

June 16, Week 3

Today: Chapter 3, Projectile Motion

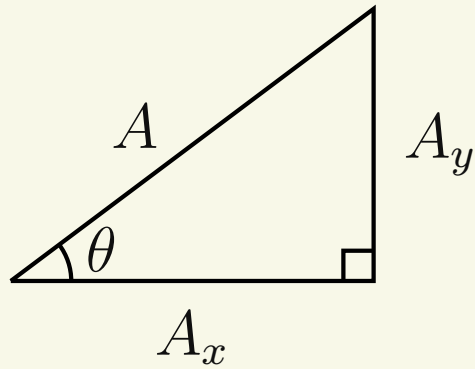
Homework #3 is now available.

Scalar Components

In physics, the sides of the right triangle are the scalar components.

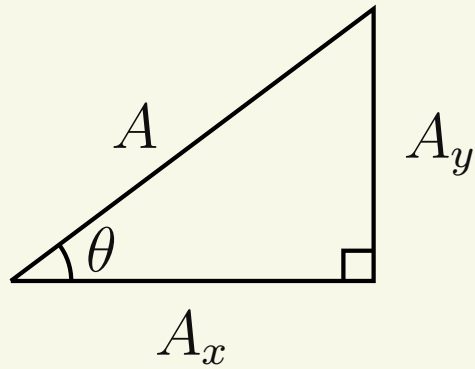
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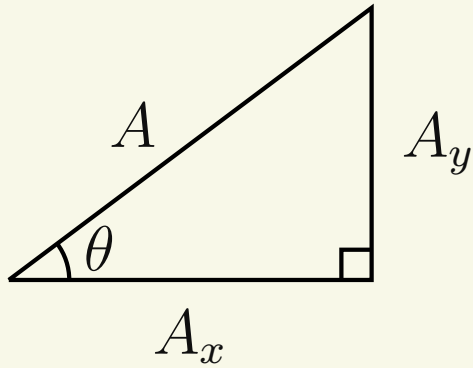
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$$\cos \theta = \frac{A_x}{A}$$

Scalar Components

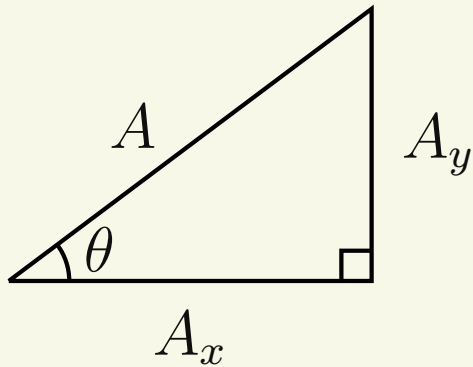
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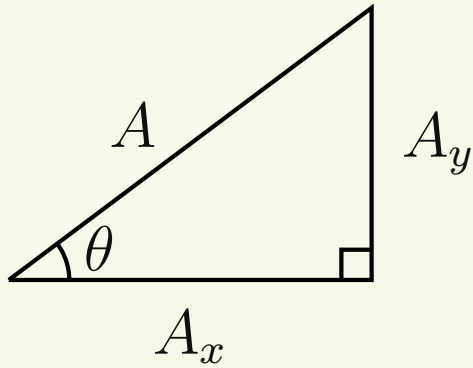


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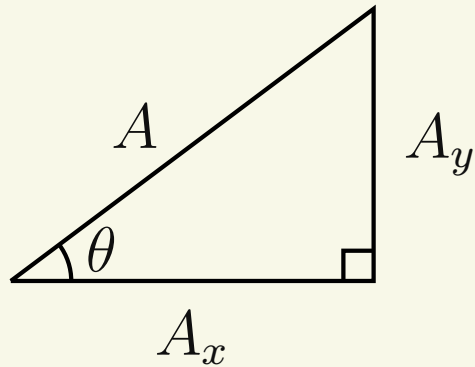


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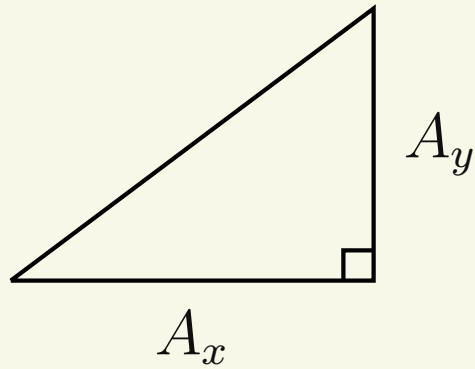
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Only guaranteed to work for the standard angle

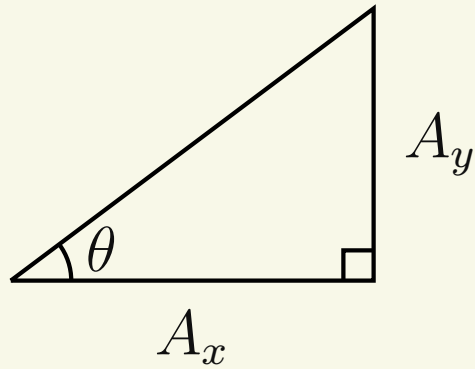
Scalar Components II

To find the magnitude and the angle *from* the components:



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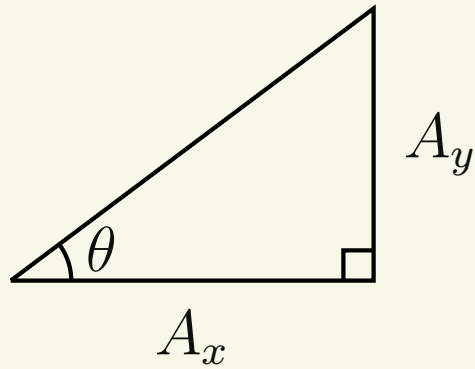
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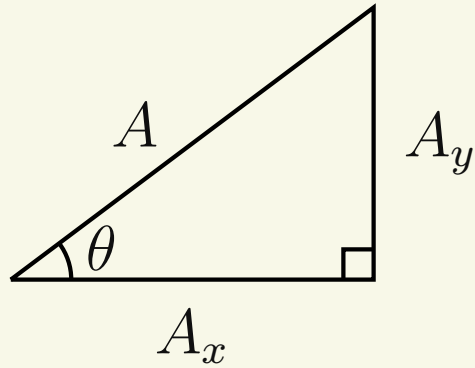
$$\tan \theta = \frac{A_y}{A_x}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

