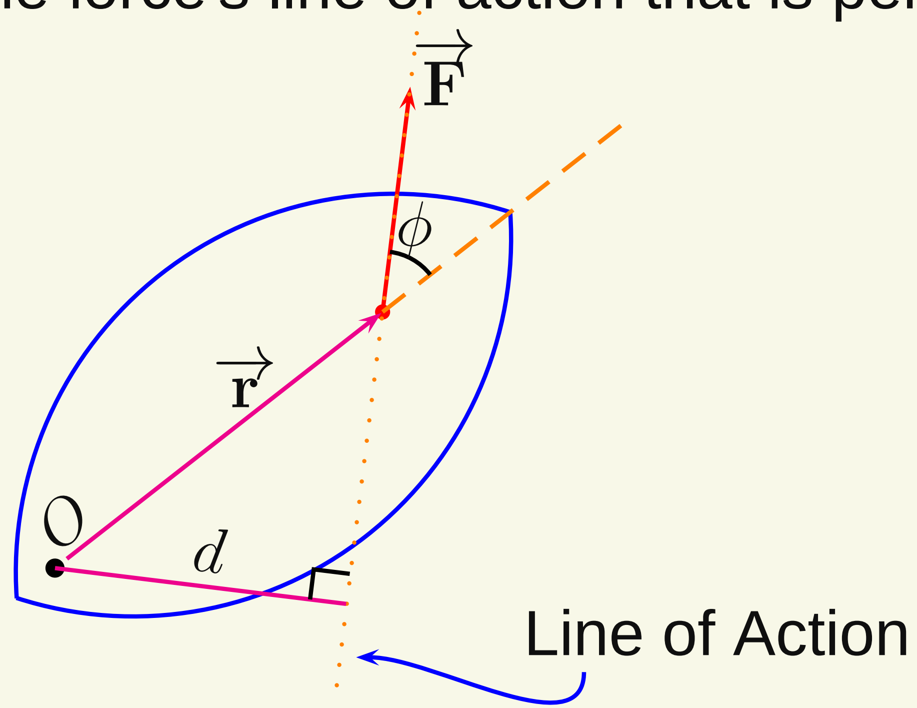

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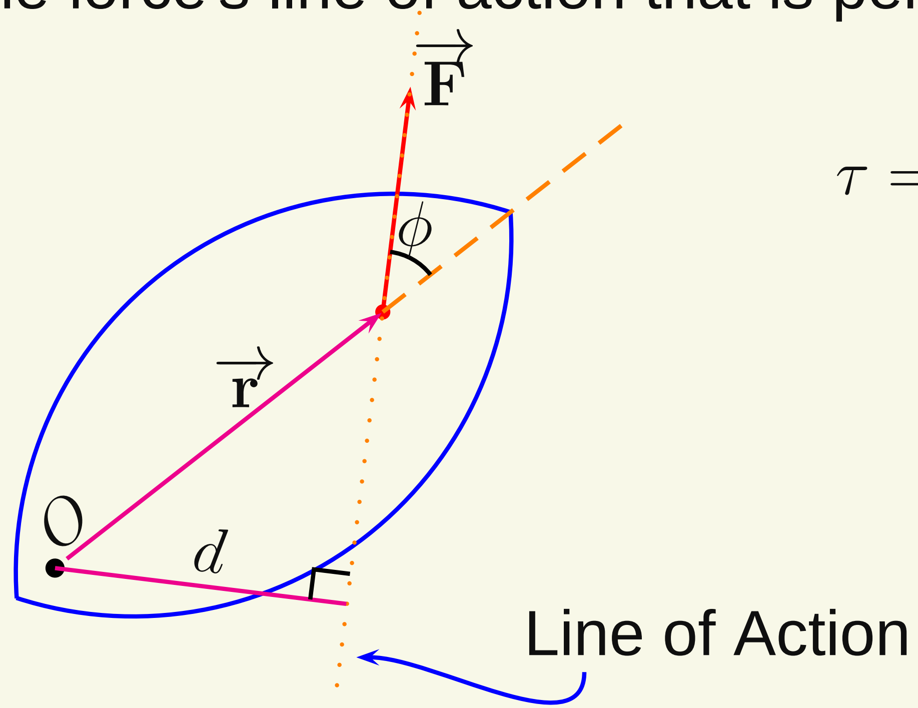

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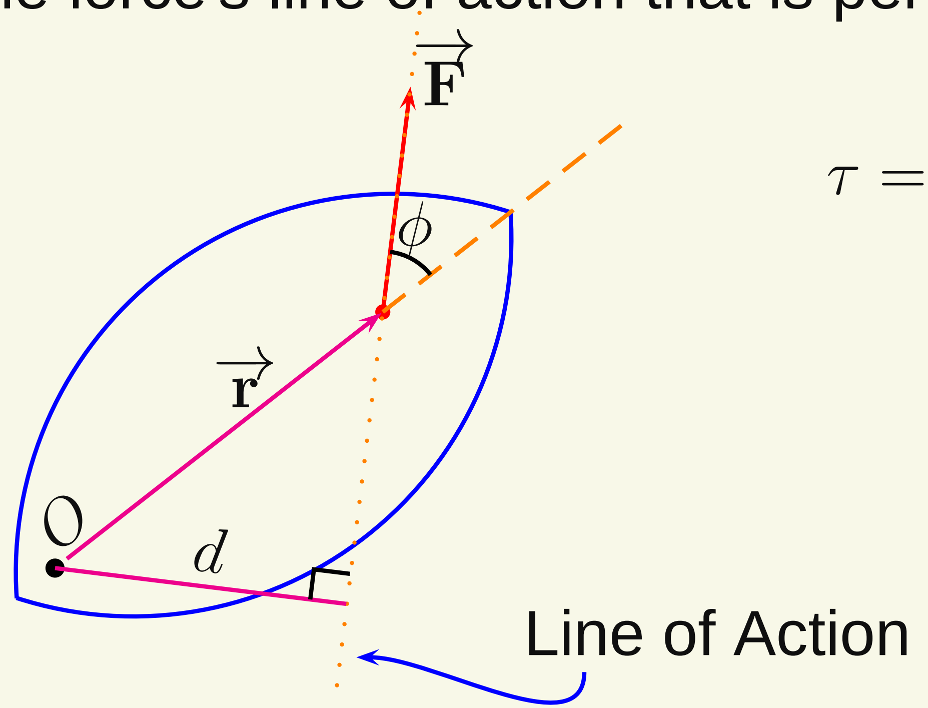

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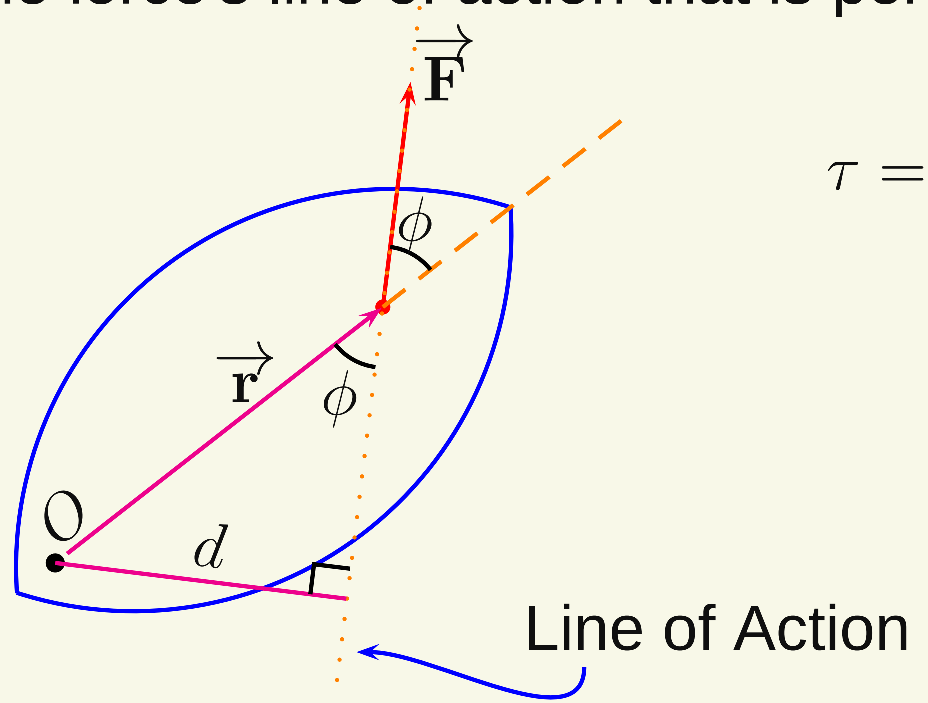

