# July 7, Week 6

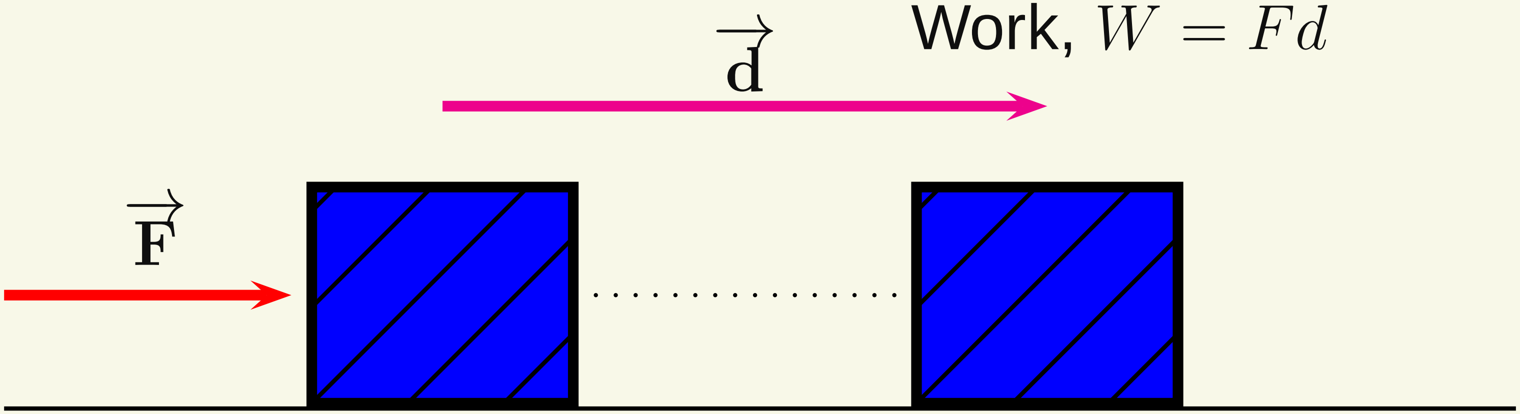
Today: Chapter 10, Gravitational Potential Energy

Homework #5 due today at 5:00PM

Office hours today: 1:00-5:00

Test #5, tomorrow.

# Restrictions




Work, $W = Fd$

This equation is correct only in the situation that:

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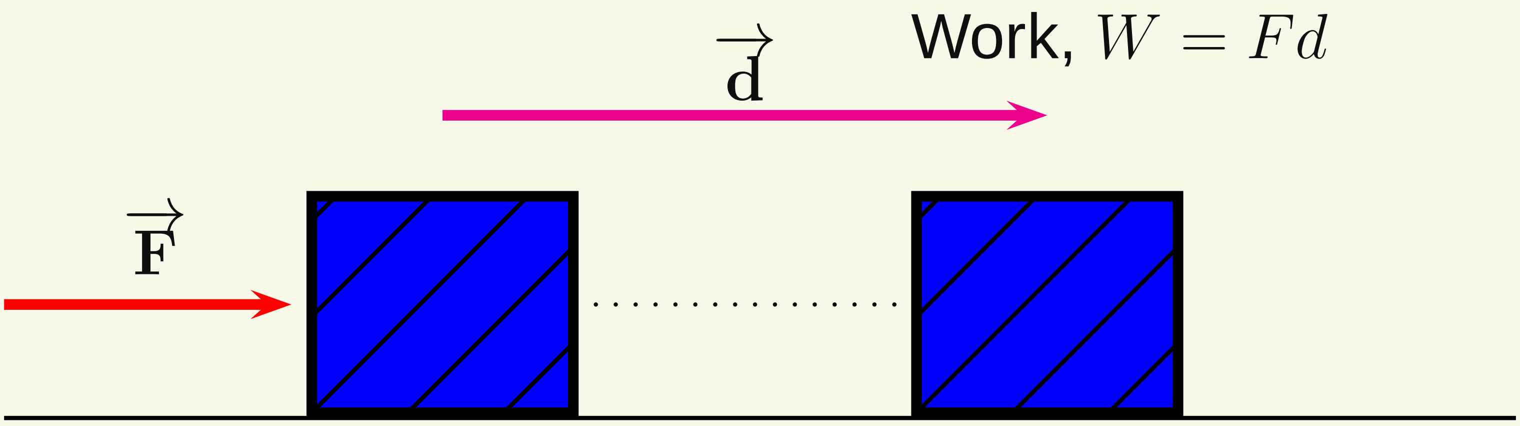


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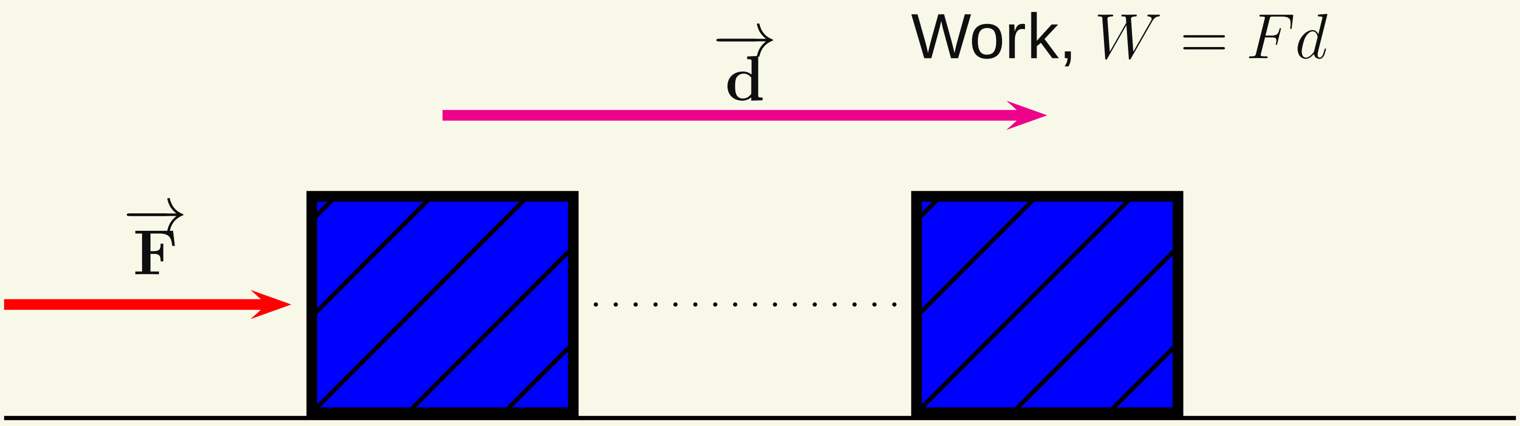


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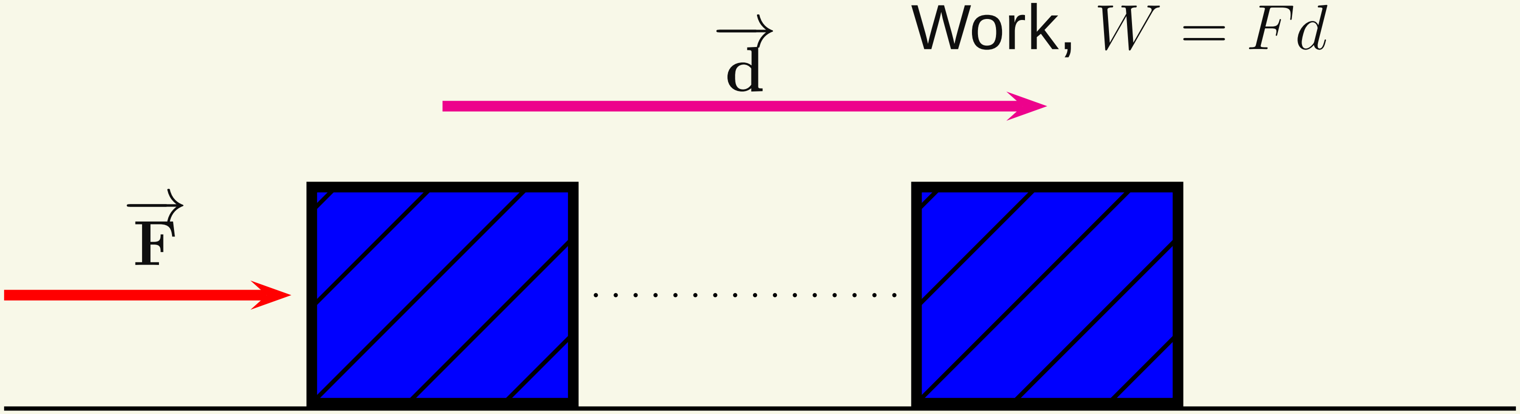


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$\overrightarrow{\mathbf{F}}$ is constant

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$\overrightarrow{\mathbf{F}}$ and $\overrightarrow{\mathbf{d}}$ are in the same direction.

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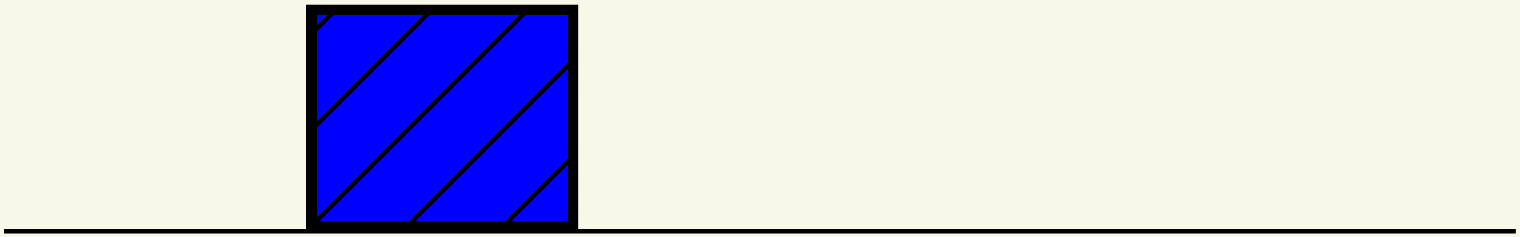

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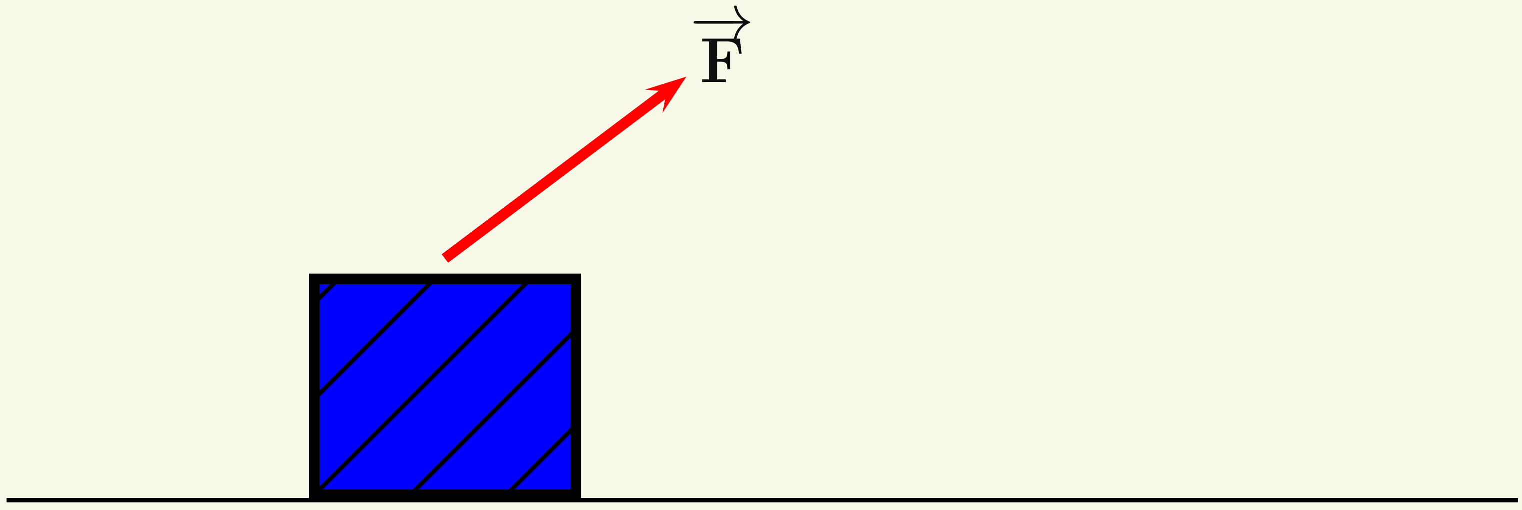