Arctangent

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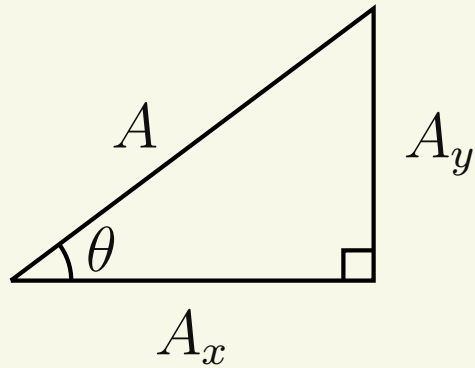


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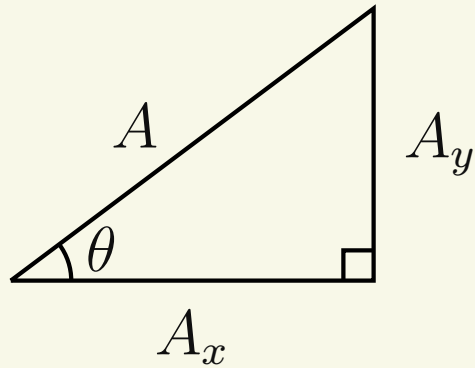
$$\tan \theta = \frac{A_y}{A_x} \Rightarrow \theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

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$$A^2 = A_x^2 + A_y^2$$

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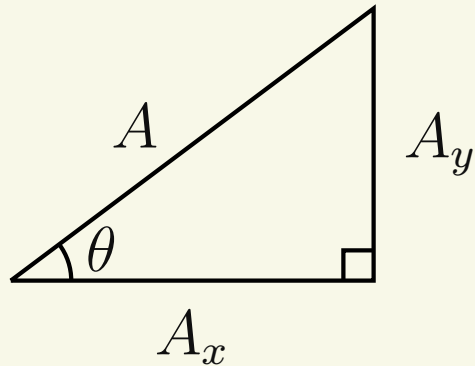
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$$A^2 = A_x^2 + A_y^2 \Rightarrow A = \sqrt{A_x^2 + A_y^2}$$

Pythagorean Theorem

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Pythagorean Theorem

Example: Find the magnitude and direction for the vector with components $A_x = 1 \text{ m}$ and $A_y = 1 \text{ m}$.

Quadrants

Sometimes your calculator will be wrong in finding angles!

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Example: Find the magnitude and direction for the vector with components $A_x = -1\text{ m}$ and $A_y = -1\text{ m}$.

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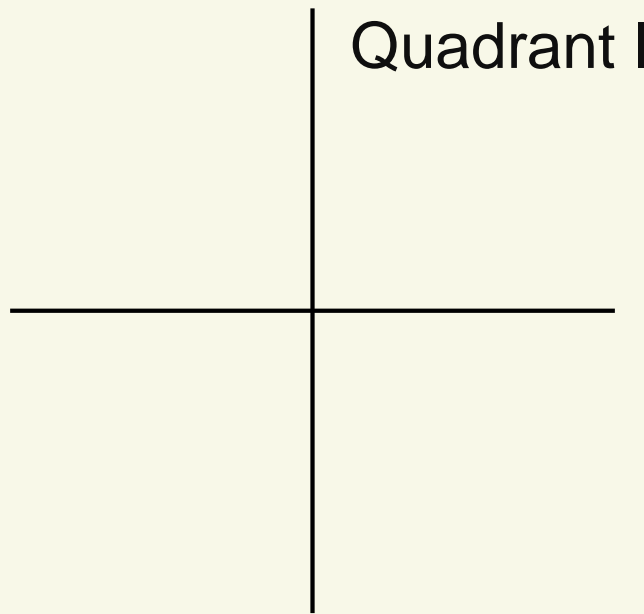
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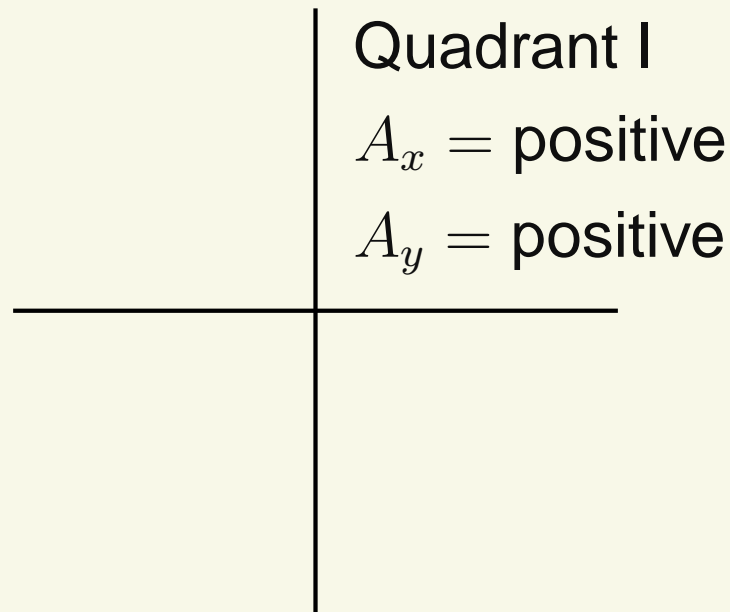


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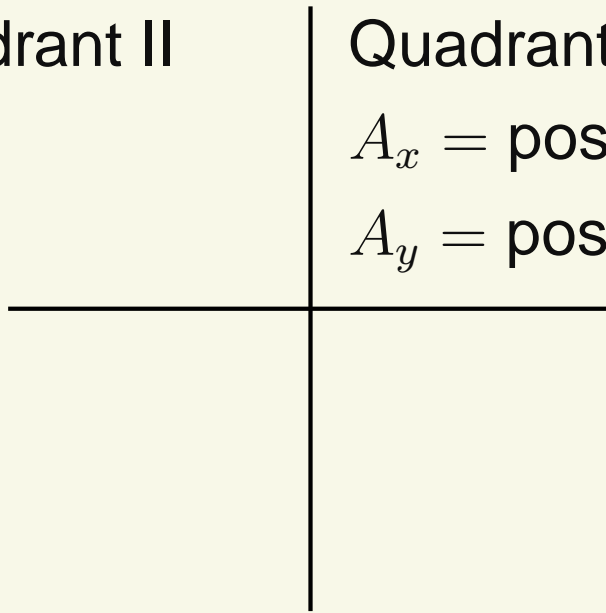
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Quadrant II