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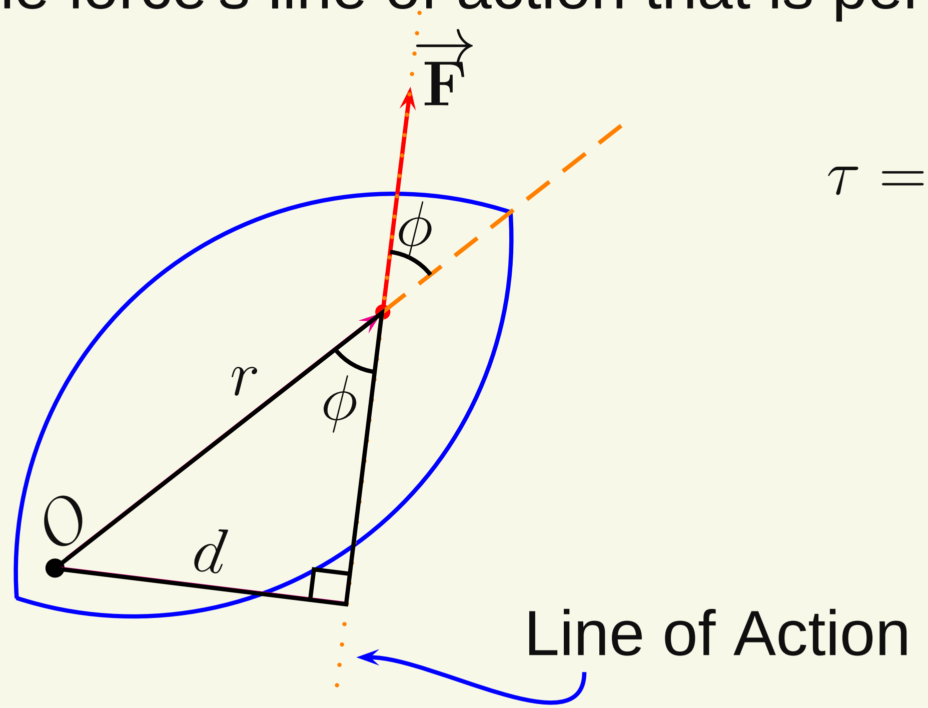

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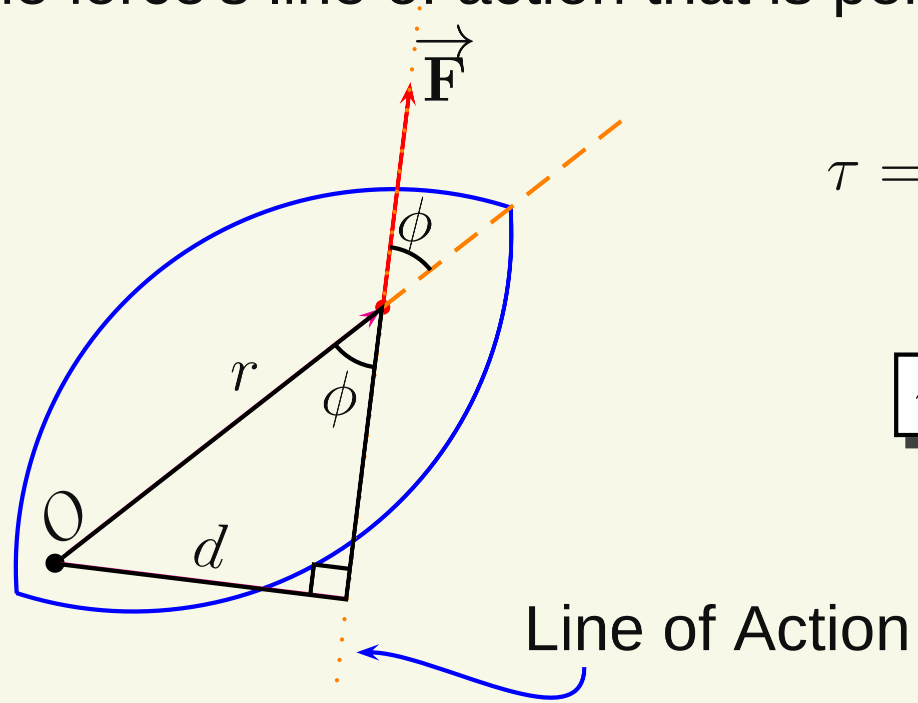

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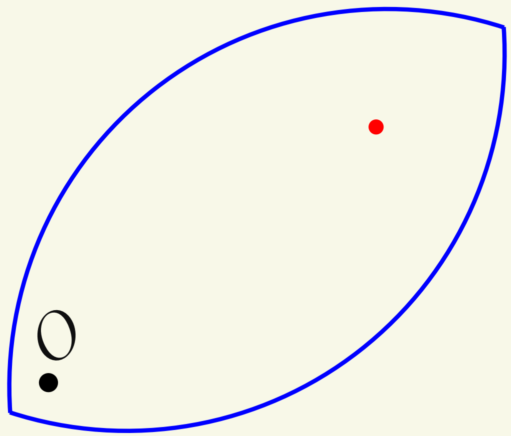

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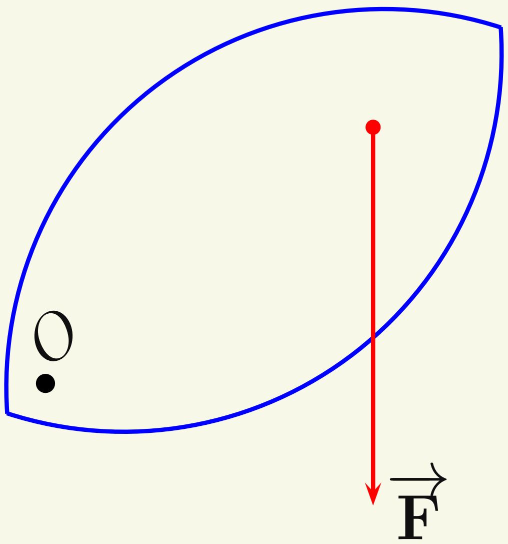

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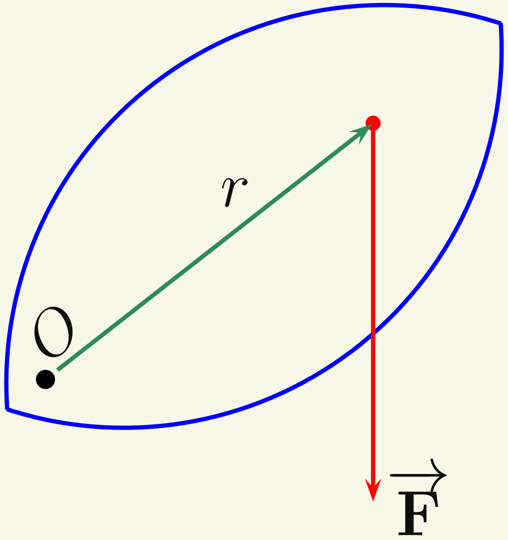

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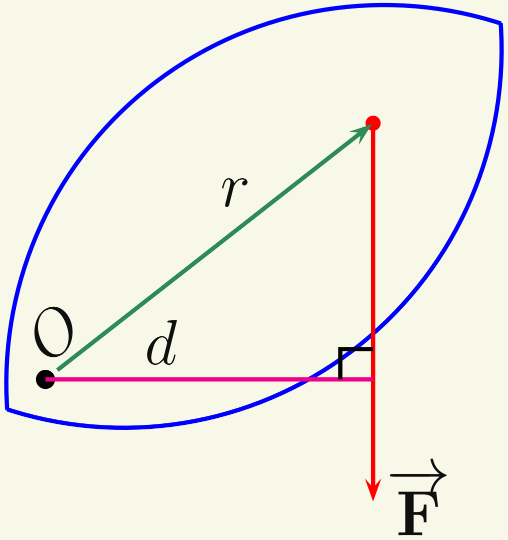

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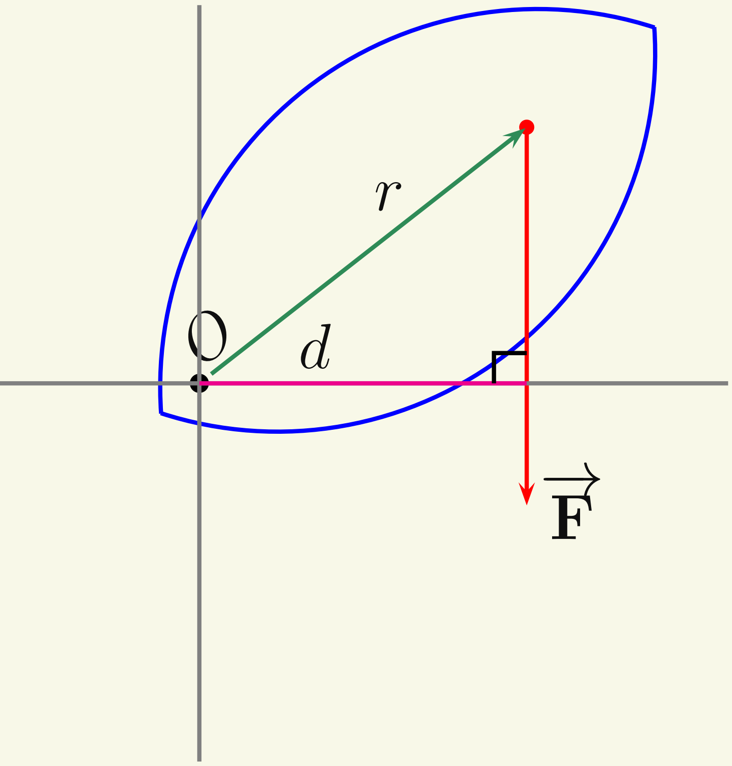