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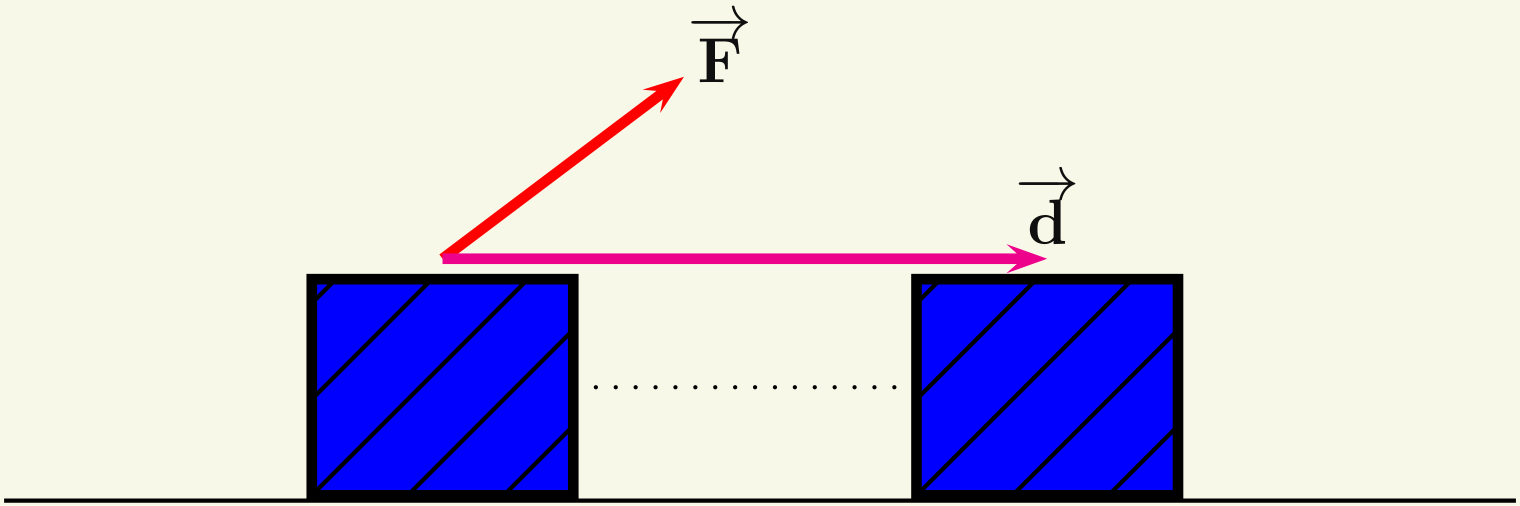

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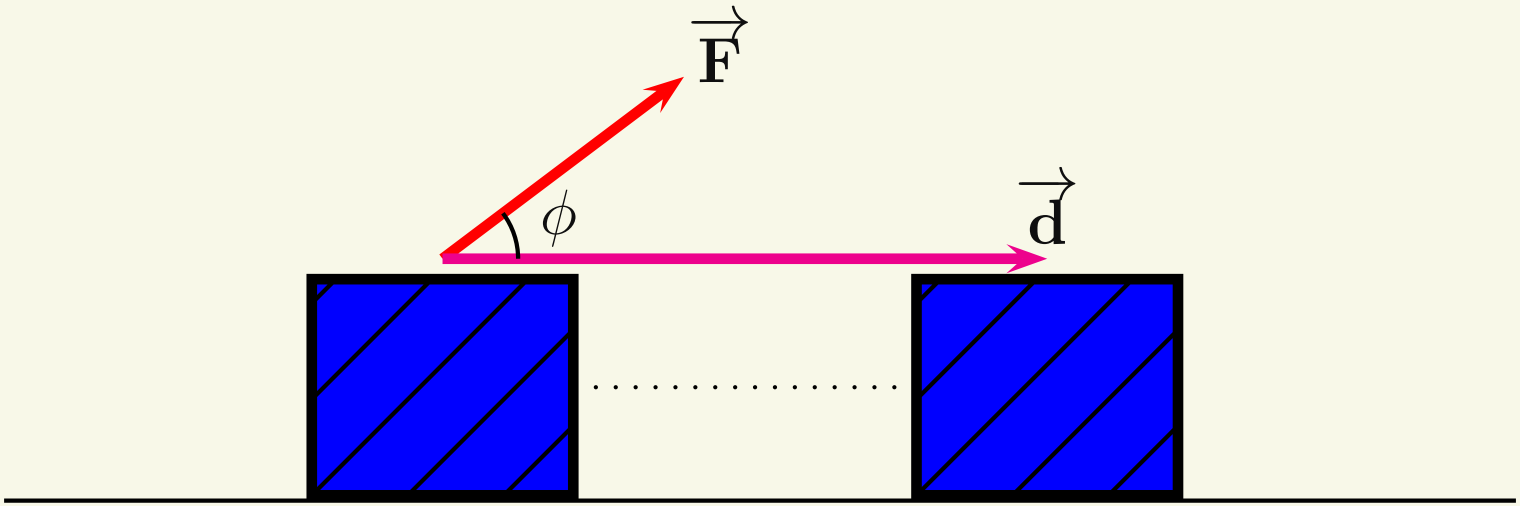

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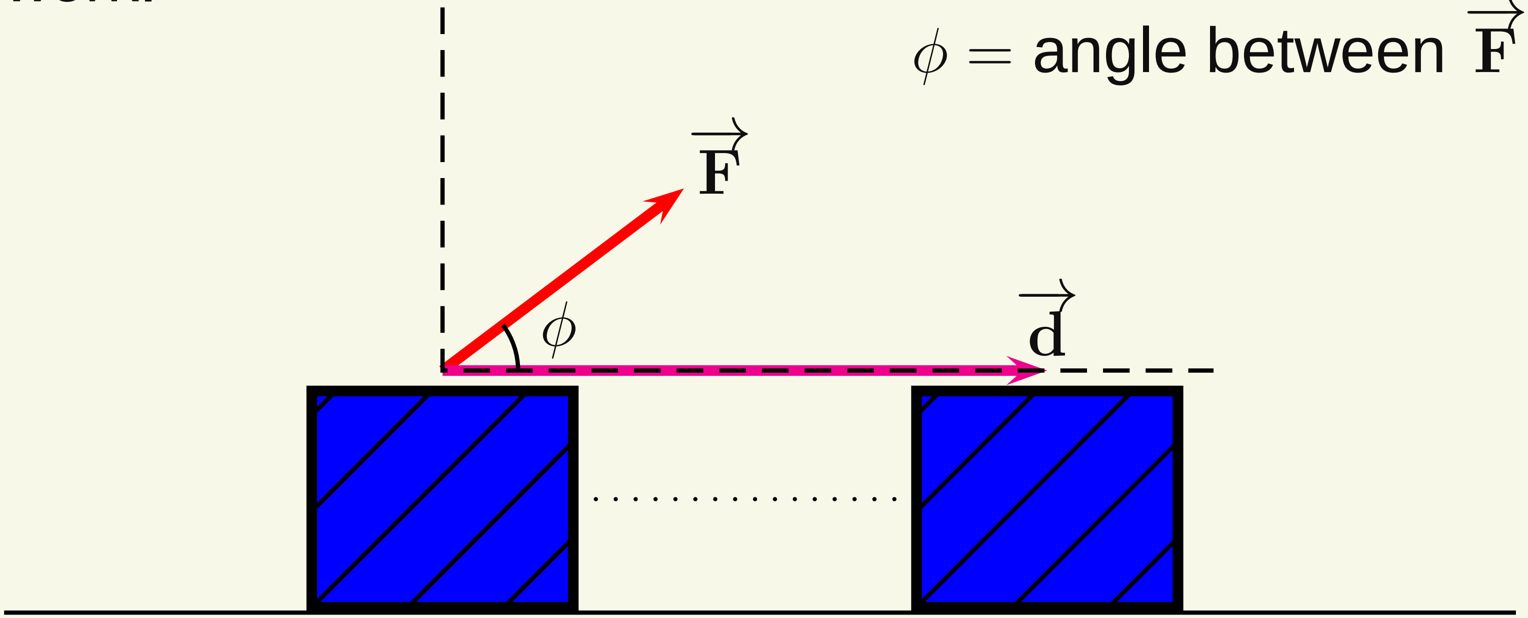

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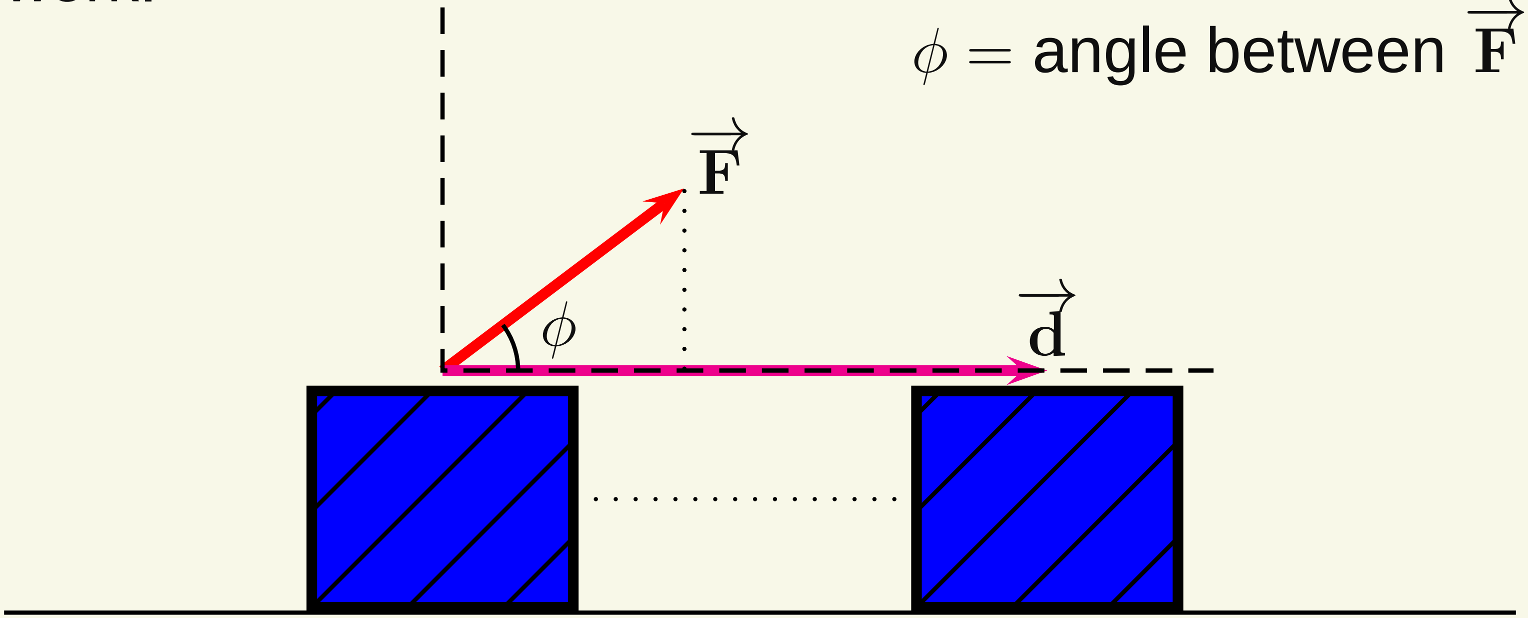