Quadrant I

$A_x = \text{positive}$

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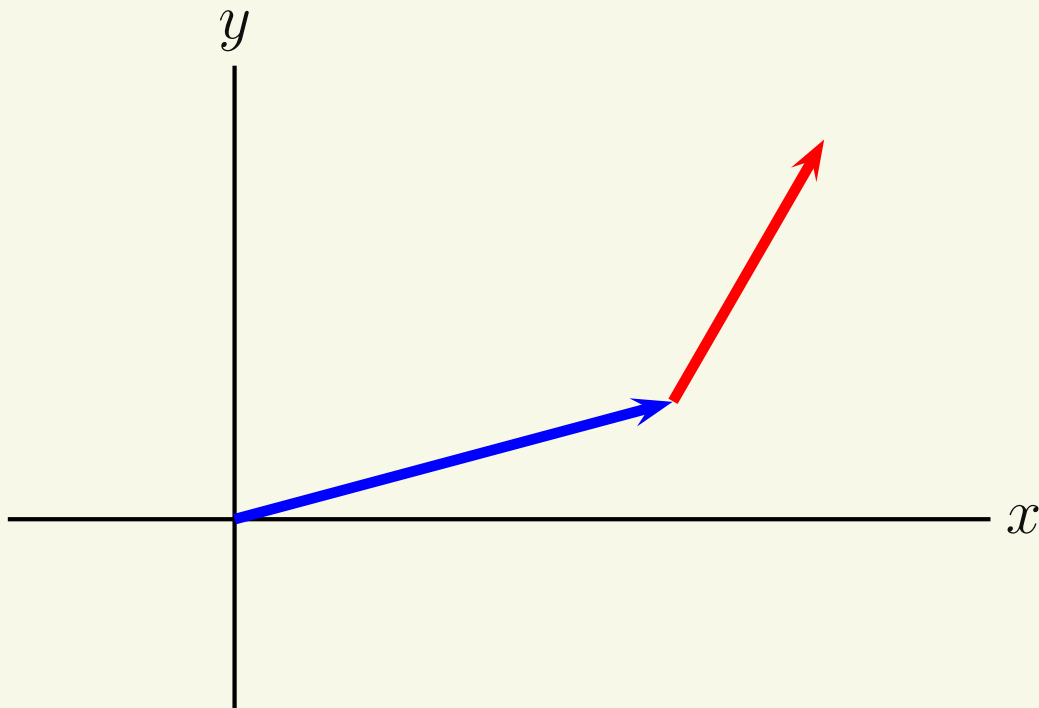
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Component Addition

While we **cannot** add the magnitudes of vectors. We can add the components.

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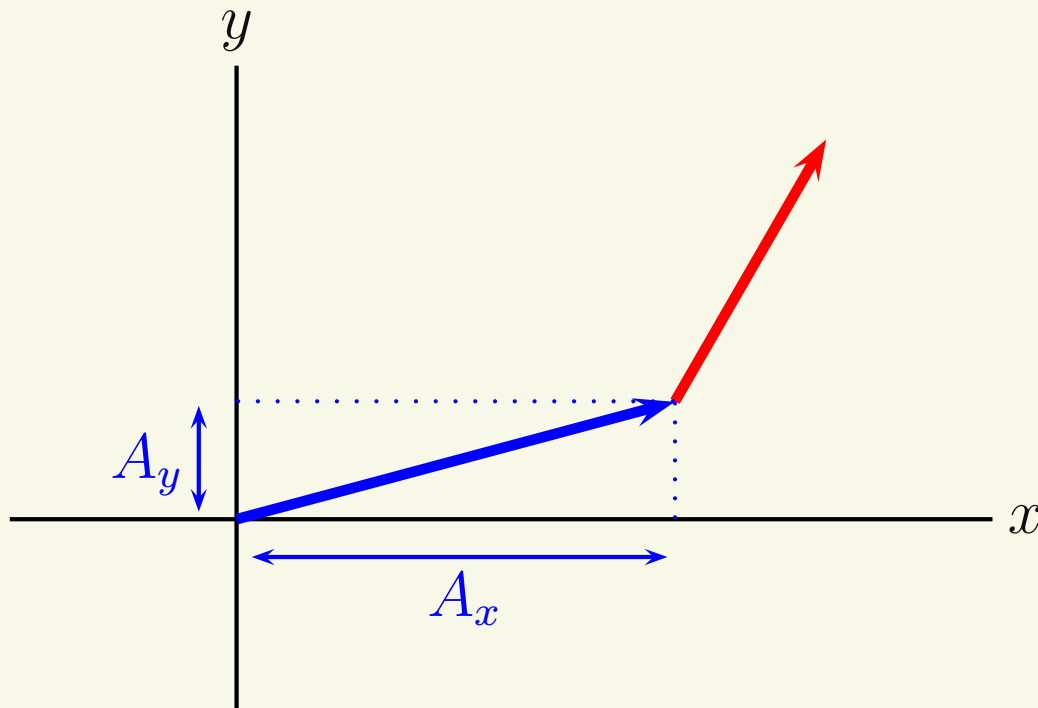
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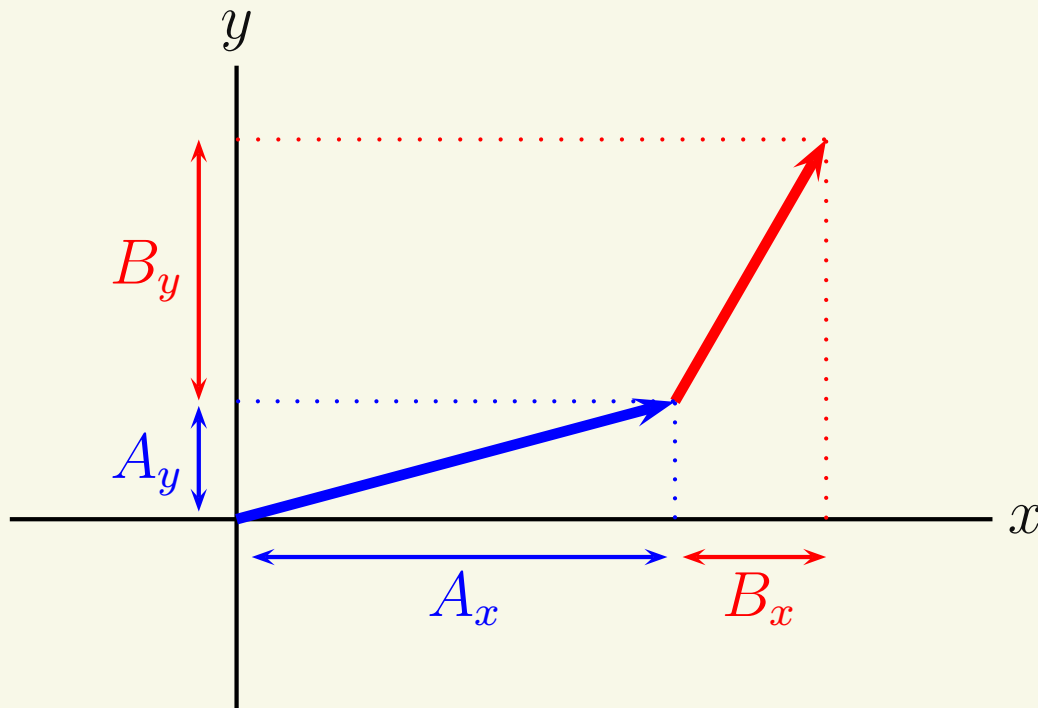