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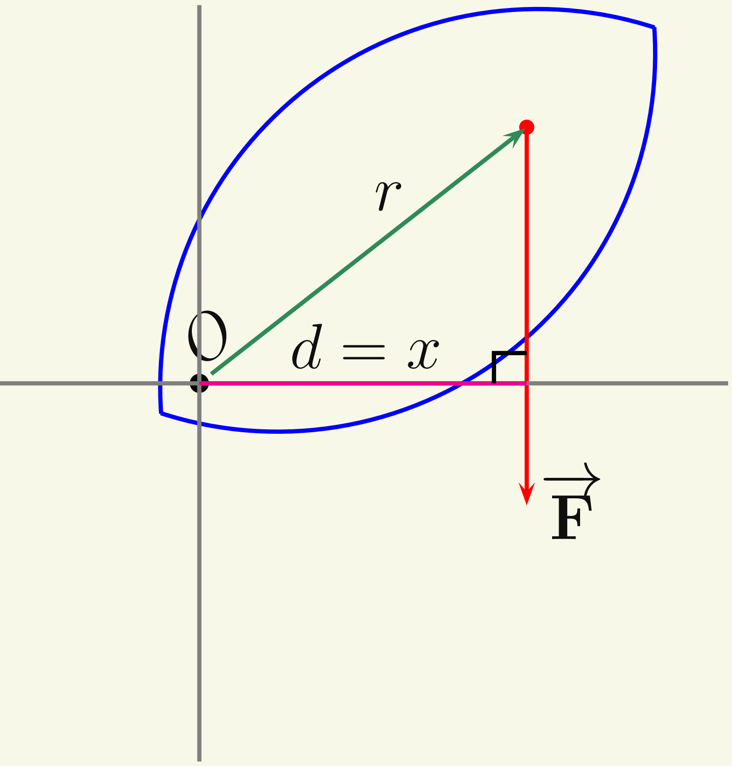

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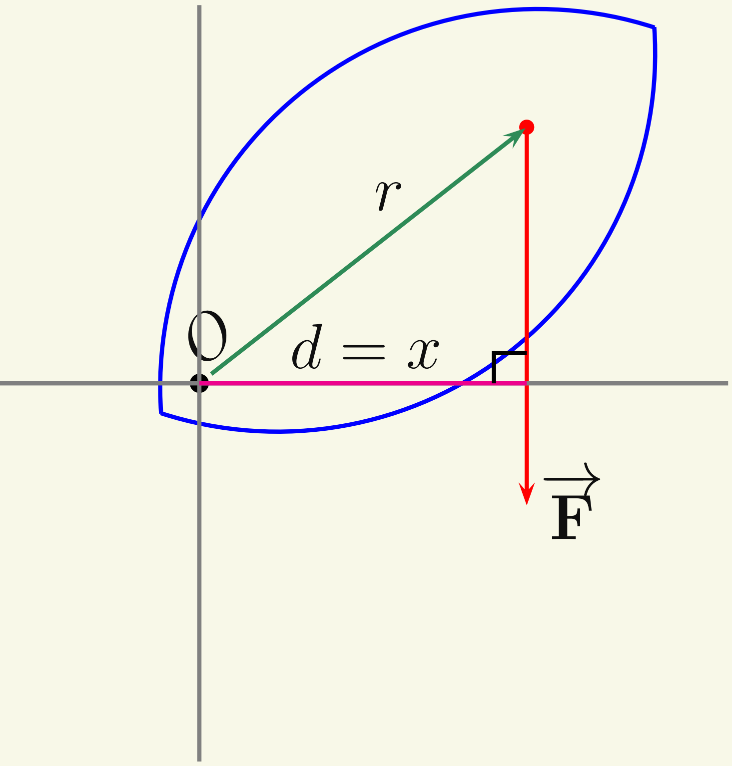

For vertical forces:

$$\boxed{\tau = xF}$$

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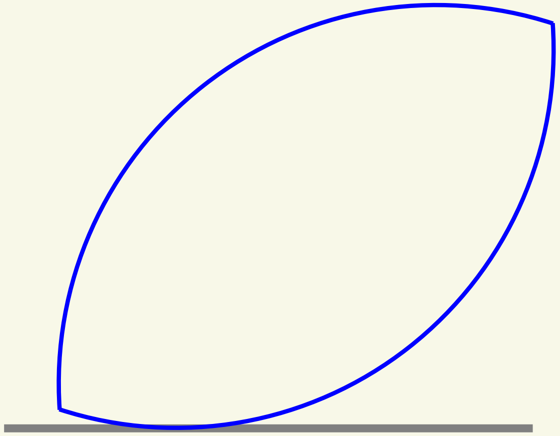

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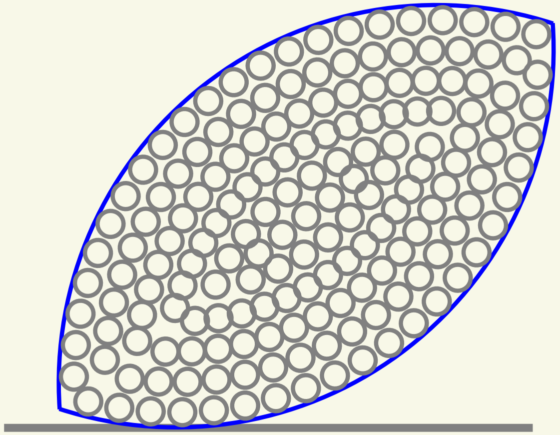

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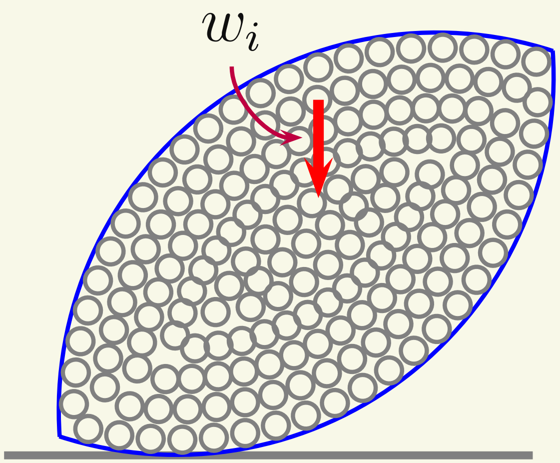

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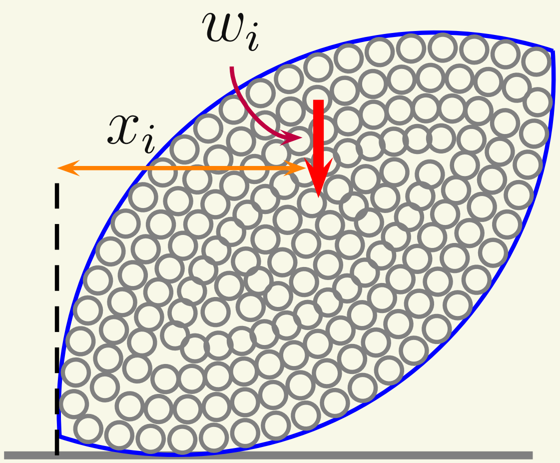

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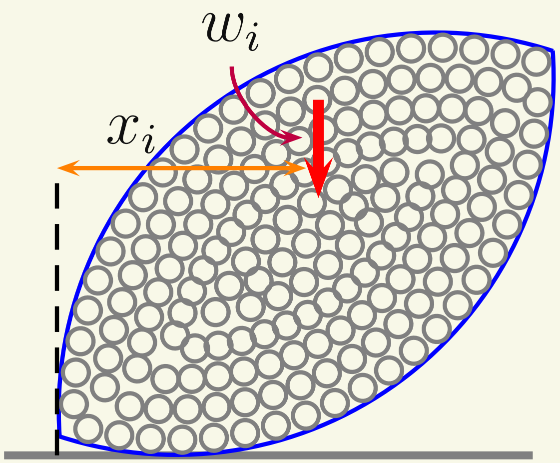

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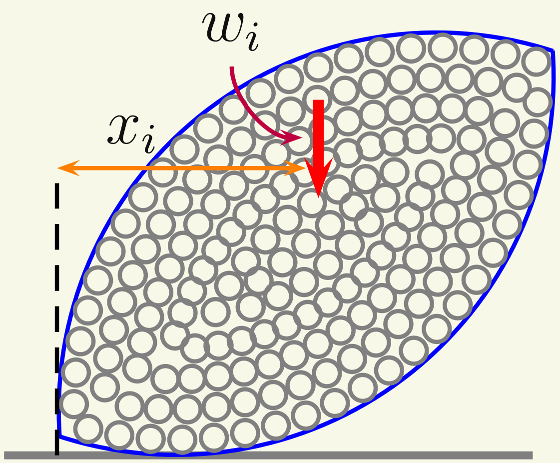

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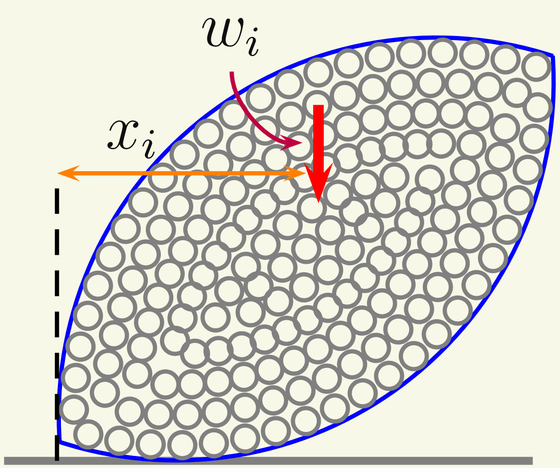