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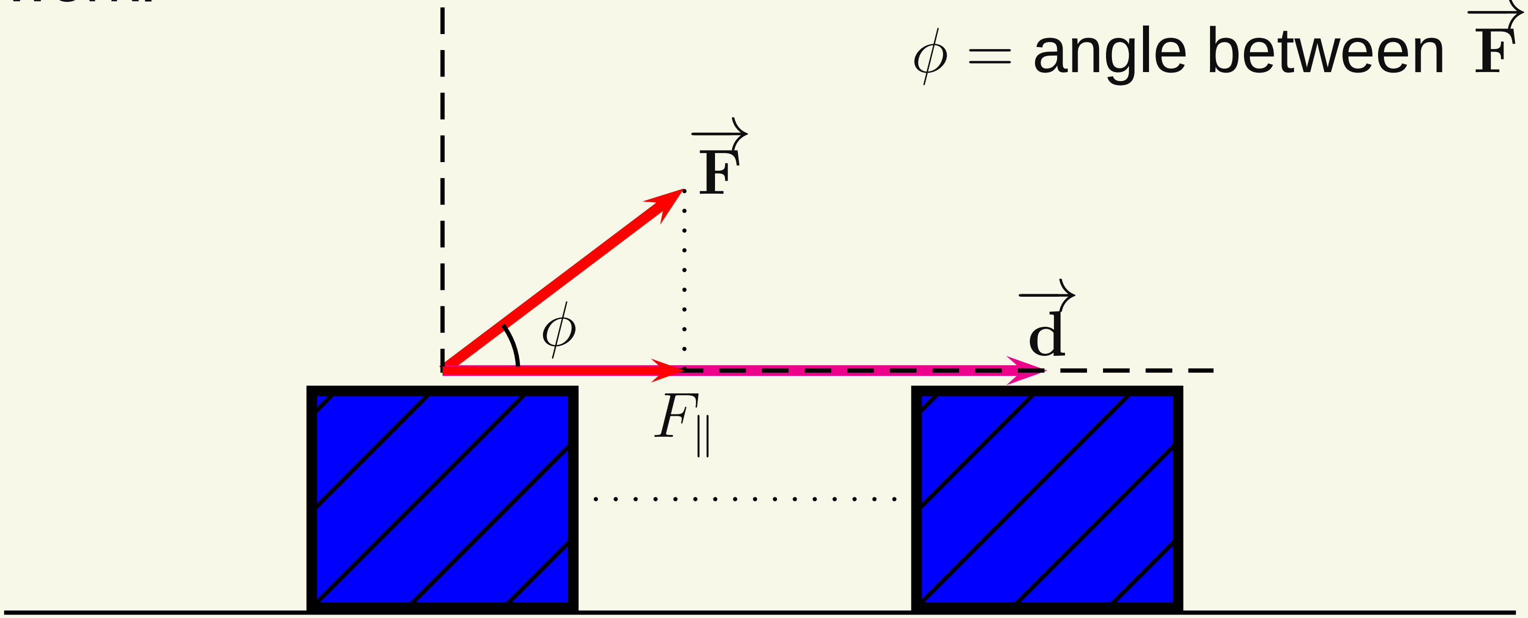

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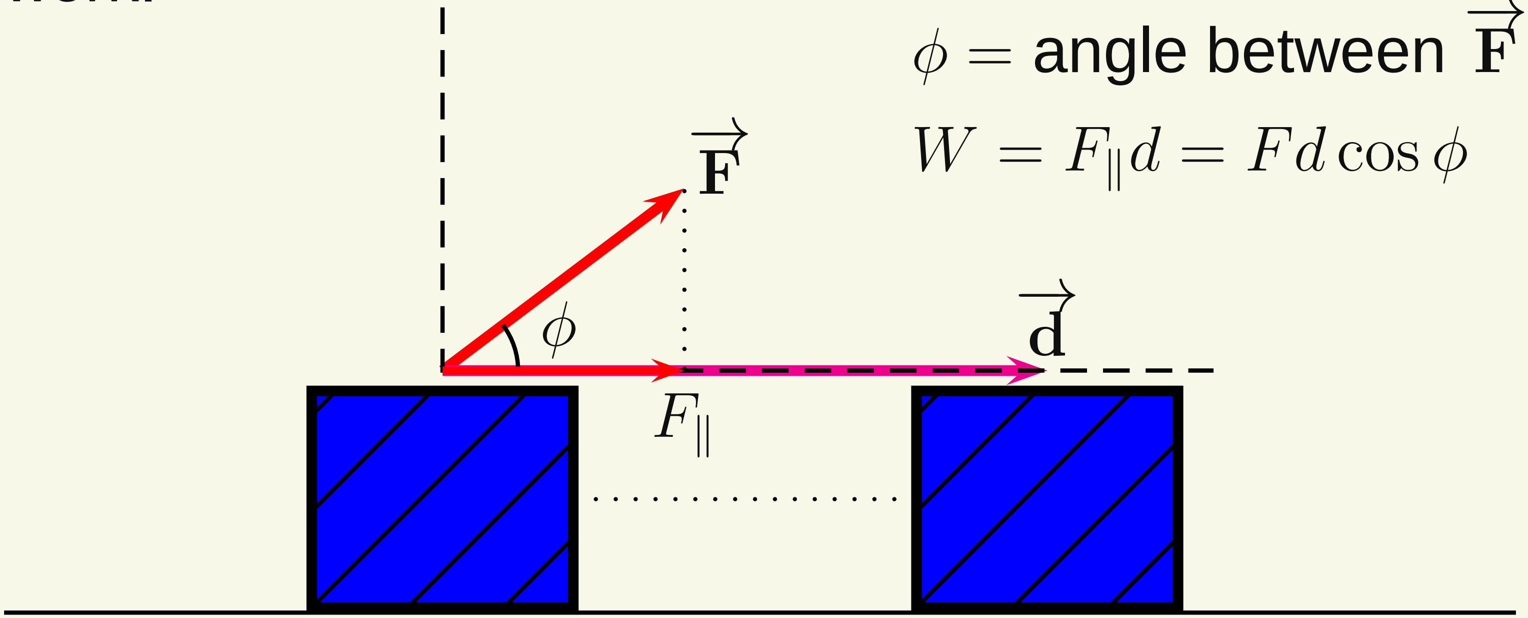

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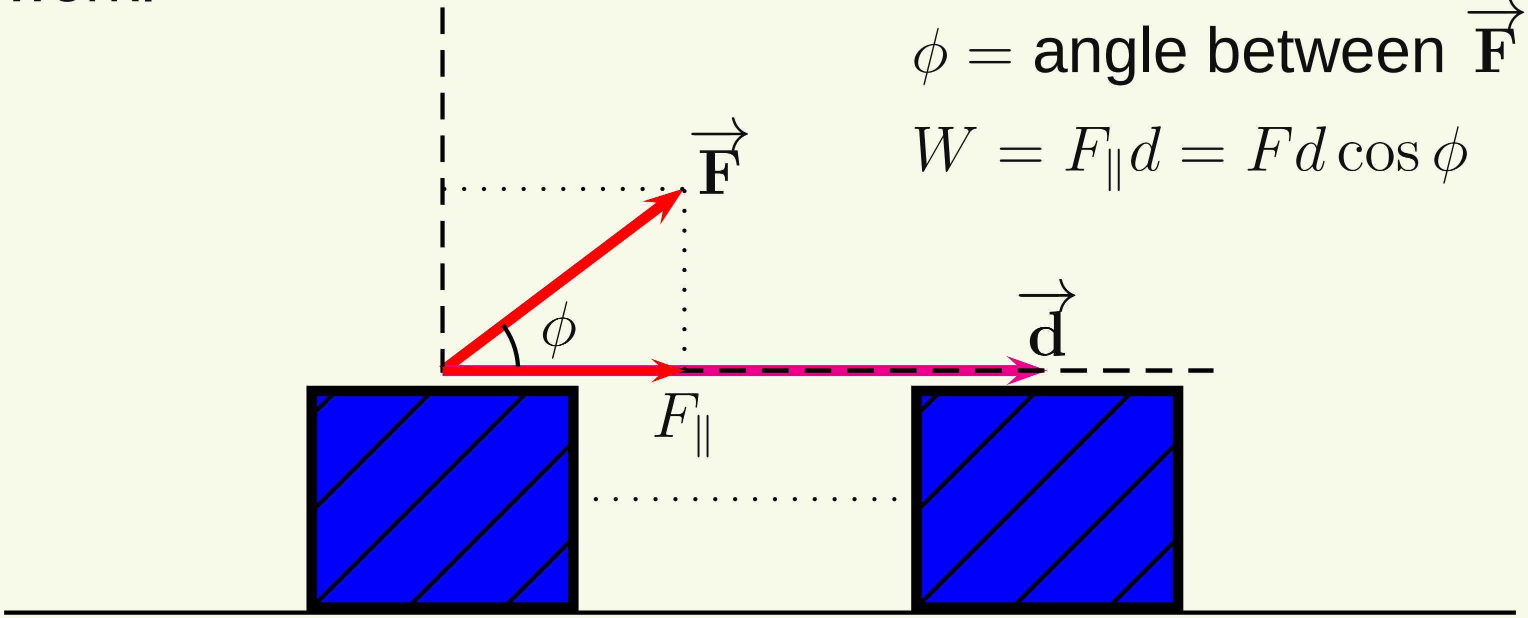


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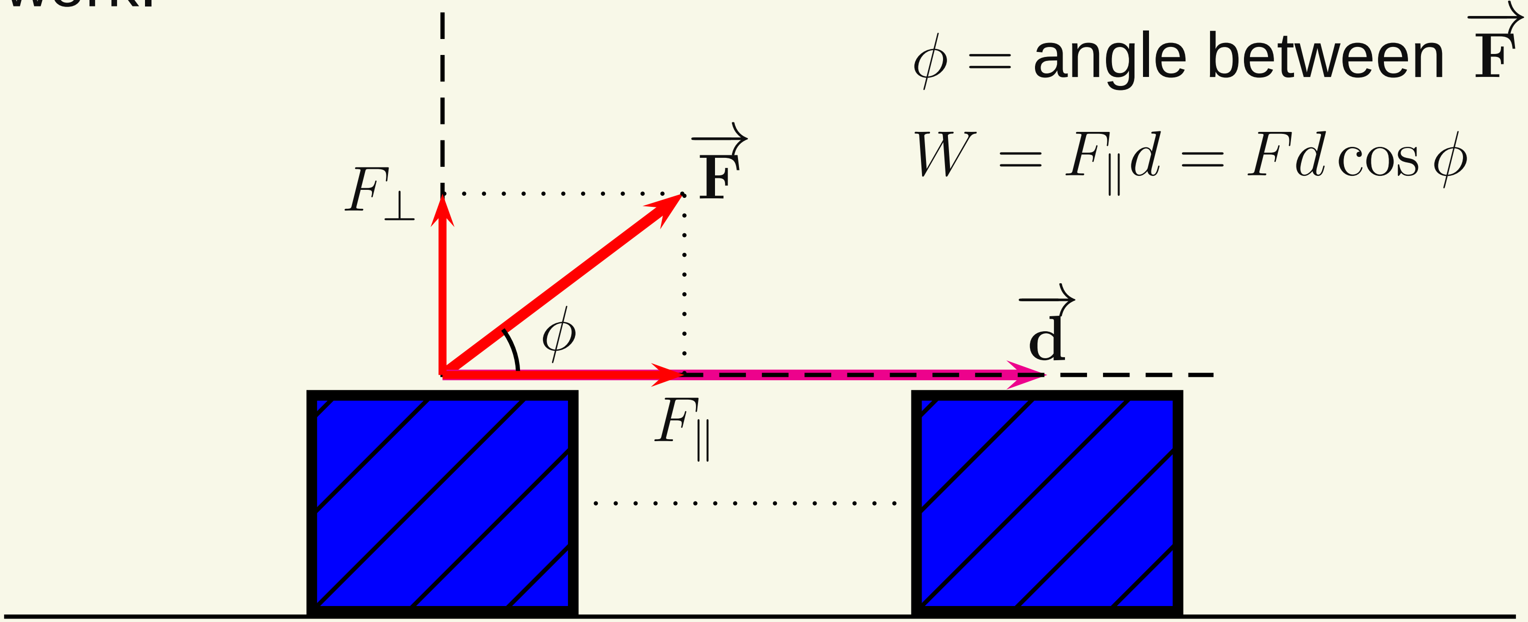