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Find the components of \vec{A}

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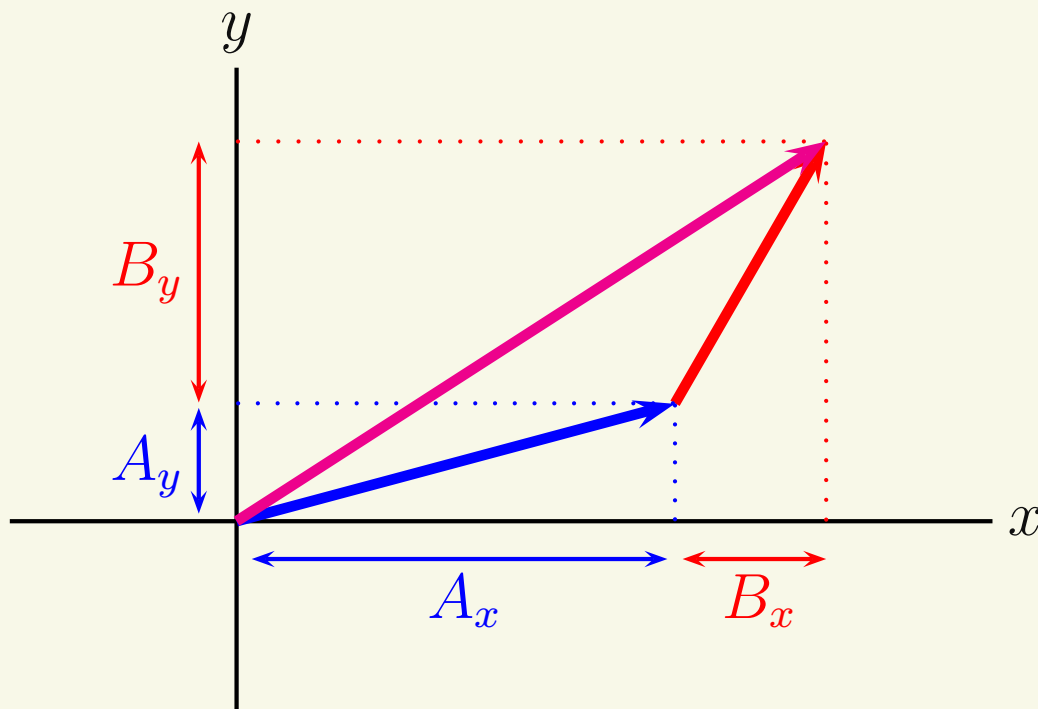
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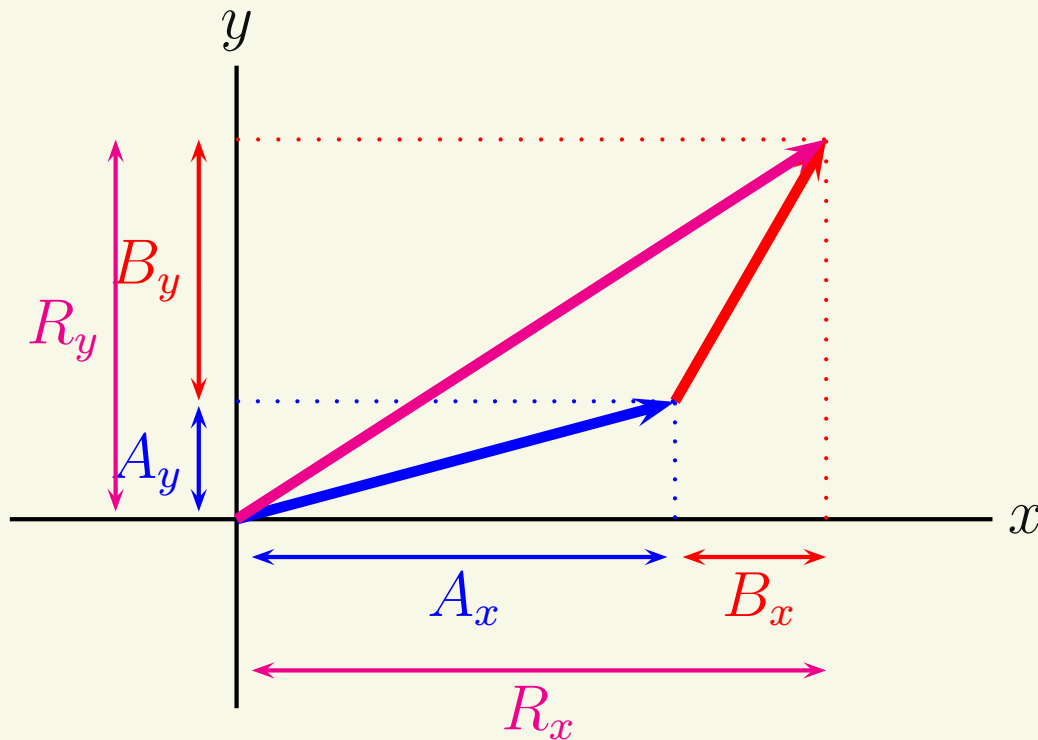
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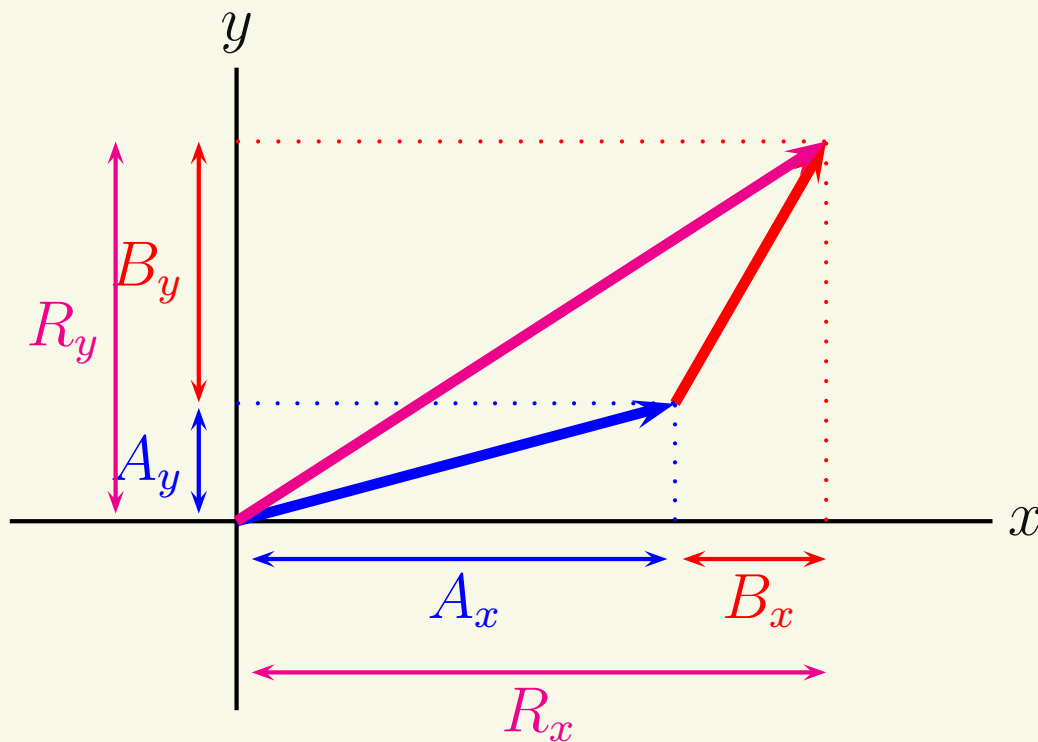
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The components of \vec{R} :