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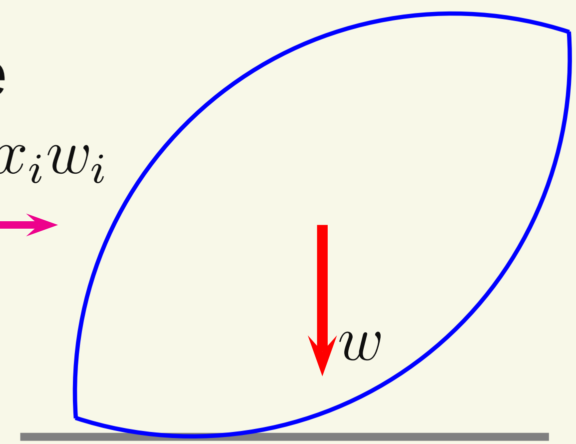

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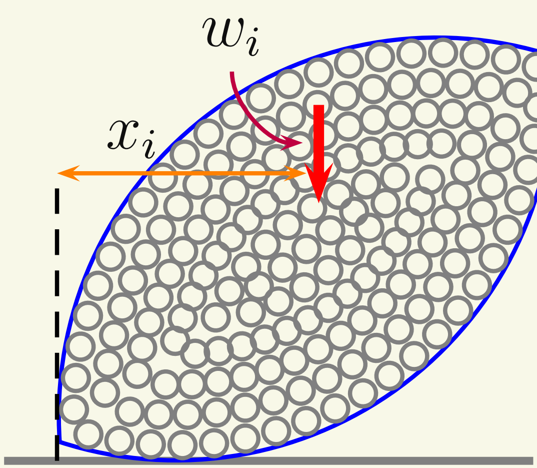

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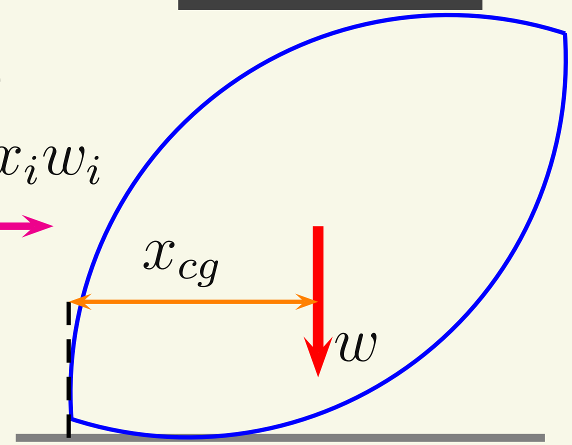

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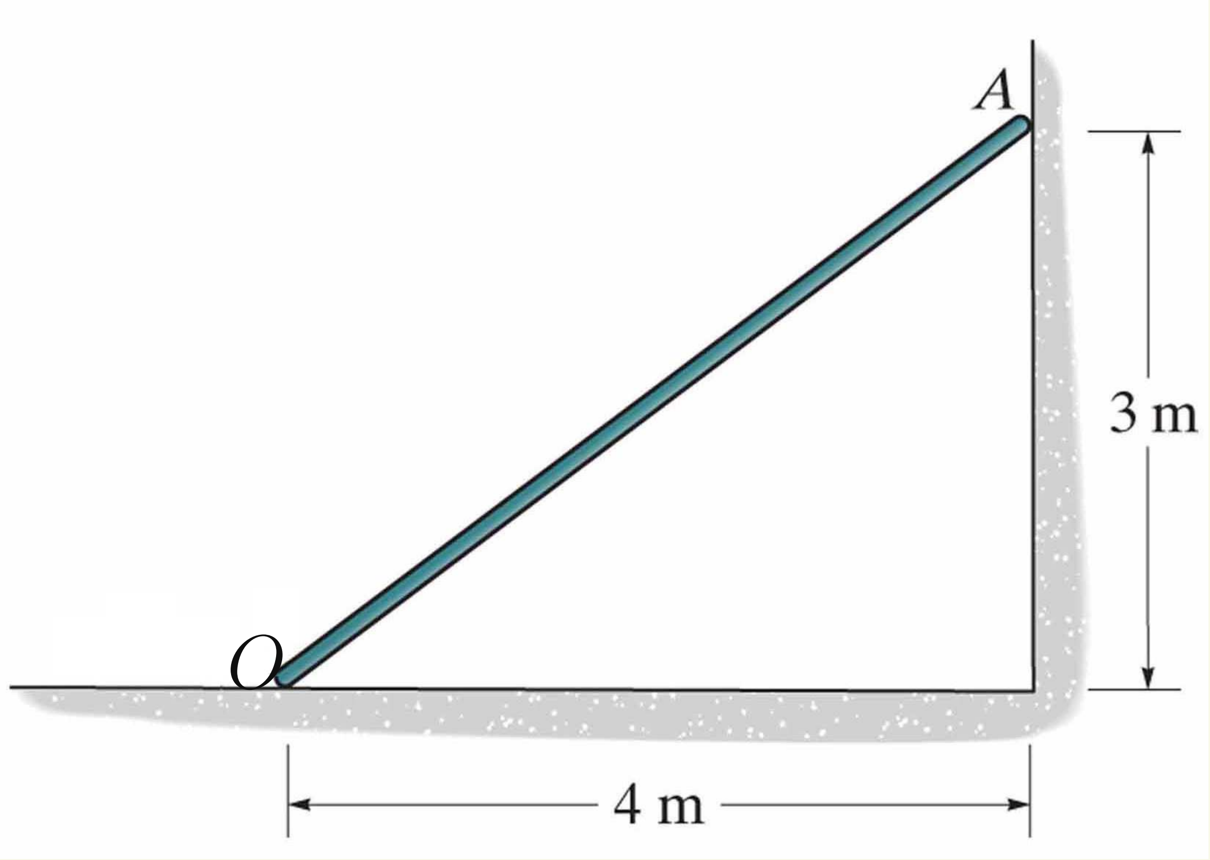
Center of Gravity Exercise

A 300-N *uniform* bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point O?



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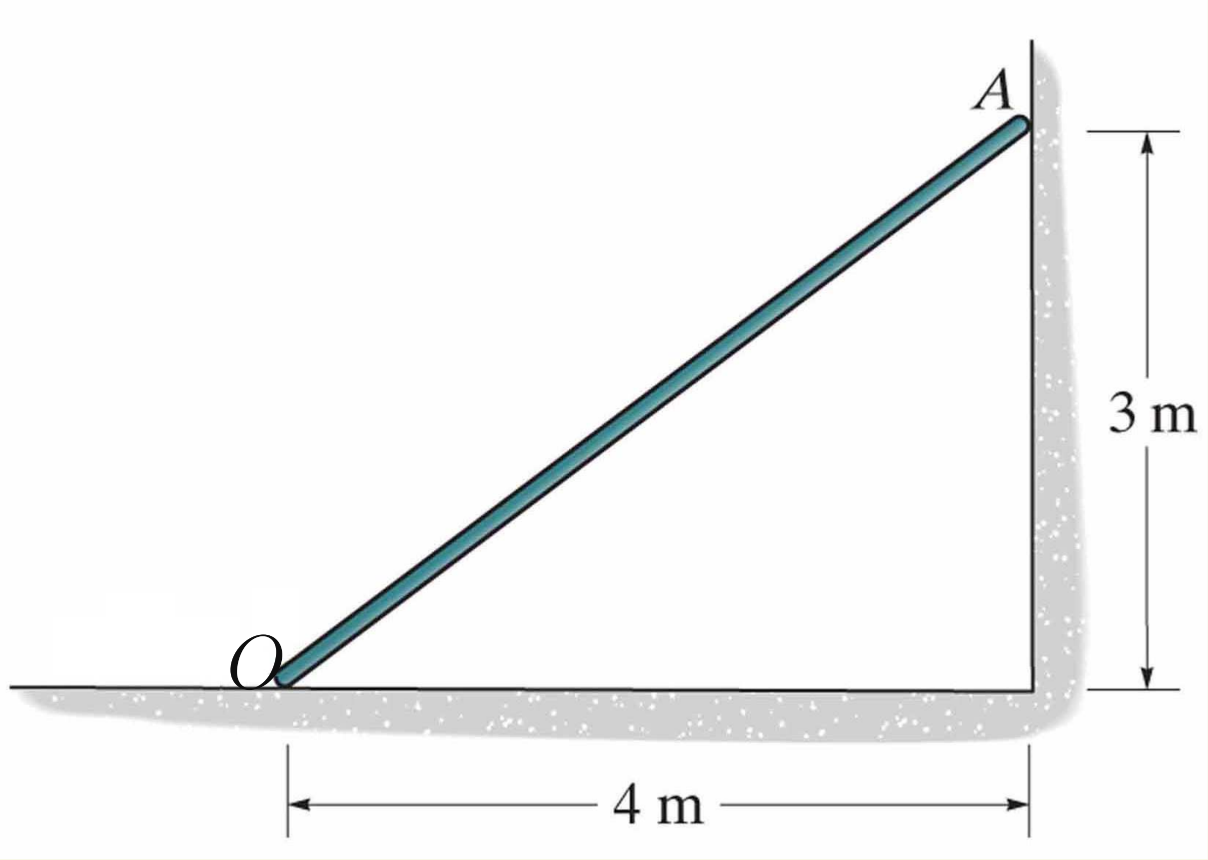