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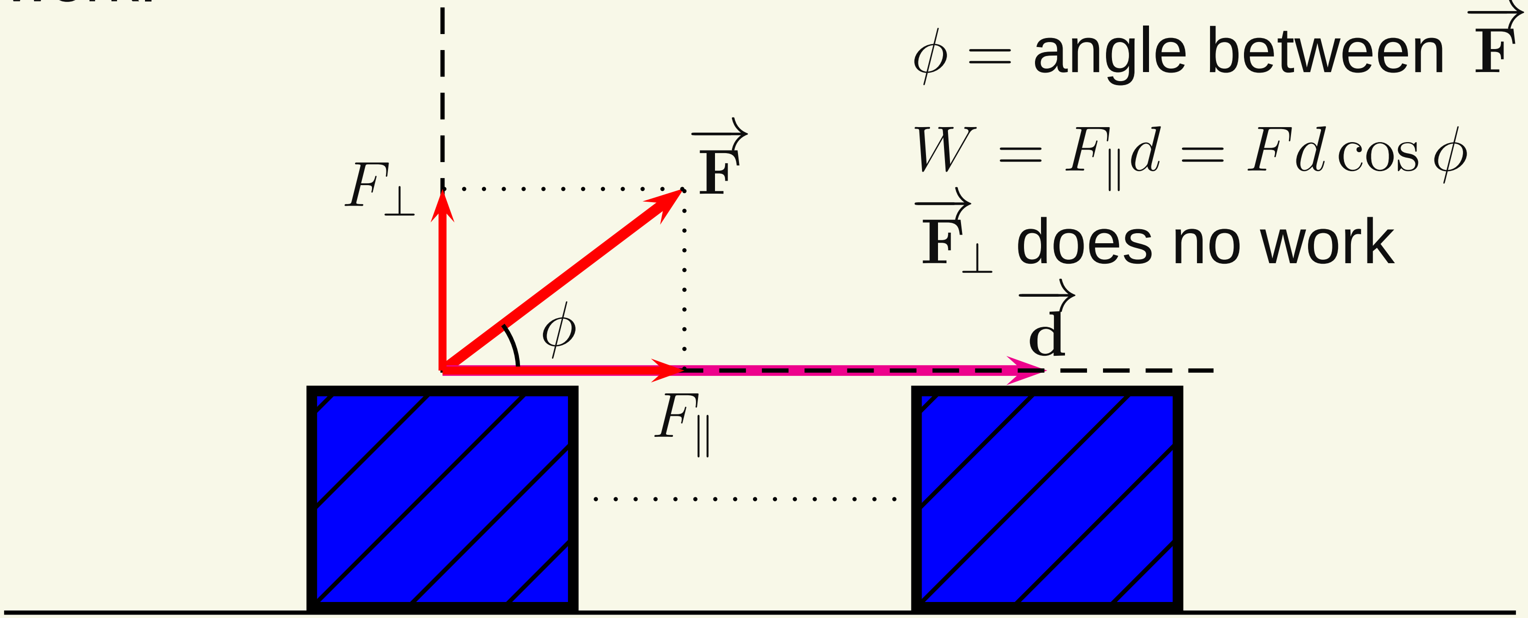

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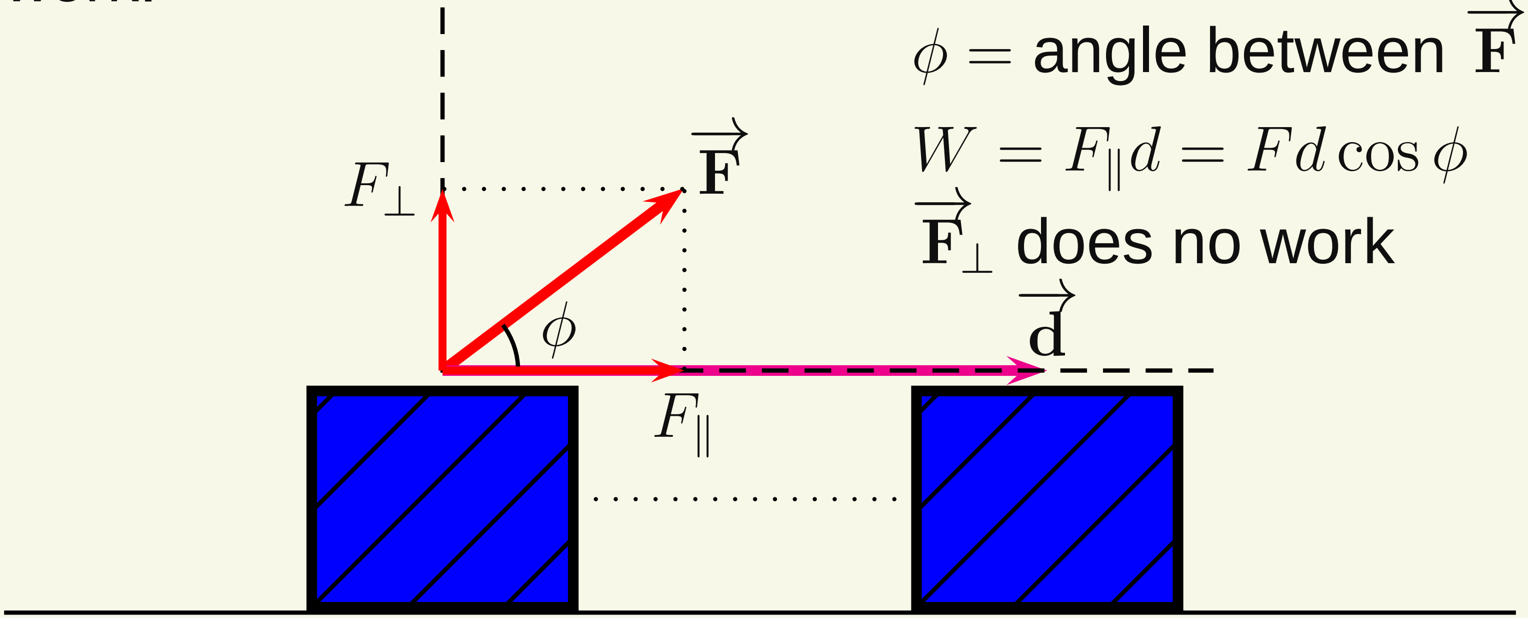


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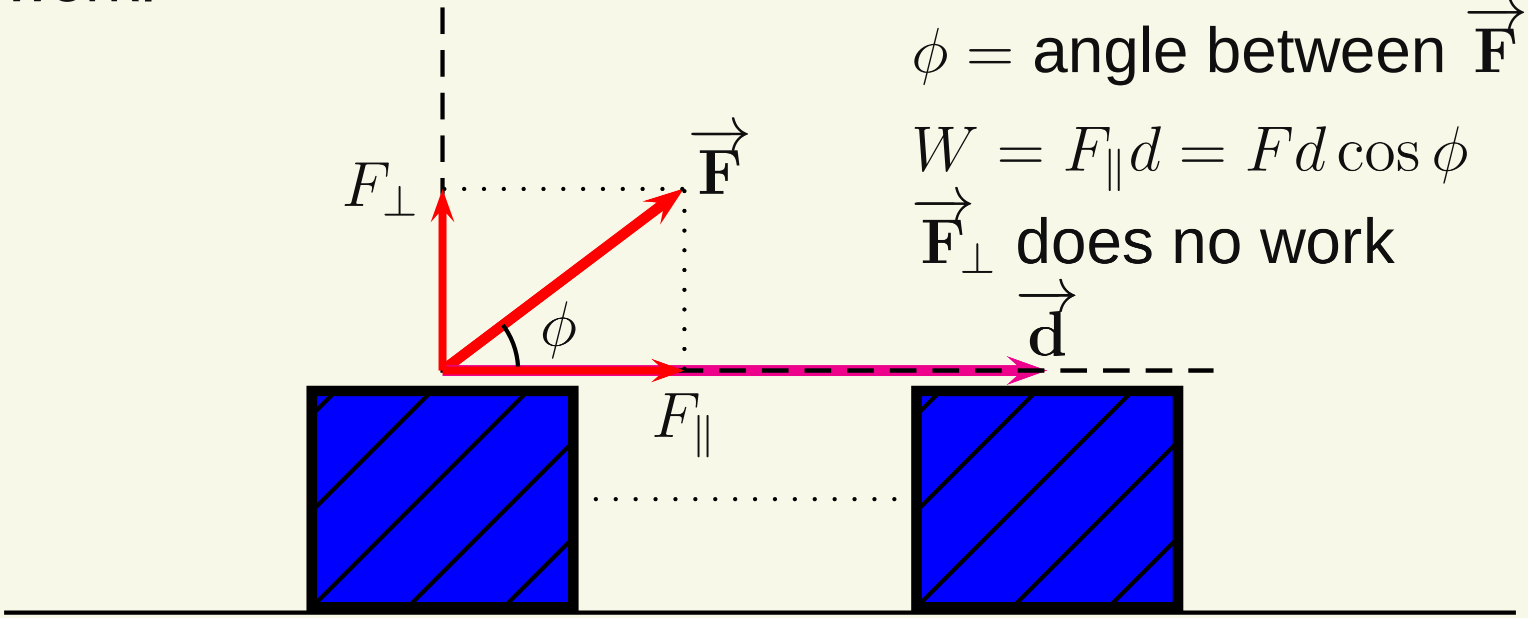


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Only correct for Constant force & Straight-line displacement

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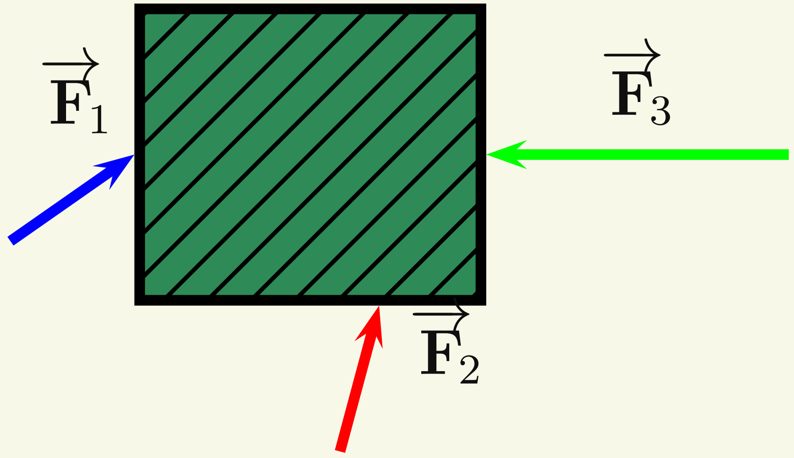


$$W_{total} = W_1 + W_2 + W_3 + \ldots$$

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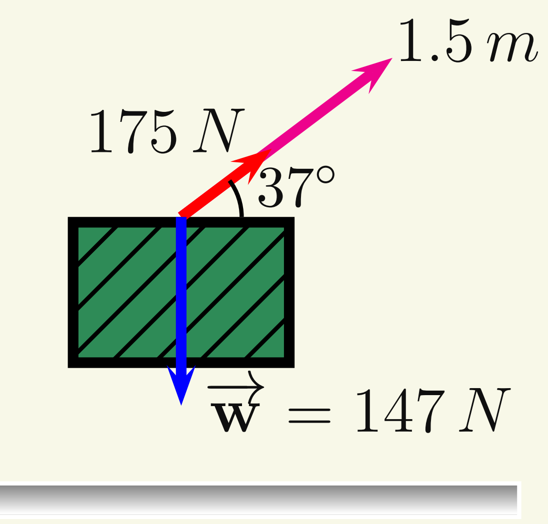
A $15\,kg$ mass is lifted $1.5\,m$ at $37°$ by applying a constant $175\,N$ force also at $37°$. What is the *total* work done?



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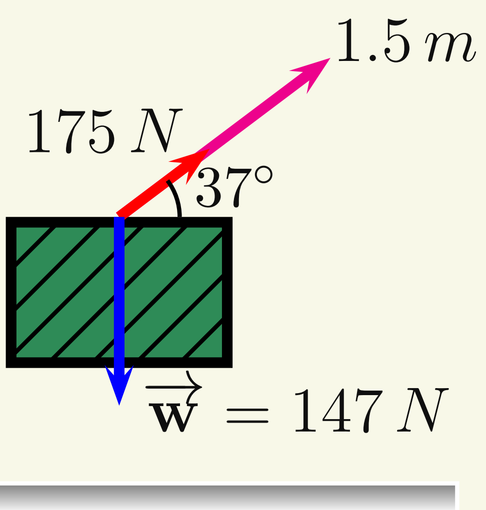

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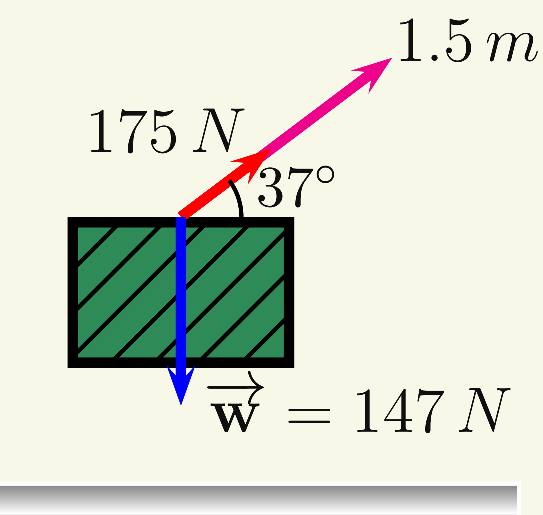