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

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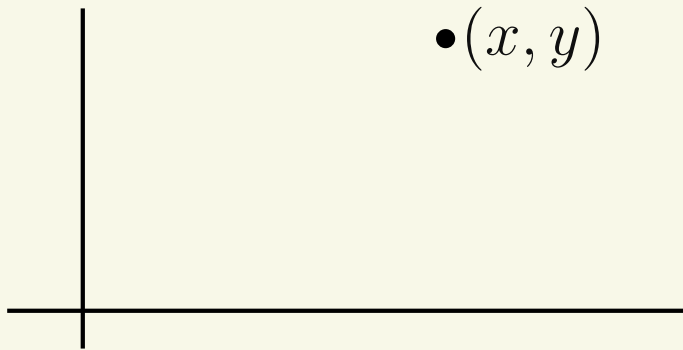
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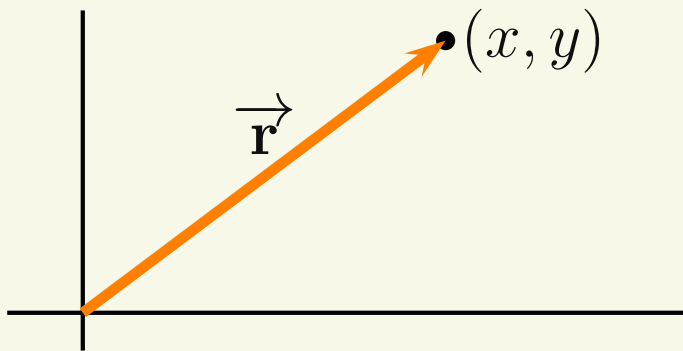


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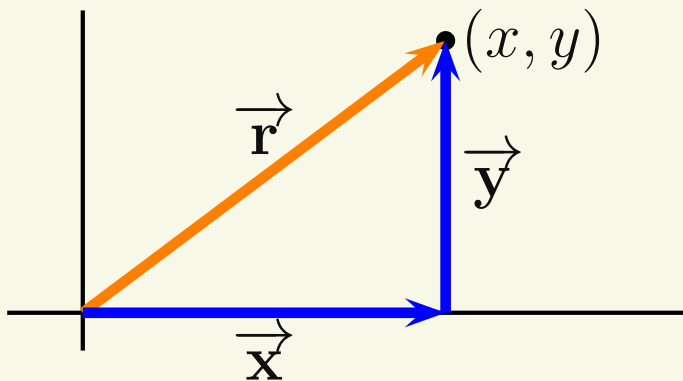


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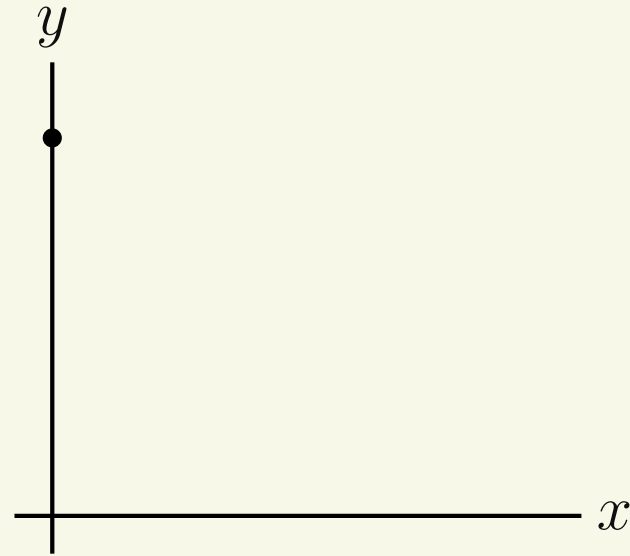
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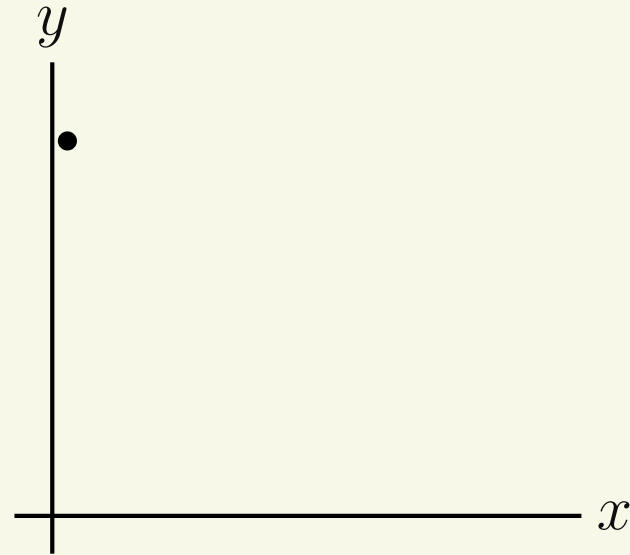
Velocity Components