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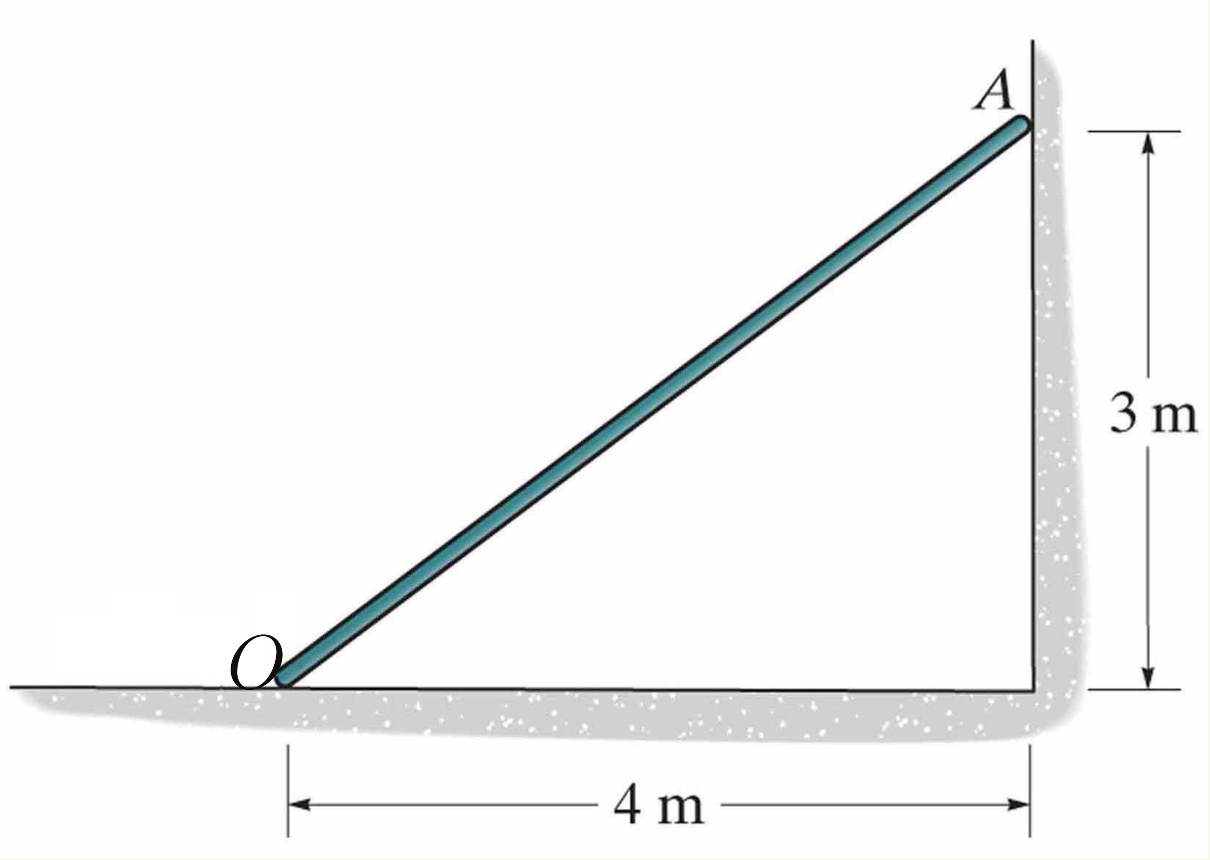

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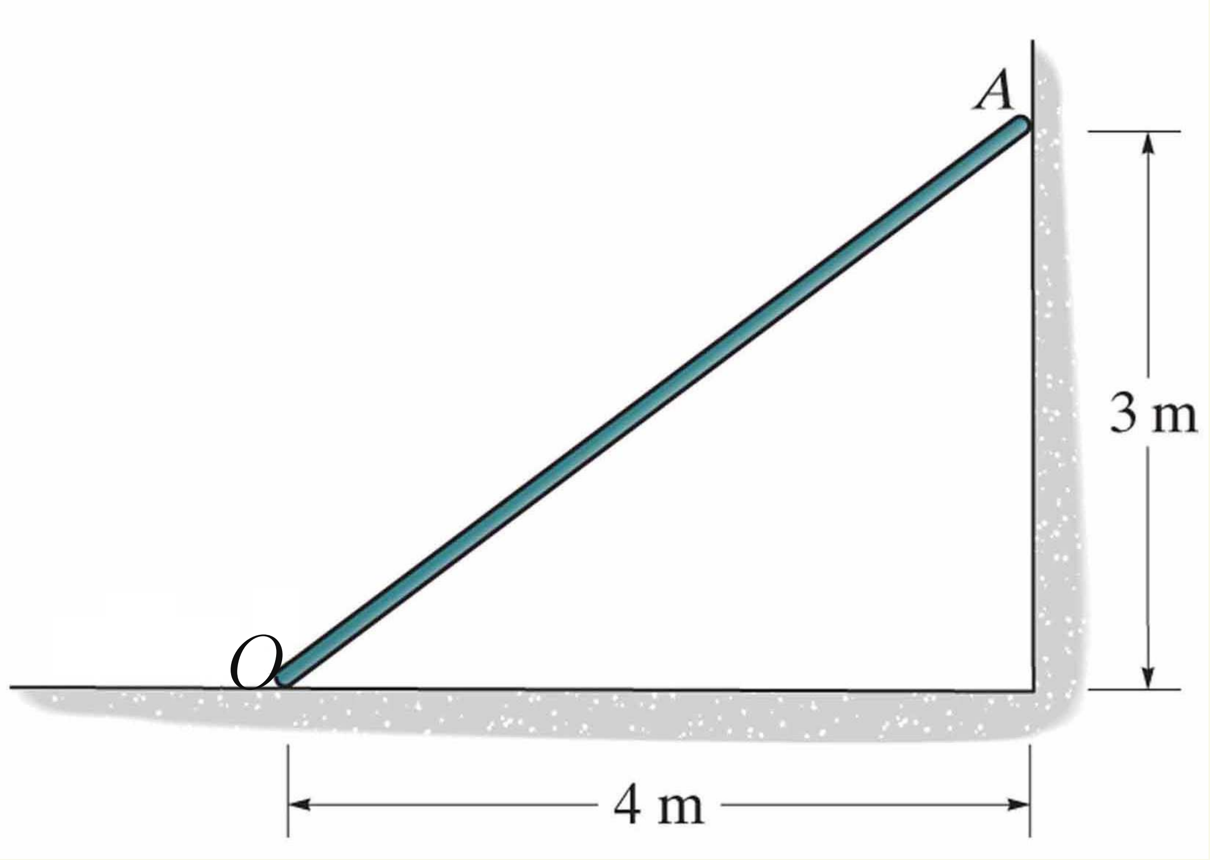

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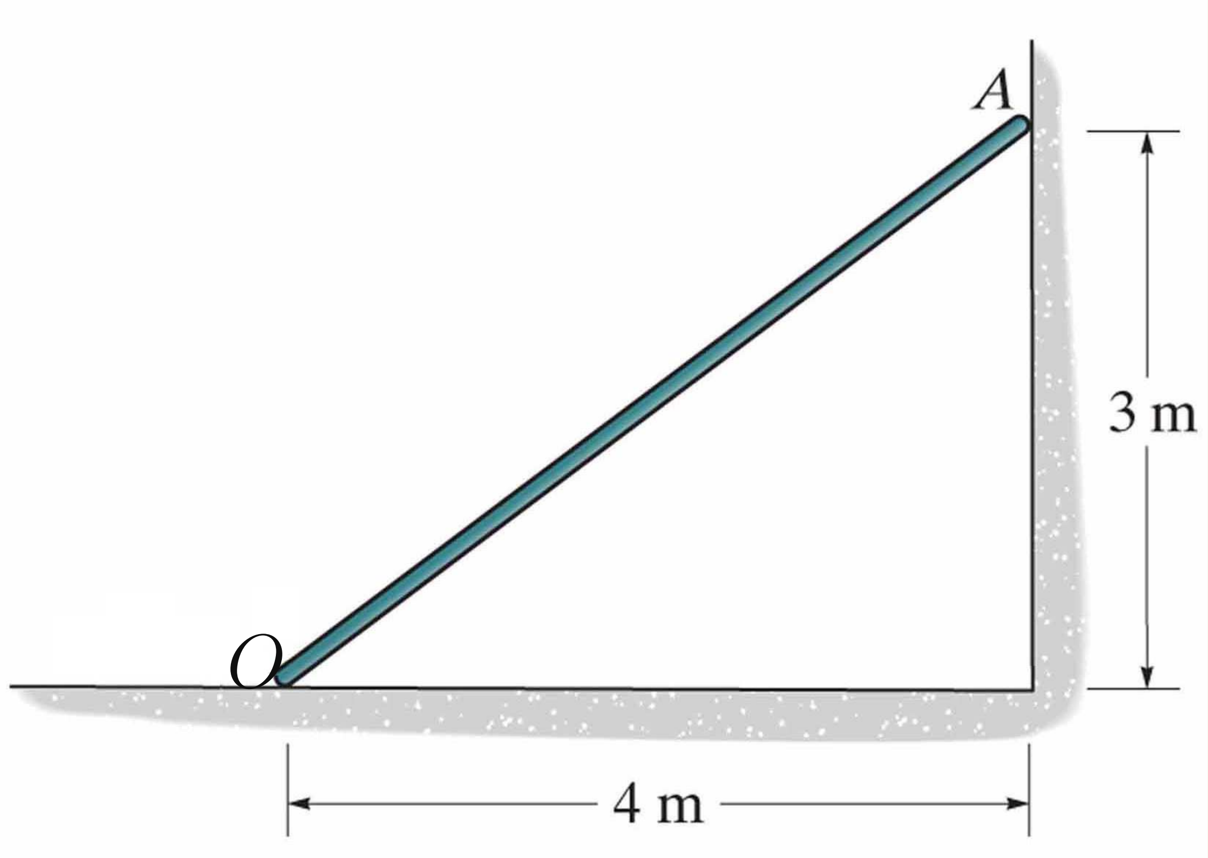