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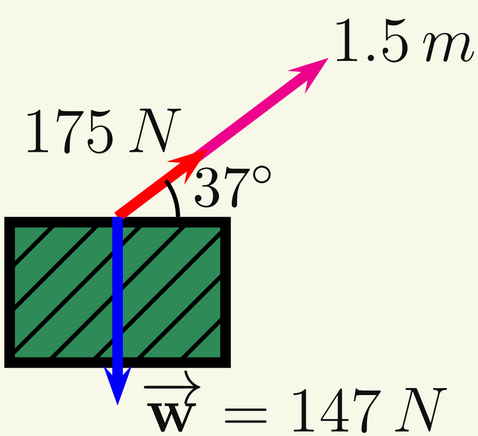

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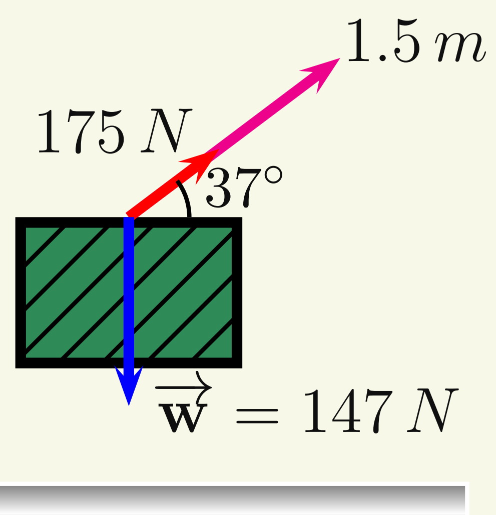


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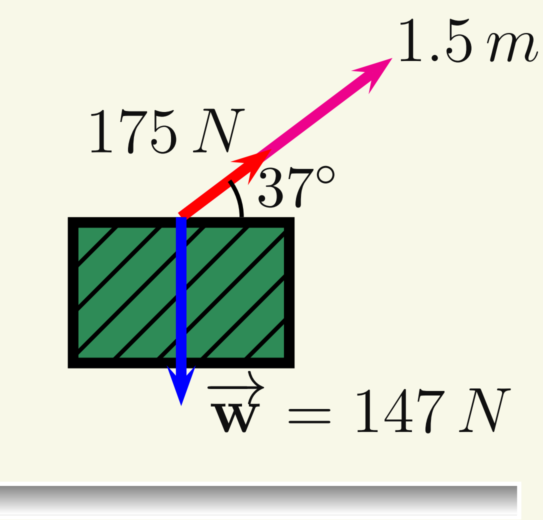


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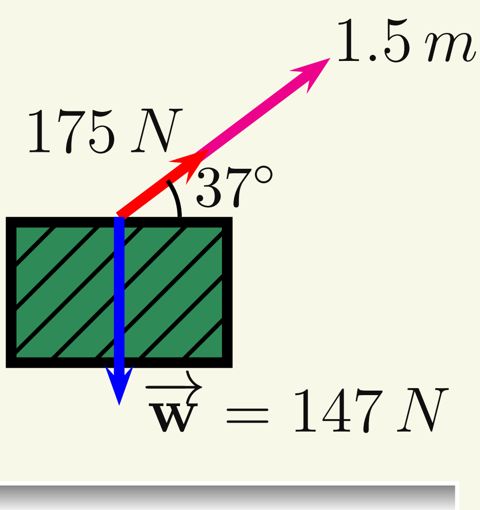


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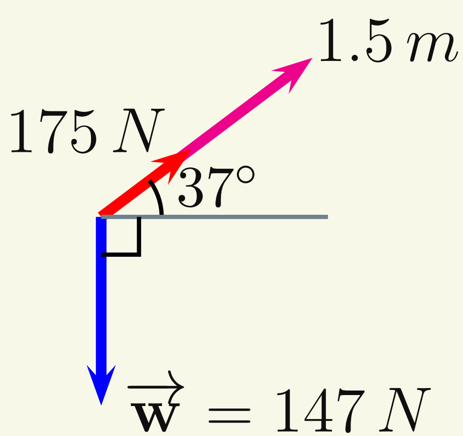


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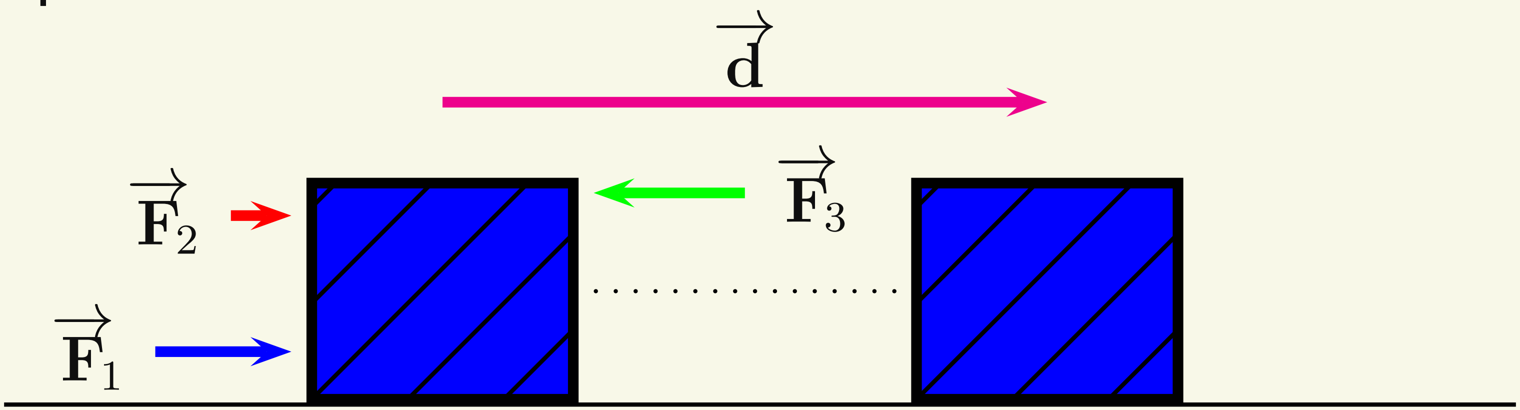

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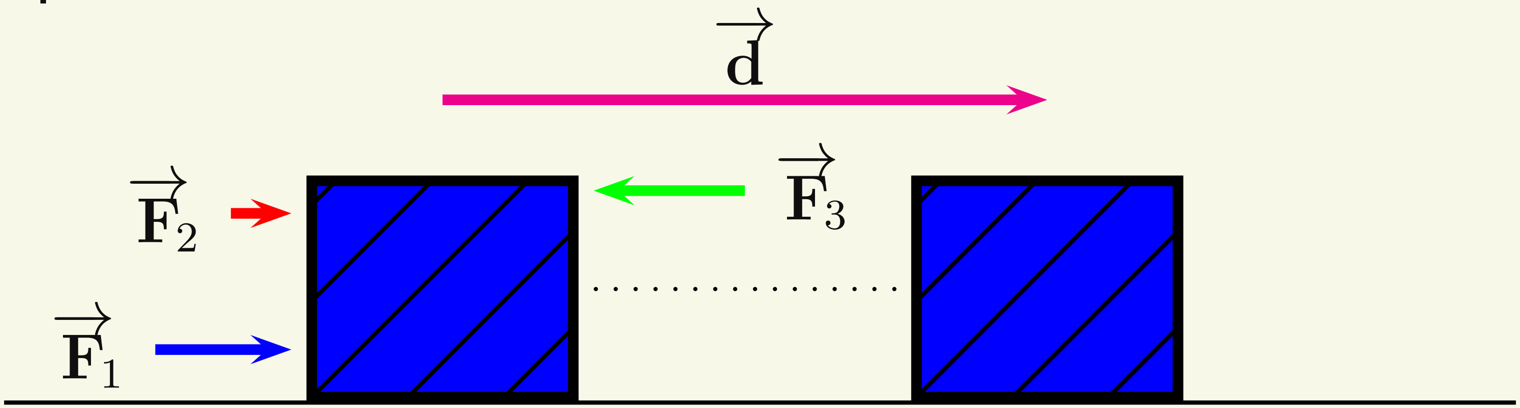


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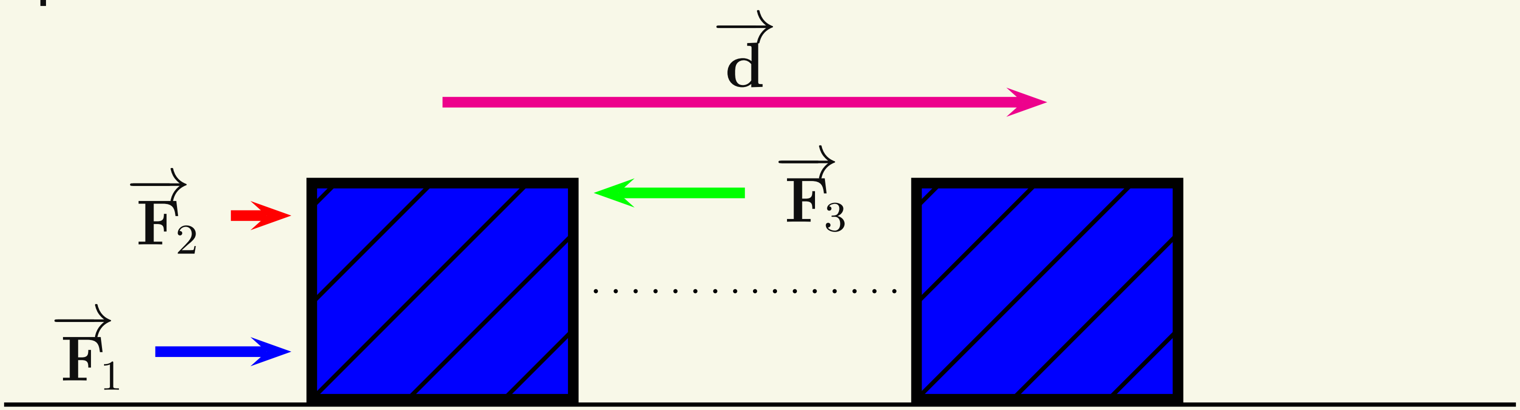


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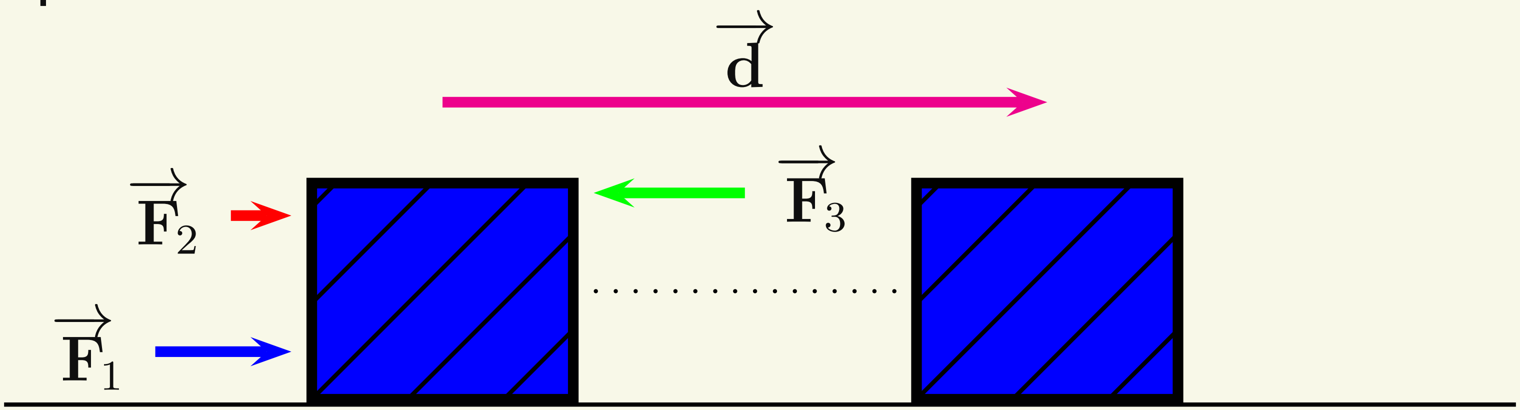