In curved motion, the path taken by a moving object is called its trajectory



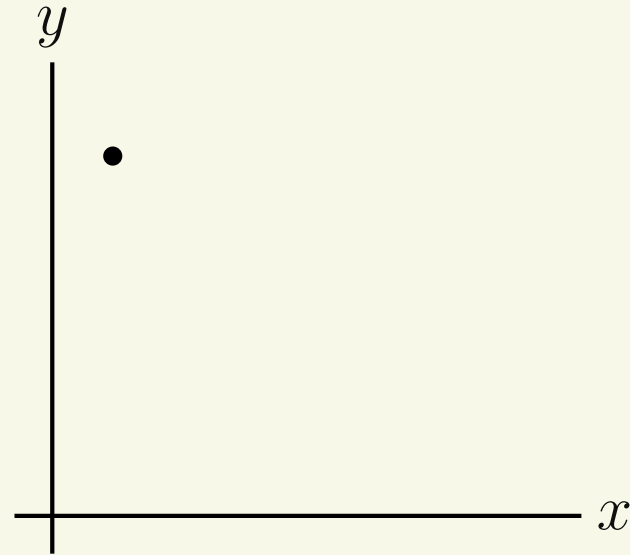
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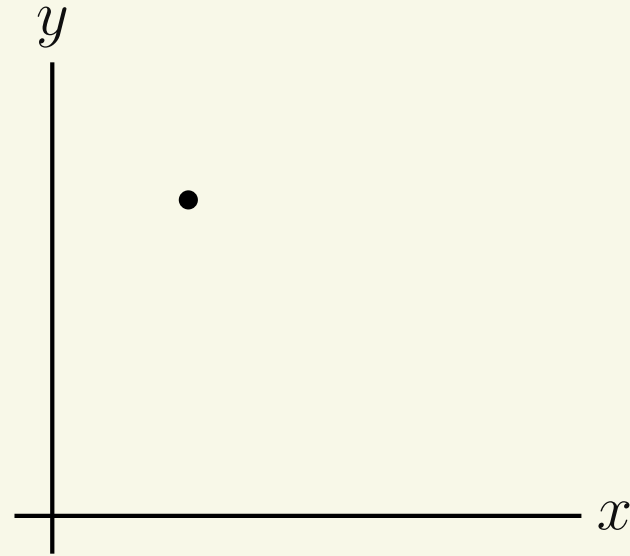
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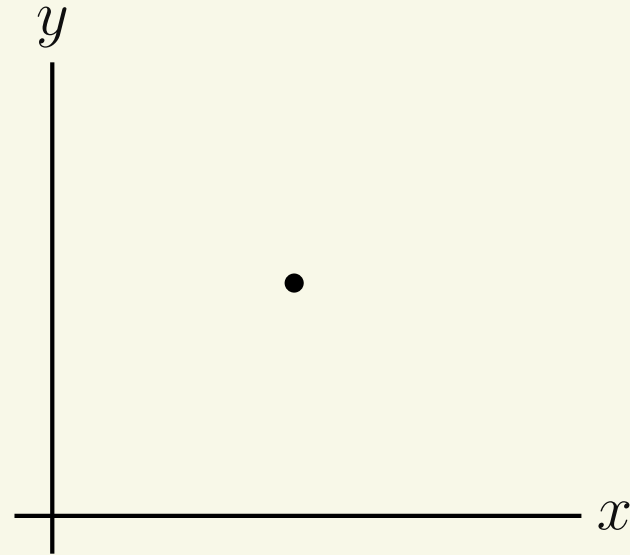
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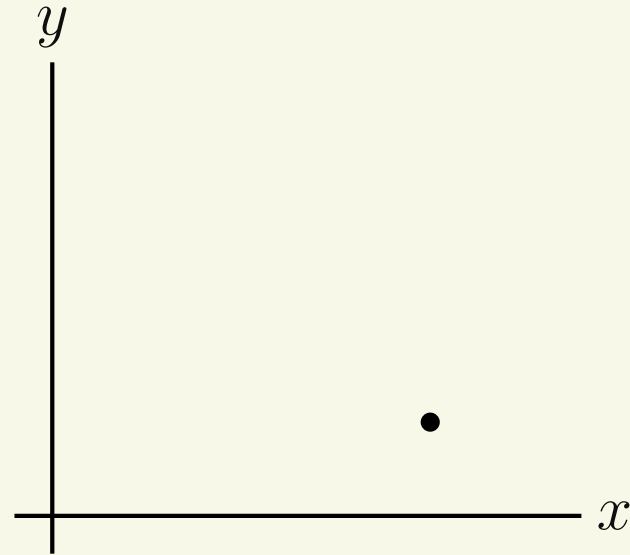
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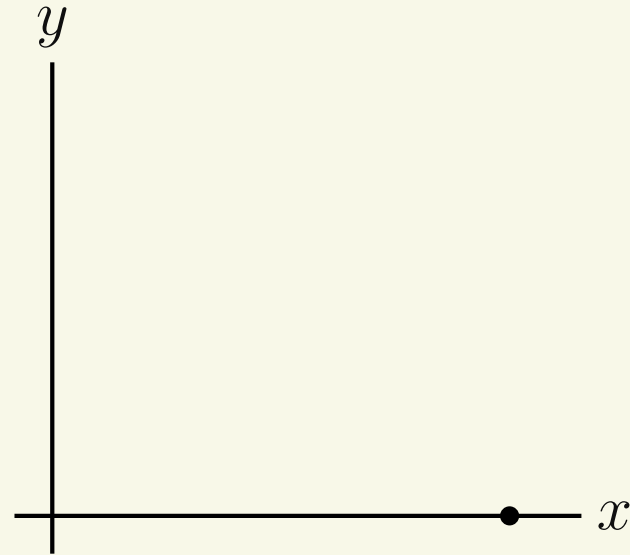