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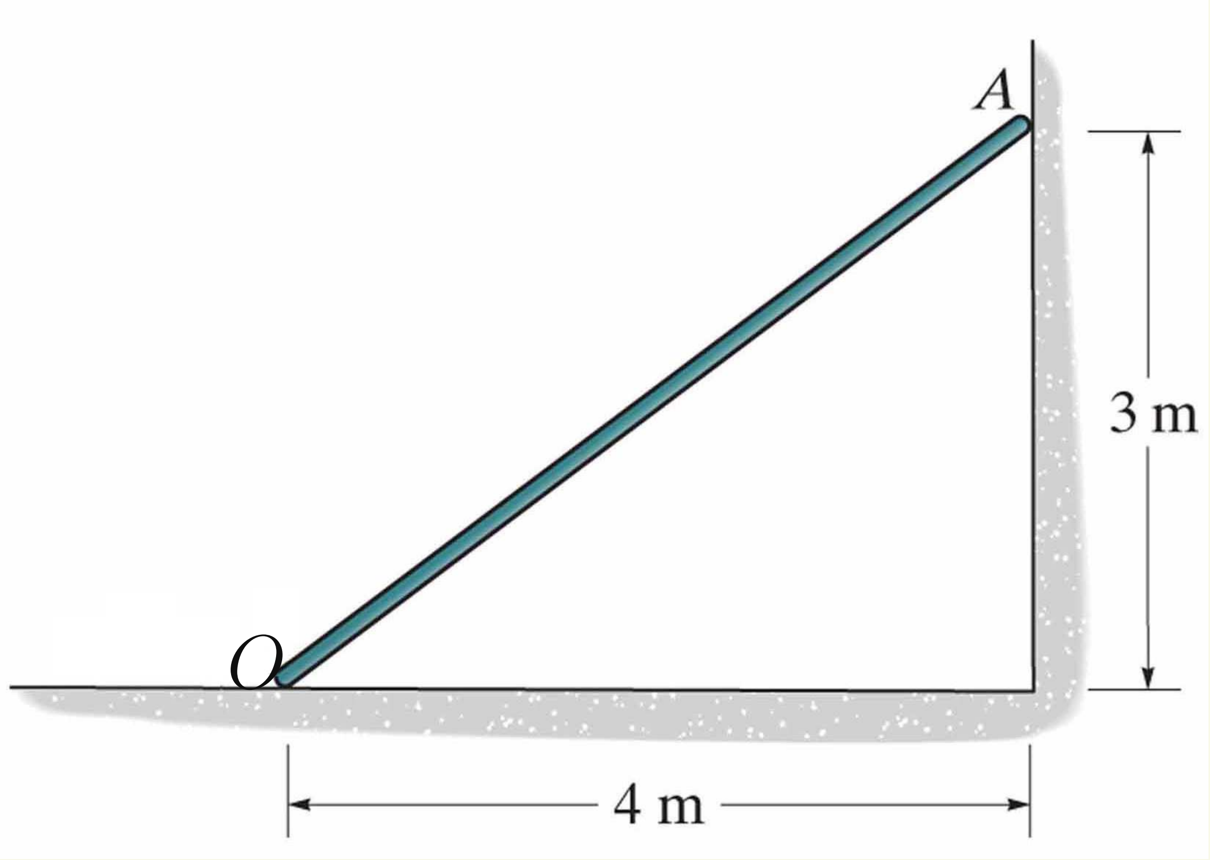

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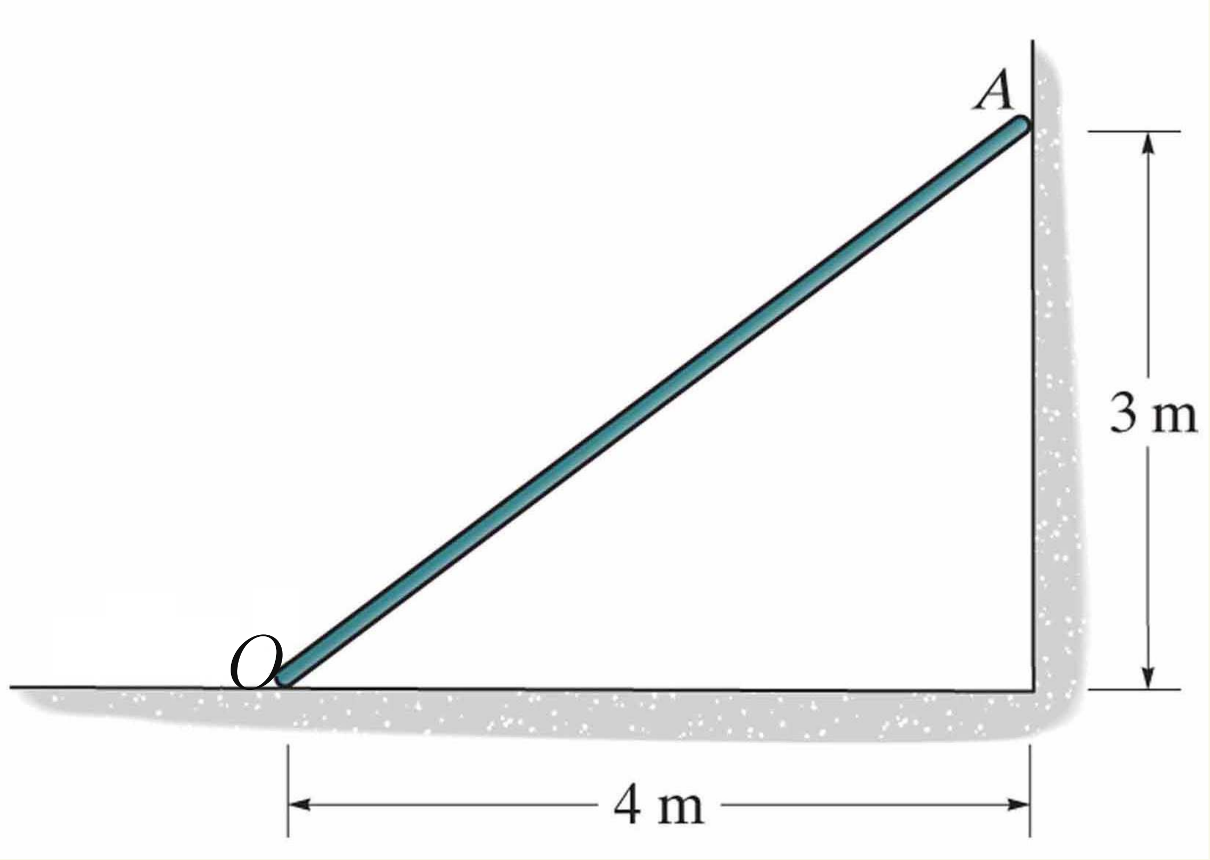