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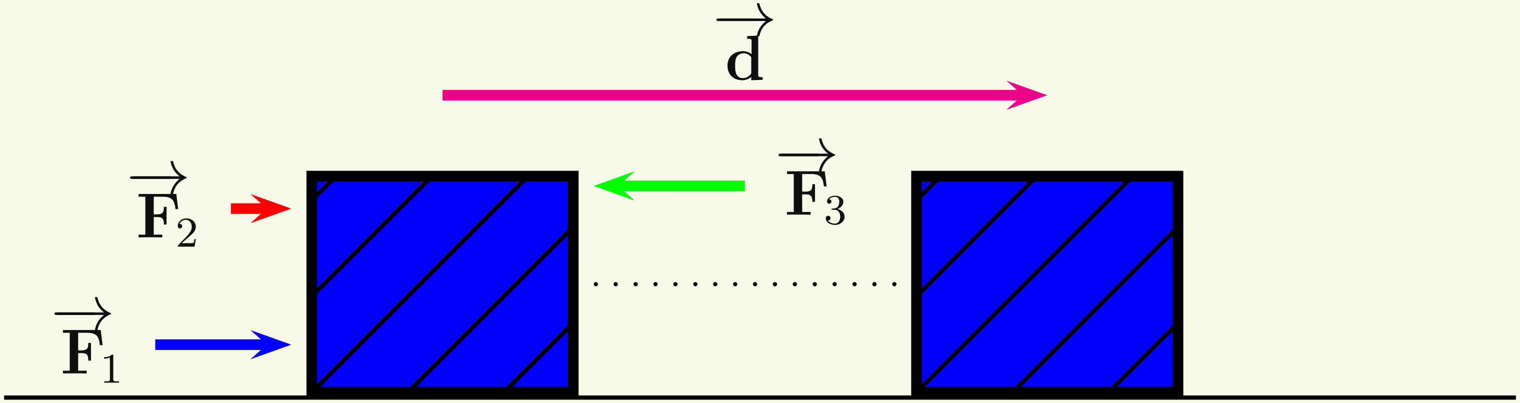


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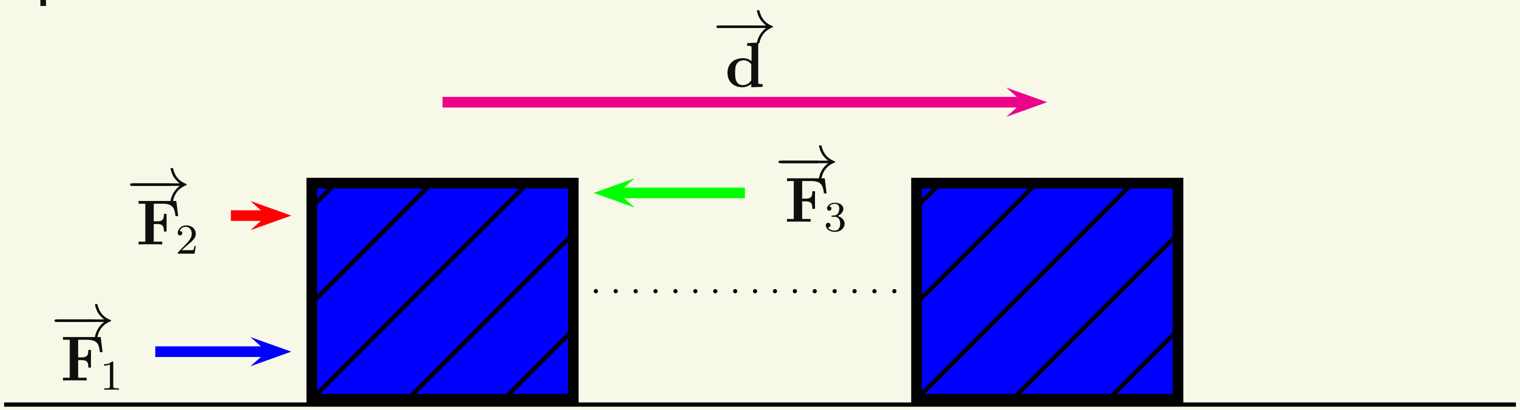


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Using $v_f^2 = v_i^2 + 2ad \Rightarrow \boxed{W_{total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$

# Work-Energy Theorem II

It can be shown that for constant forces in *ANY* direction that:

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Example: A $15\,kg$ box is lifted, from rest, by applying a $175\,N$ force at $37°$. How fast will the box be going after it has moved $1.5\,m$? (Remember that the total work done was $129.8\,J$.)

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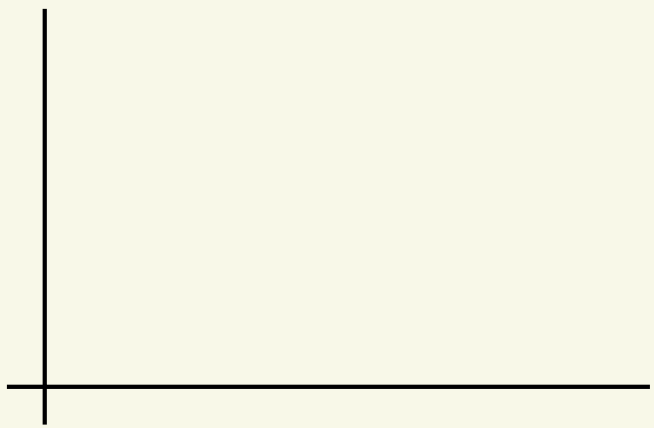

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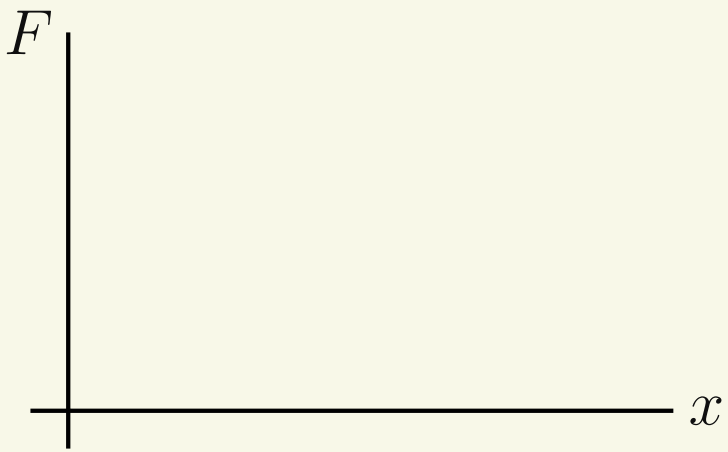

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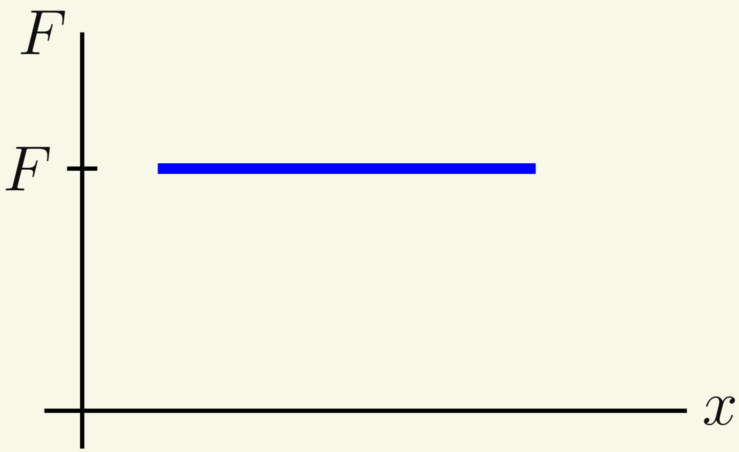

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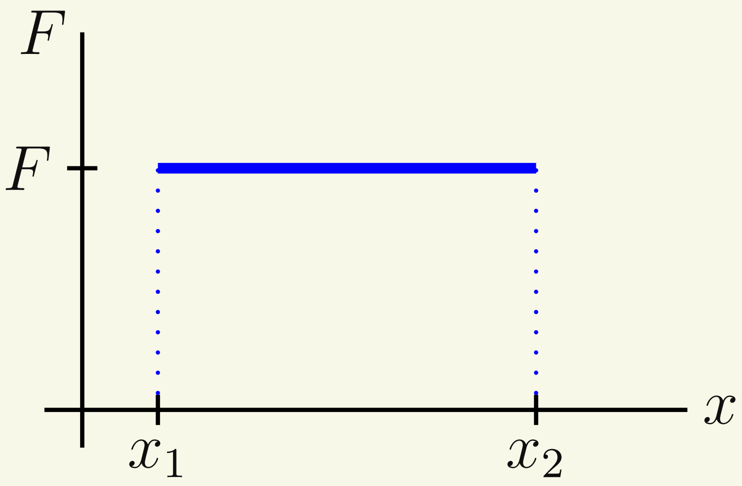

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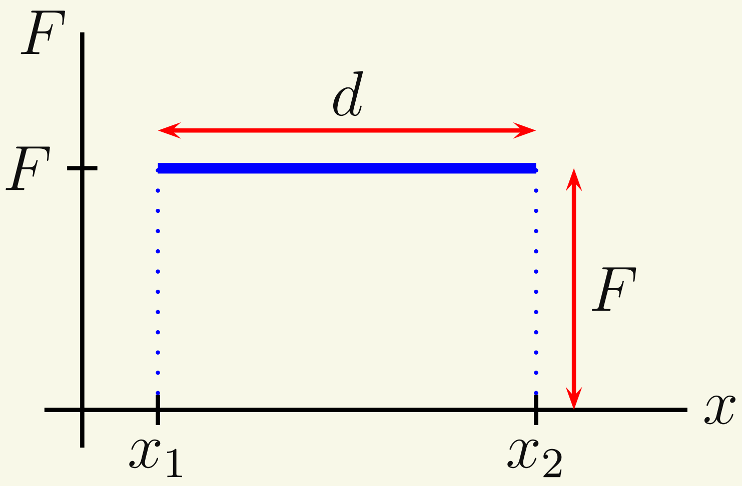

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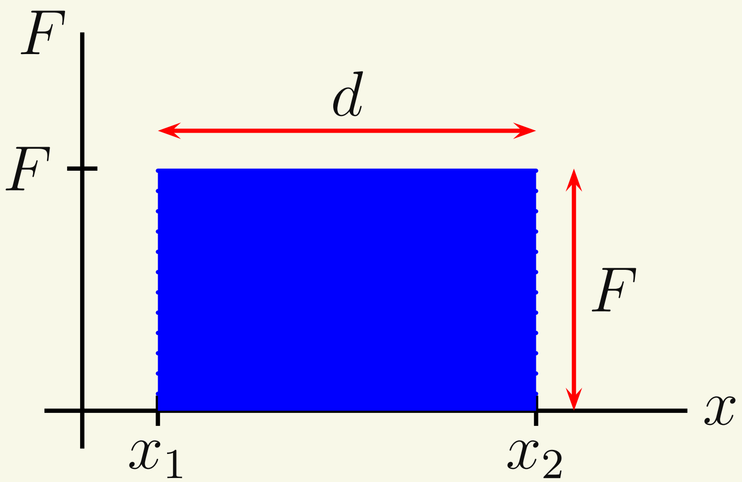

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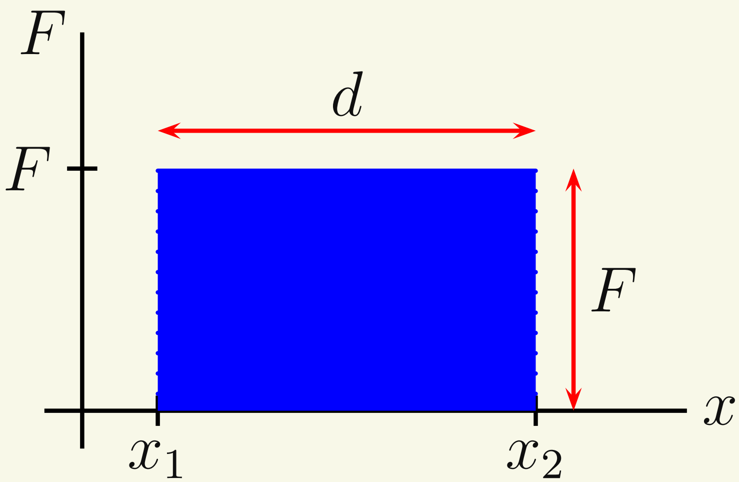


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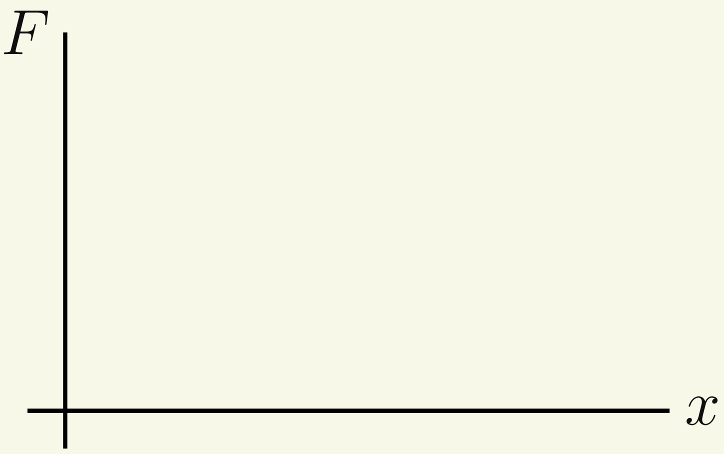