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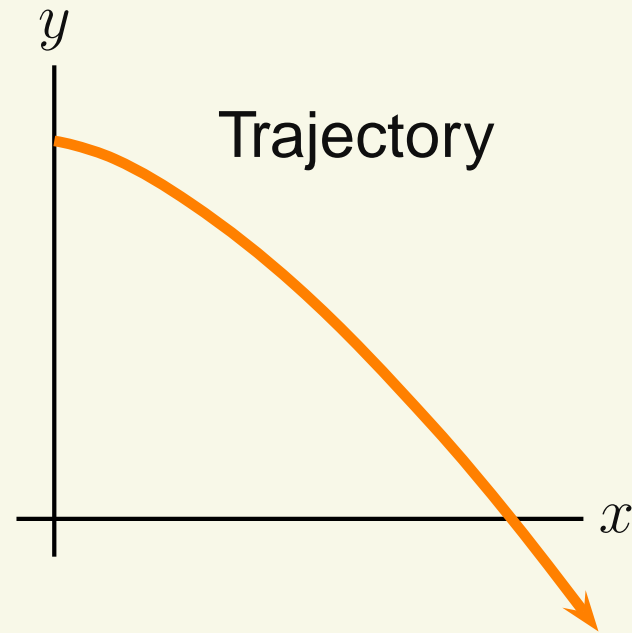
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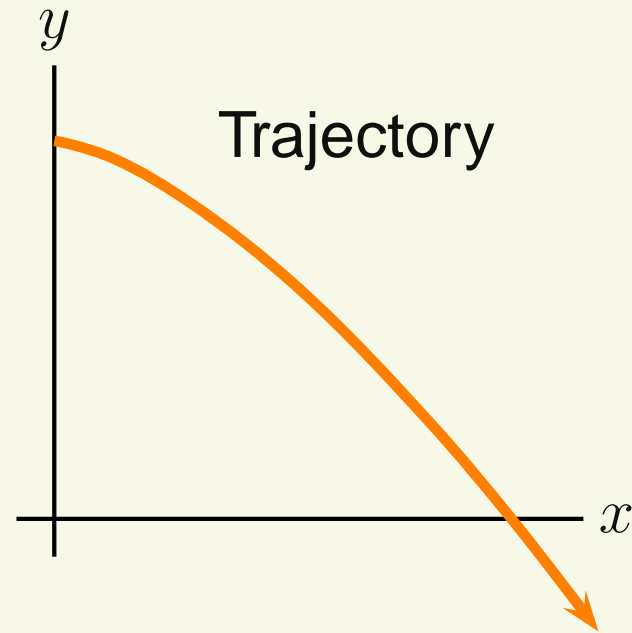
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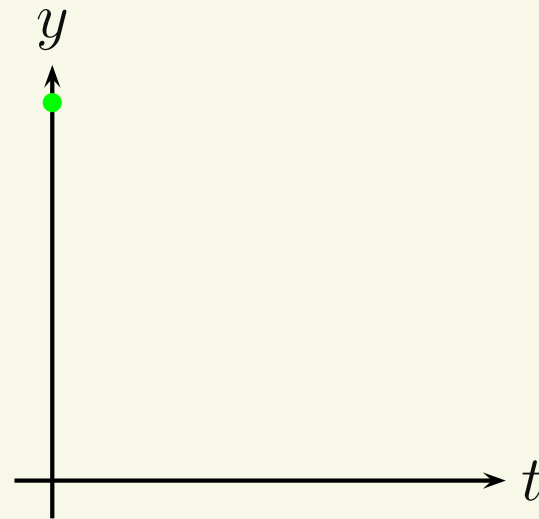
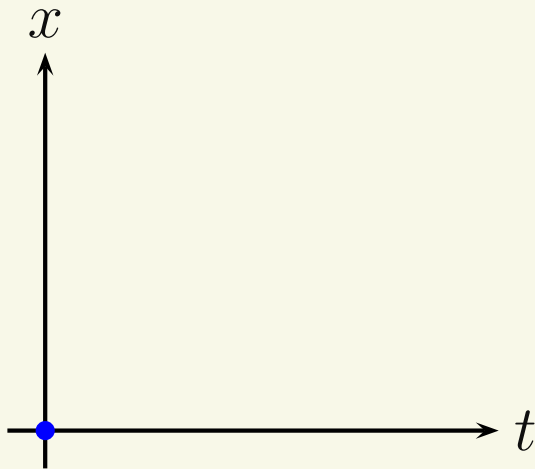
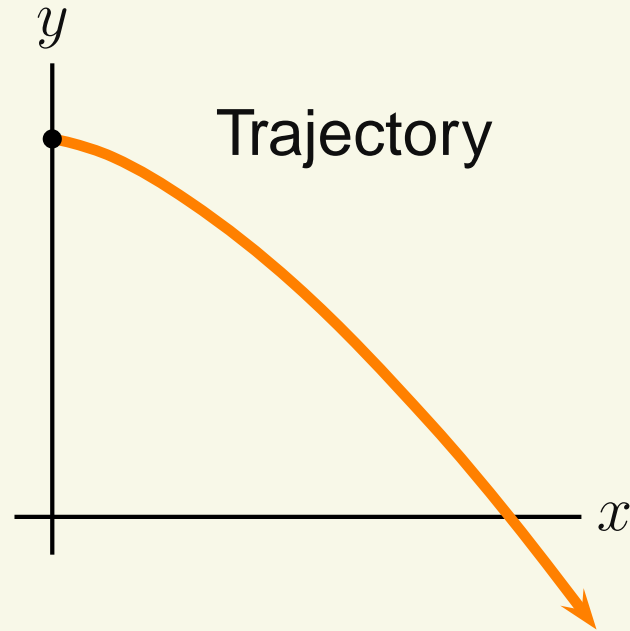
There are two separate position plots which give the velocity vector's components