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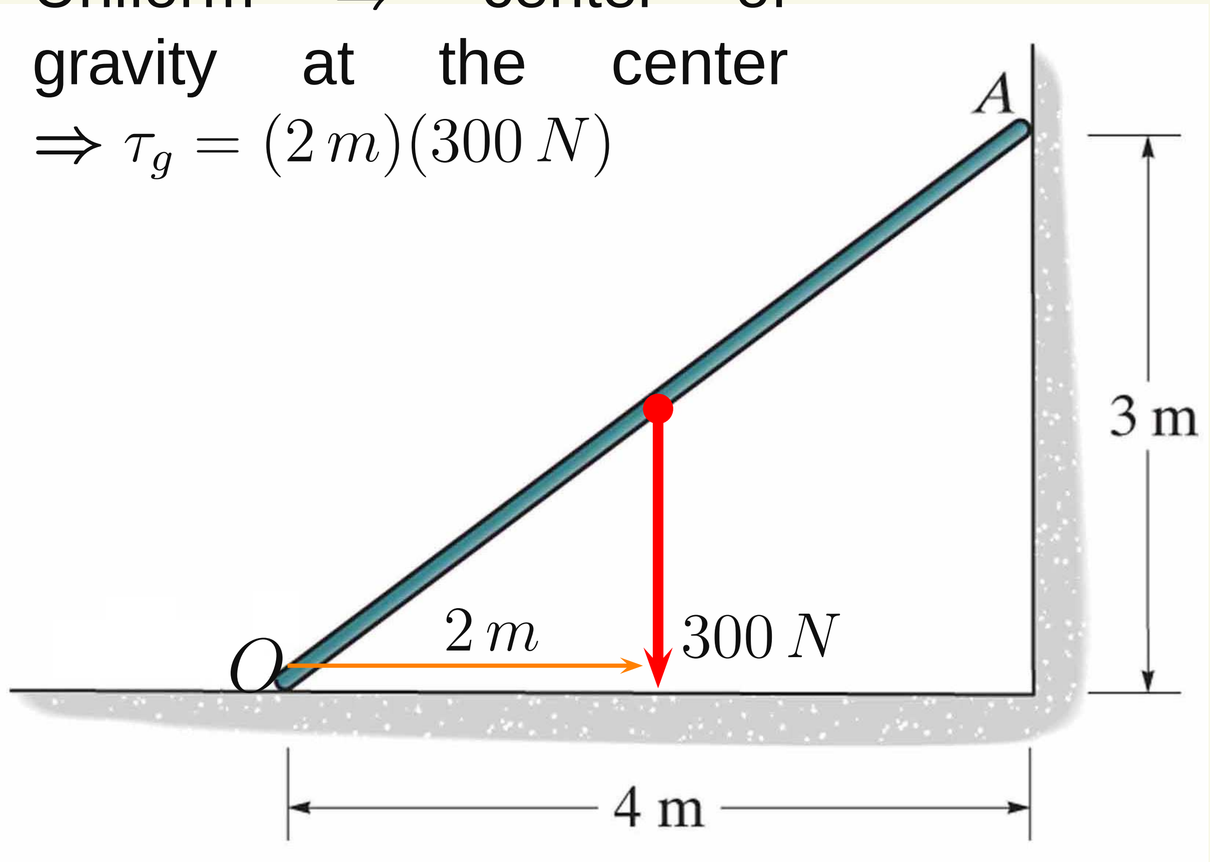
Uniform $\Rightarrow$ center of gravity at the center $\Rightarrow \tau_g = (2\,m)(300\,N)$



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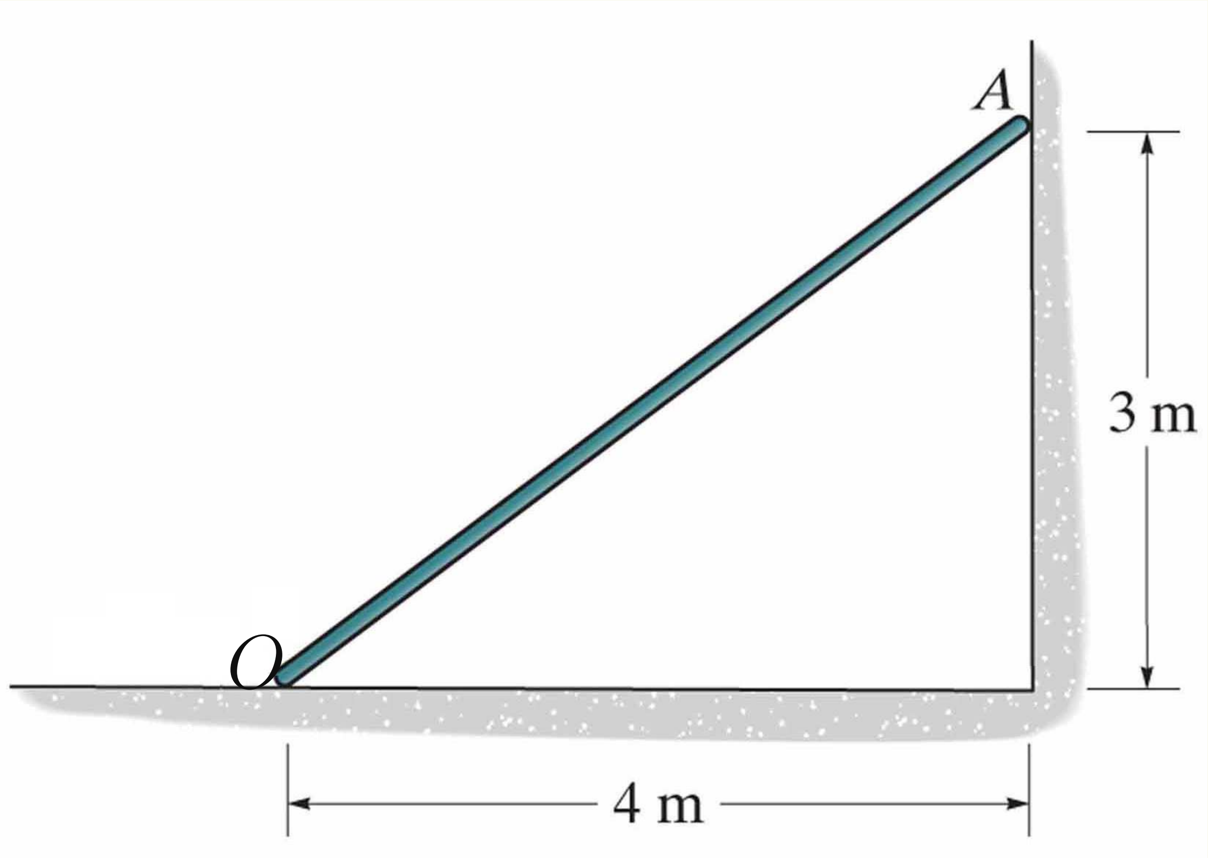
Note: The sum of the torques about *any* point must be zero.

Equilibrium Example

A 300-N uniform bar is resting against a wall as shown. There is no friction between the bar and the wall at point A, but how much friction must be acting on the bar at O?



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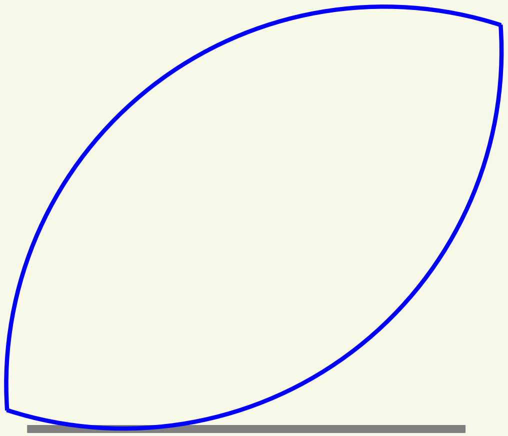

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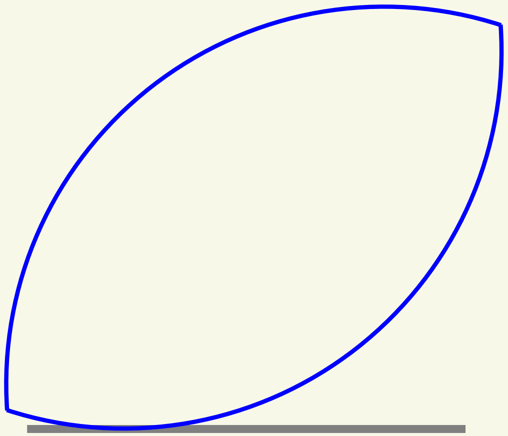

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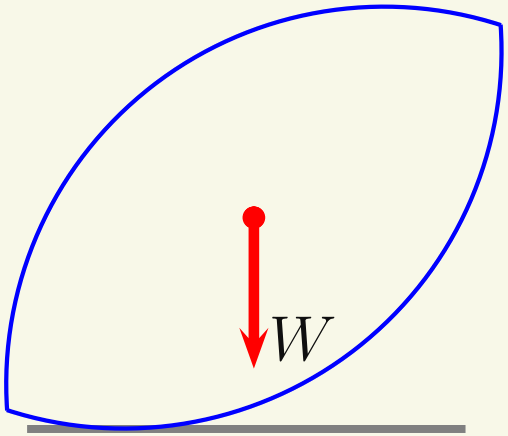

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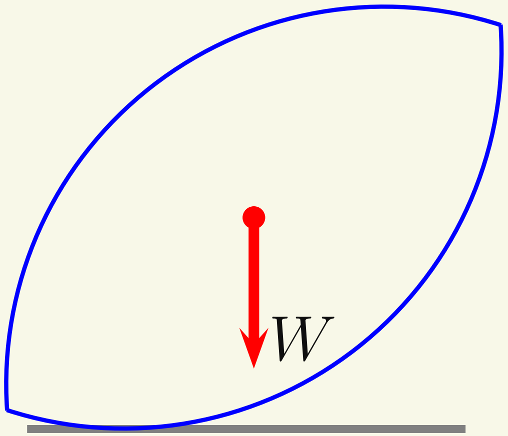