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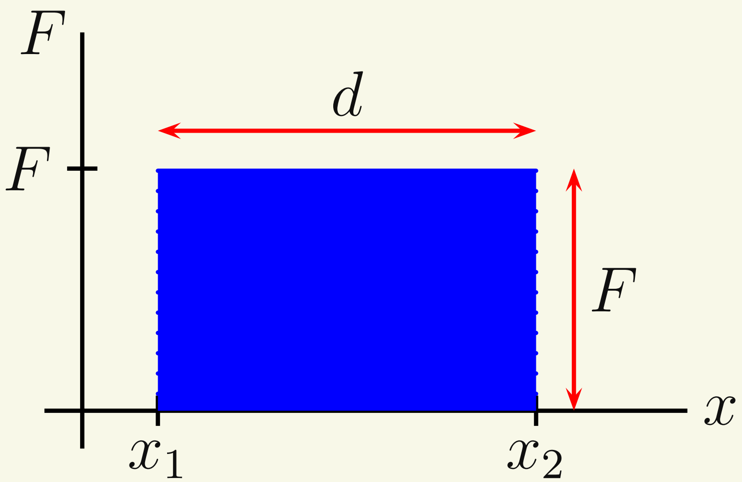


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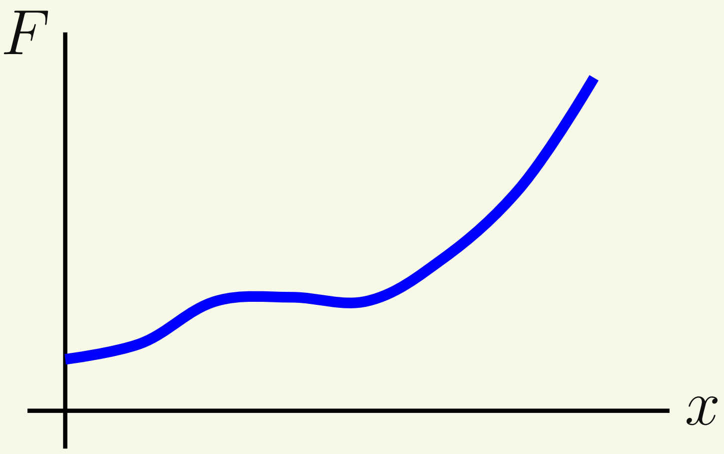

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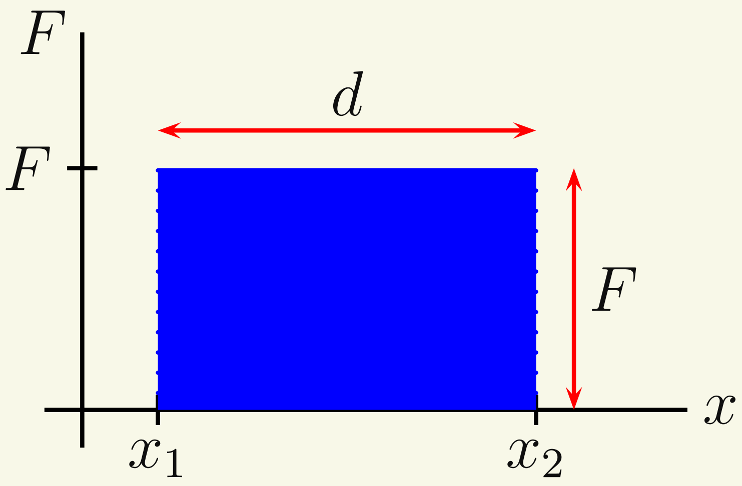


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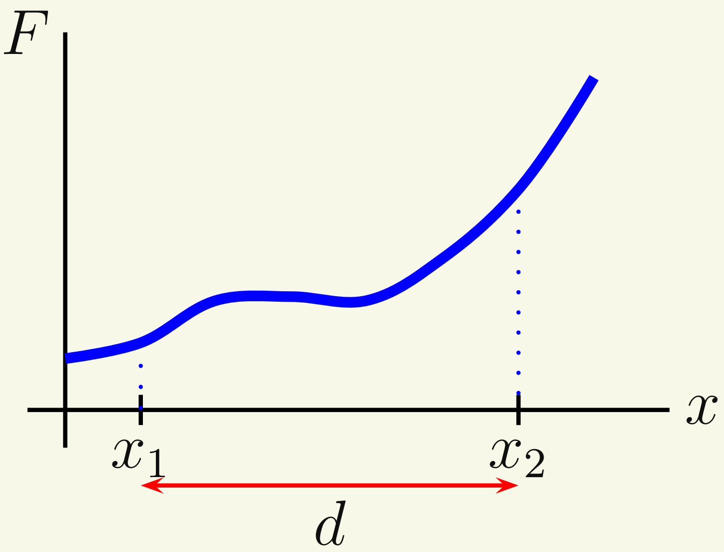

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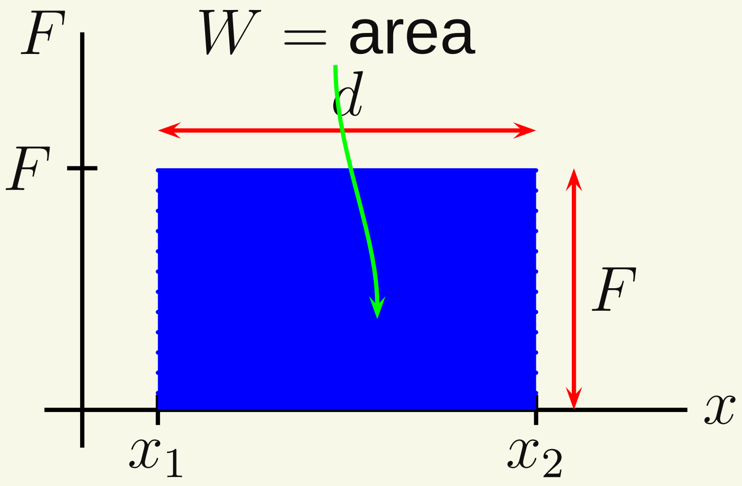


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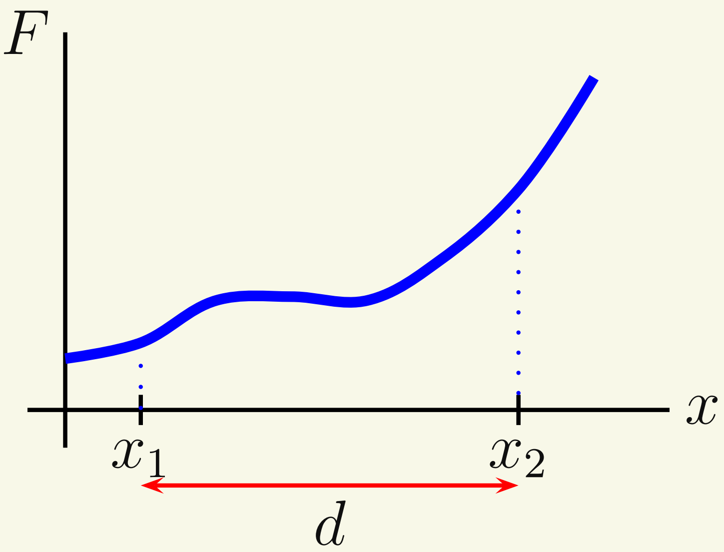

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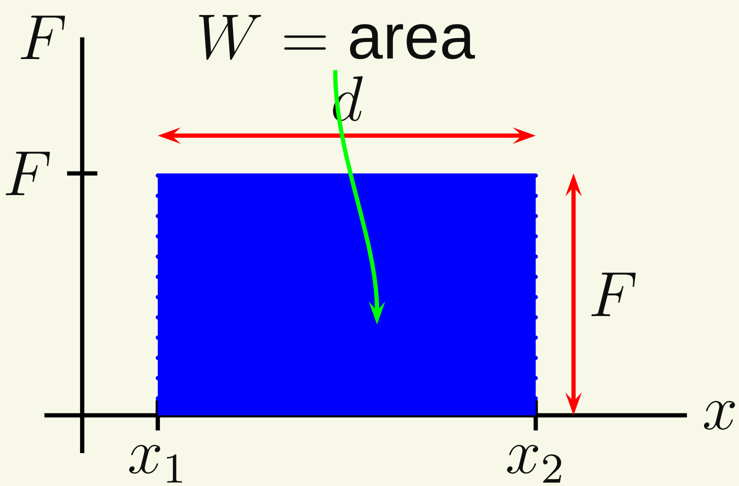


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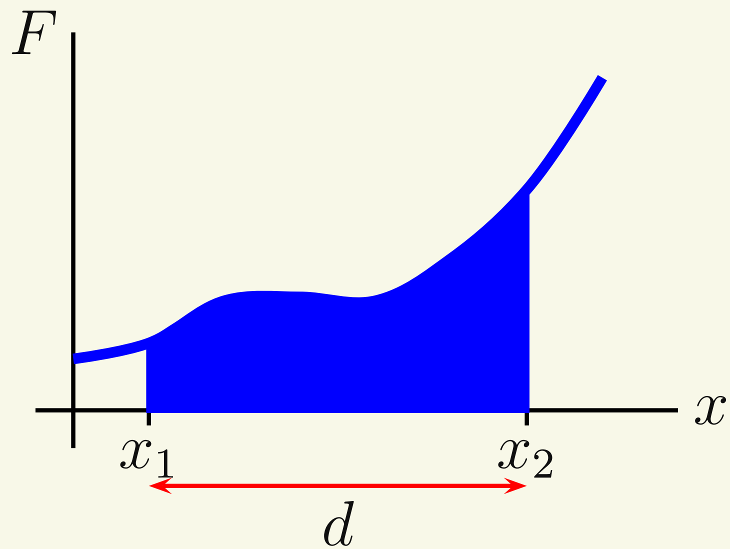

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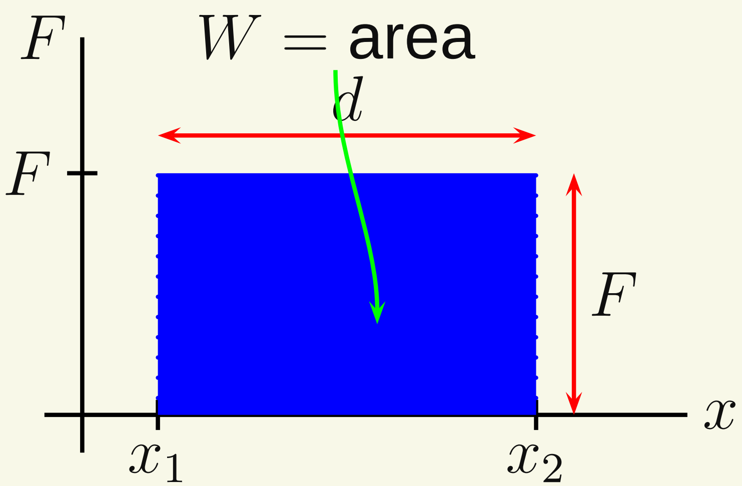


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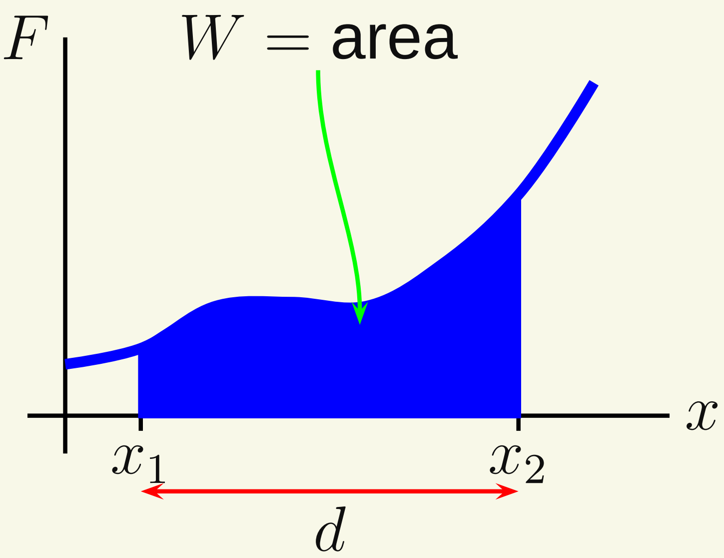

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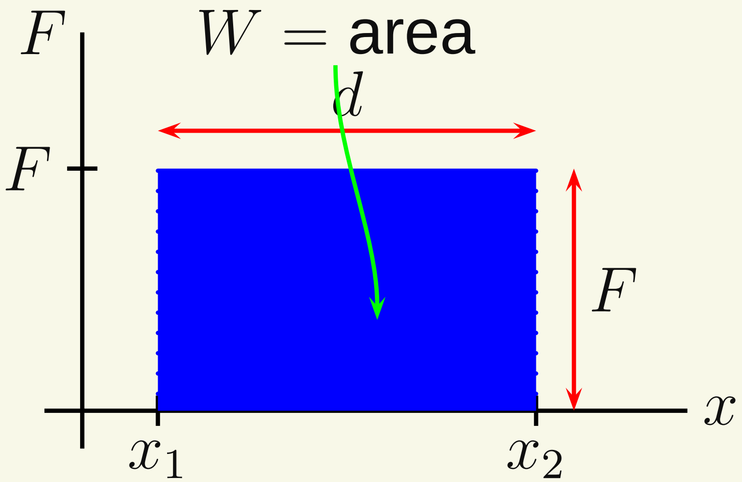


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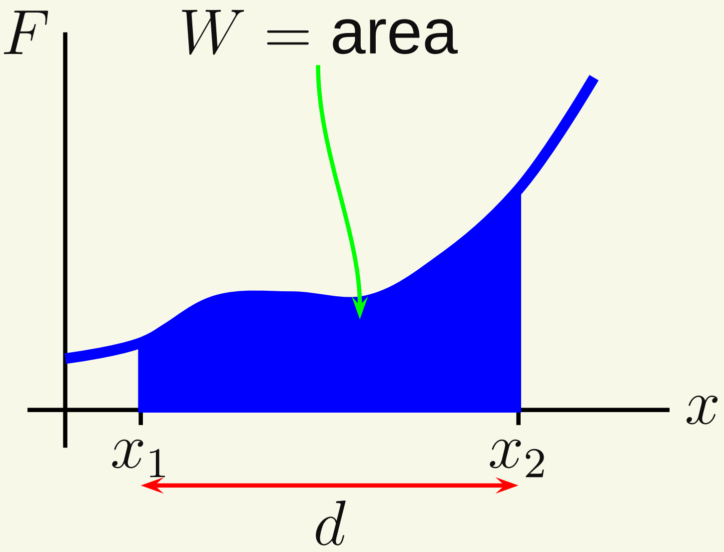


For variable forces, it can be shown that the work-energy theorem still holds! $\boxed{W_{total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$

# Potential Energy

Some forces do work that can be saved or stored.

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<u>Conservative Forces</u> - Forces that create potential energy.

Conservative forces are rare. Only gravity and the spring force are conservative. (You'll learn two more next term - the electric and magnetic force.) For a force to be conservative, the work it does must be independent of path.

# Conservation of Energy

For a conservative force,

$$W = -\Delta U$$

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Conservation of Energy - If only conservative forces do work on an object, its total energy cannot change.

Total Energy, E = the sum of kinetic and potential energy.

$$\boxed{E = K + U}$$

# Conservation of Energy II

Proof: If a conservative force is the only force doing work on an object then:

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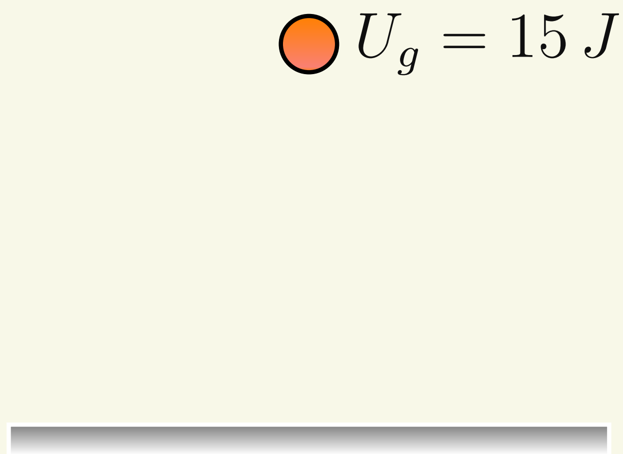
$$\Rightarrow E_i = E_f$$

# Energy-Conservation Exercise

A block having $15\,J$ of gravitational potential energy is dropped from rest. When the block hits the ground, it has $15\,J$ of kinetic energy. If gravity is the only force acting on the block, how much potential energy does the block have when it hits the ground?



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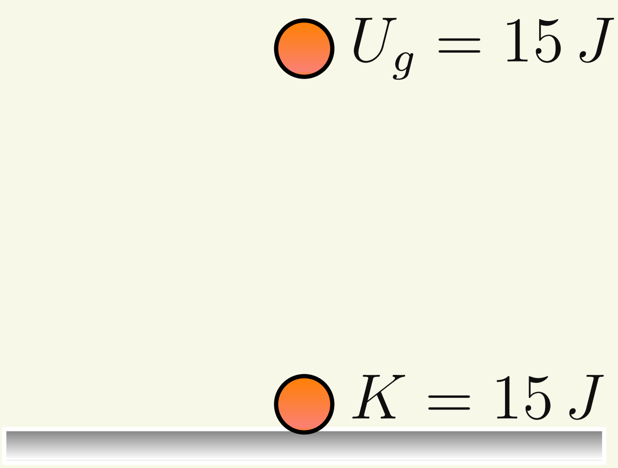

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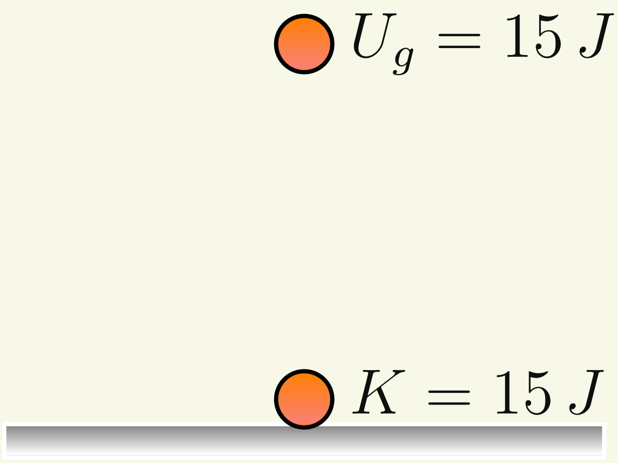


(a) $0\,J$

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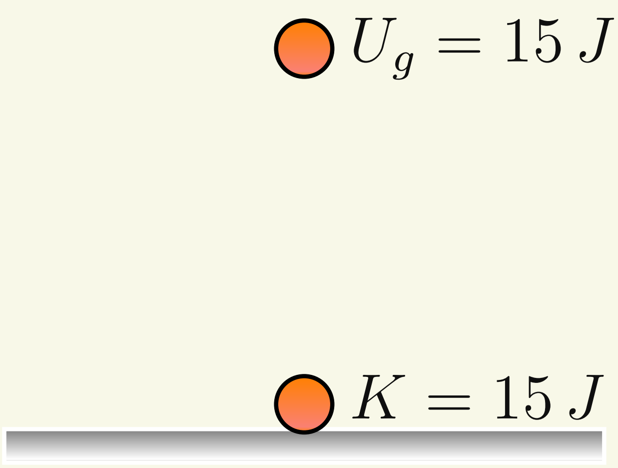


(a) $0\,J$

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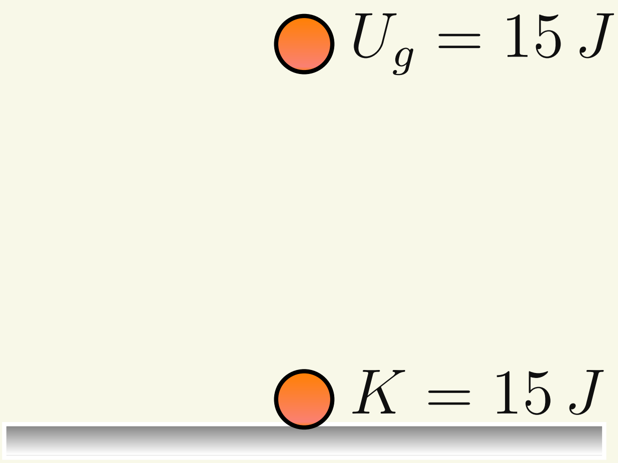


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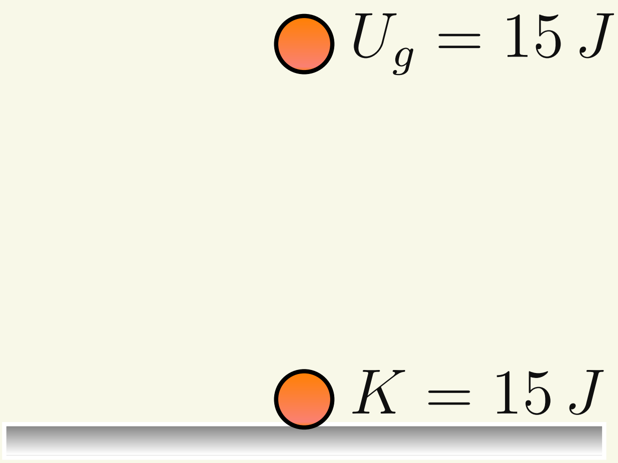


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(c) $15\,J$

(d) $30\,J$

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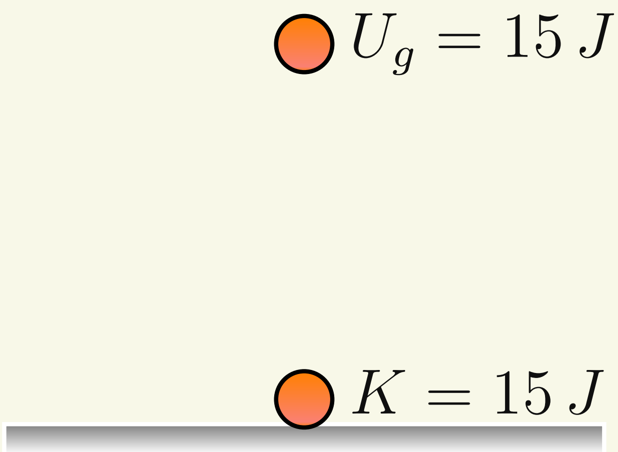


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(e) Cannot be determined

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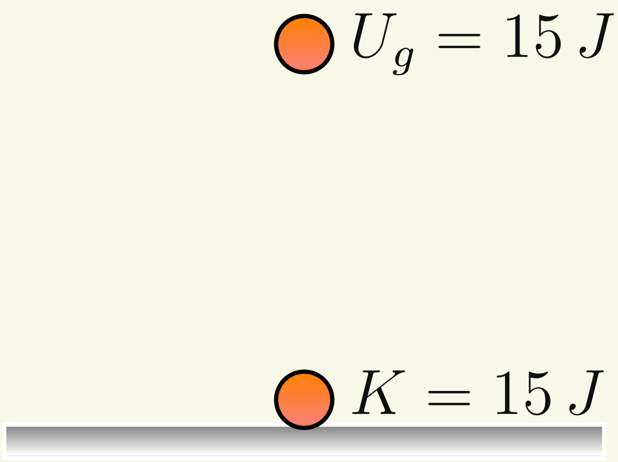


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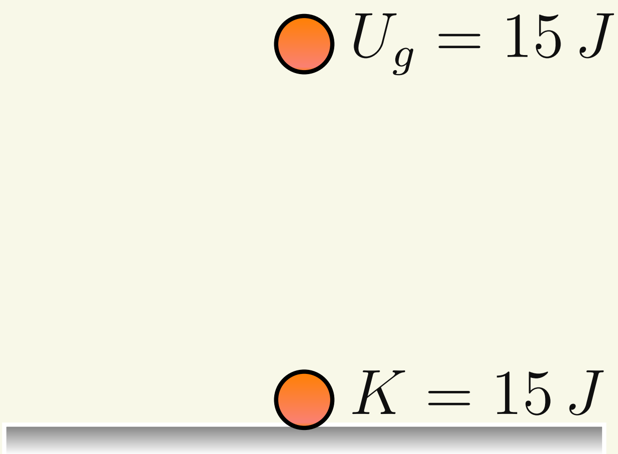


$$K_i + U_i = K_f + U_f \Rightarrow$$

$$0 + 15\,J = 15\,J + U_f$$