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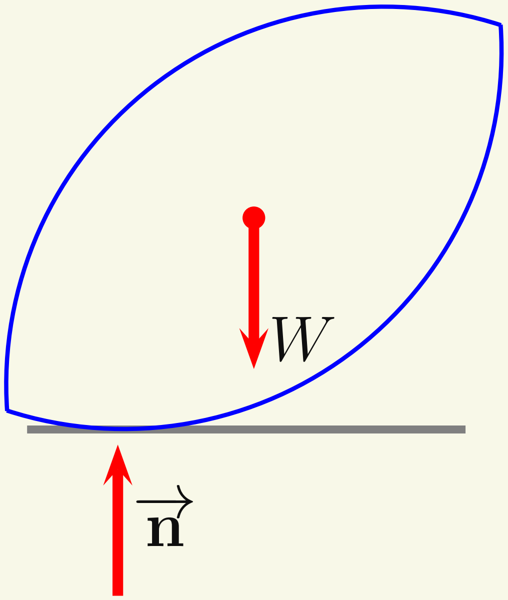

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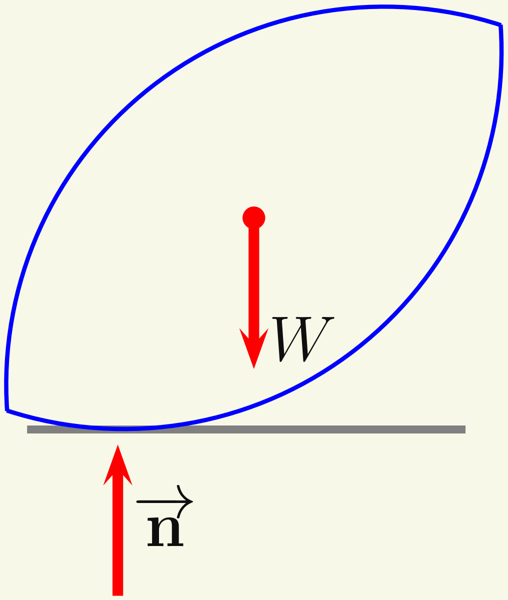

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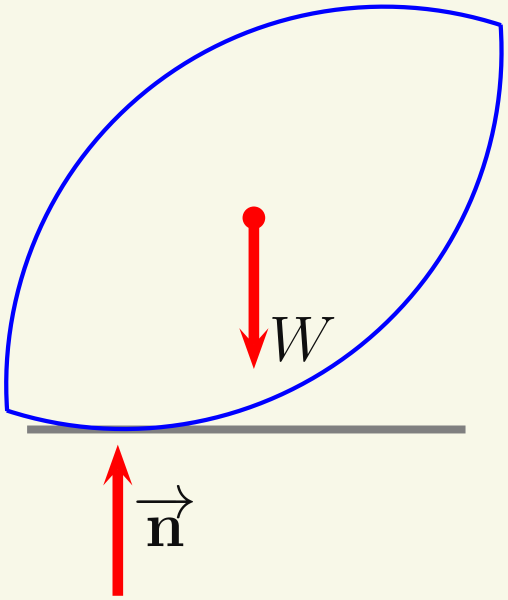

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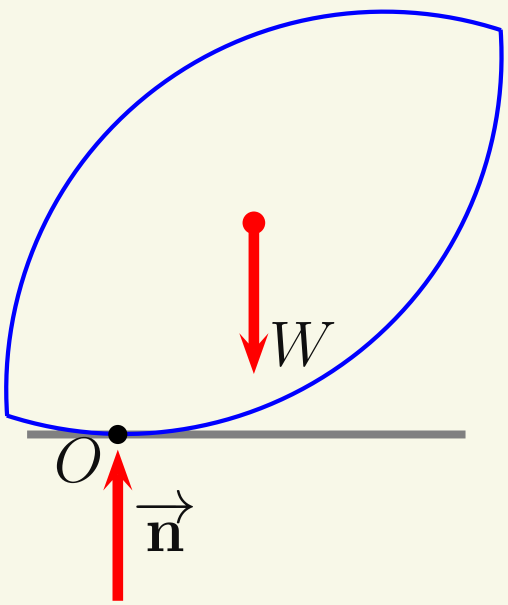

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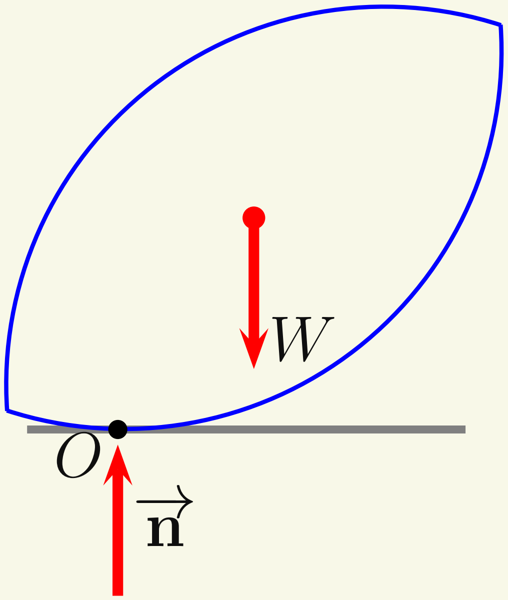

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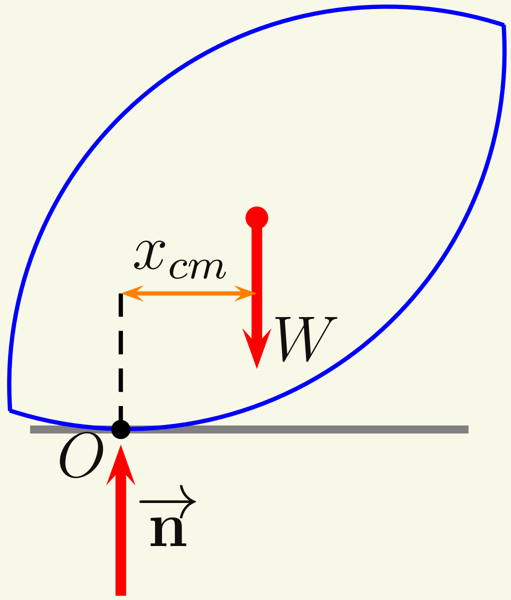

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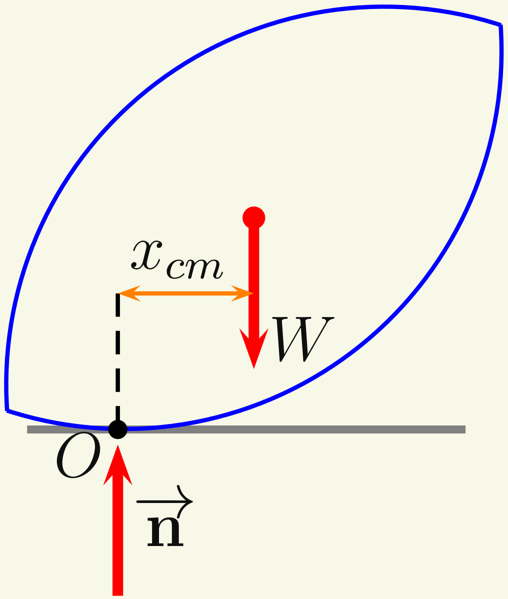

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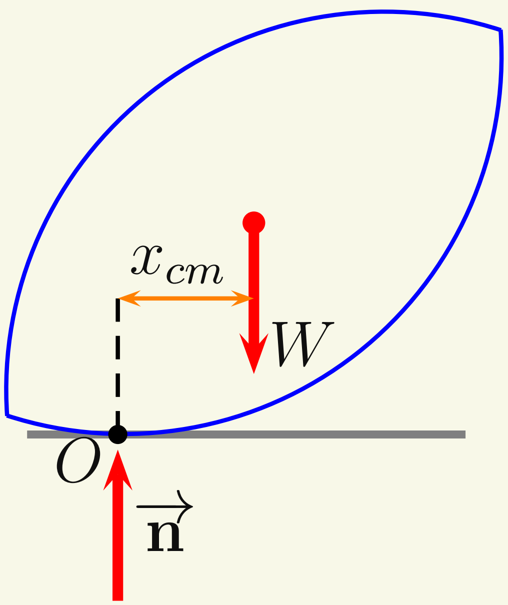

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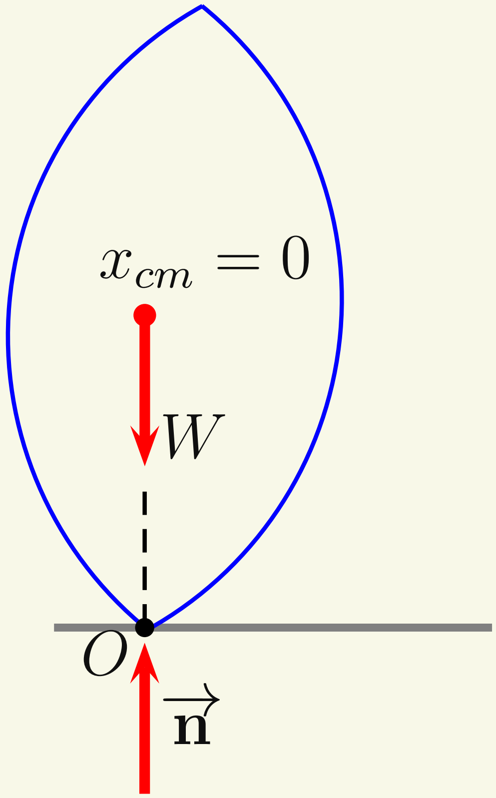

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