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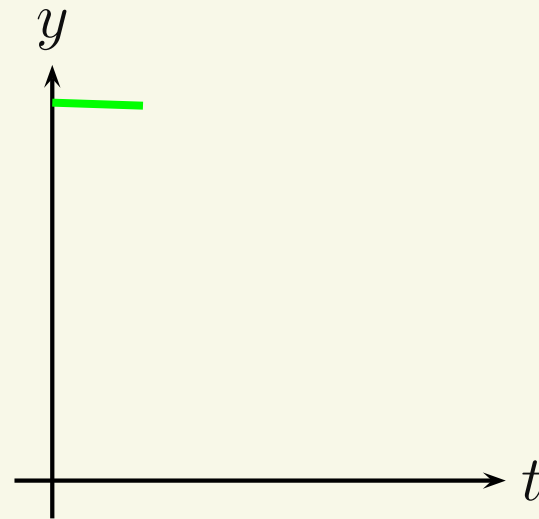
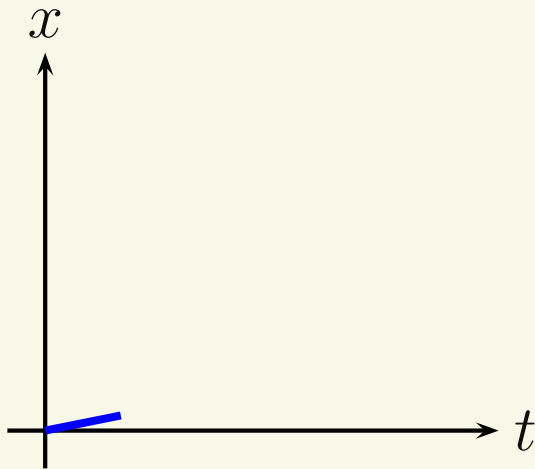
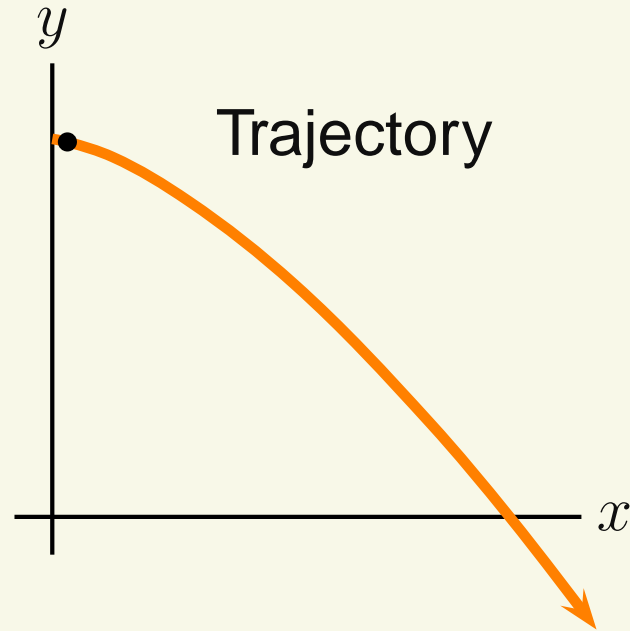
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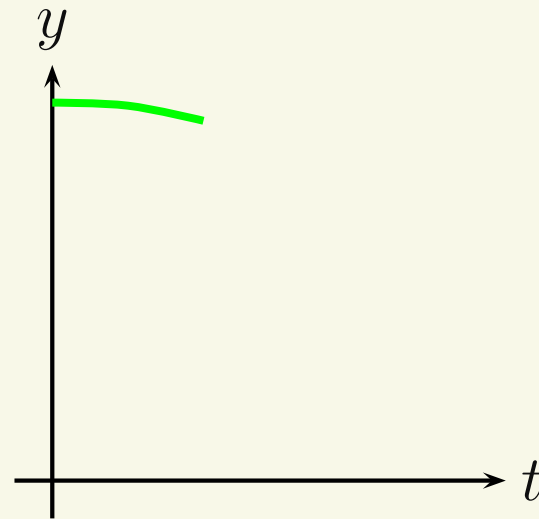
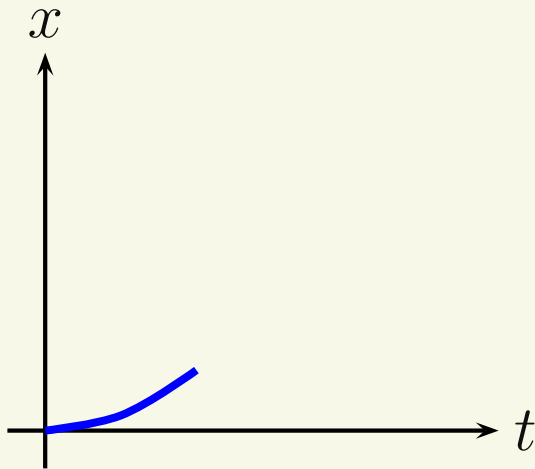
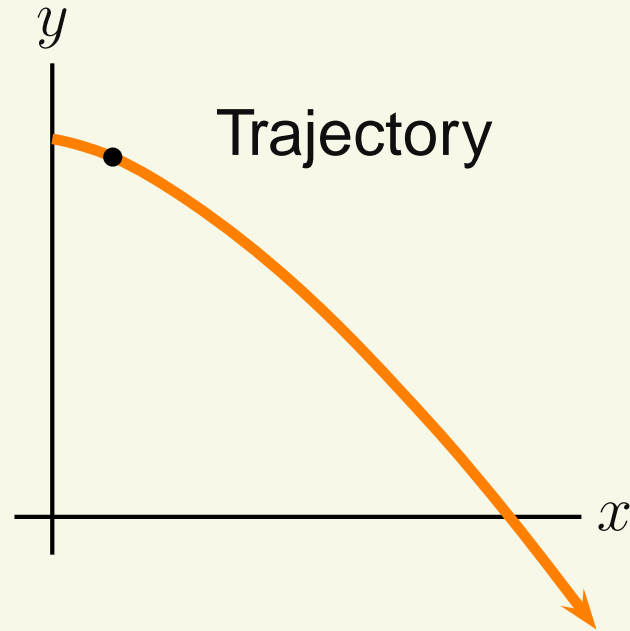
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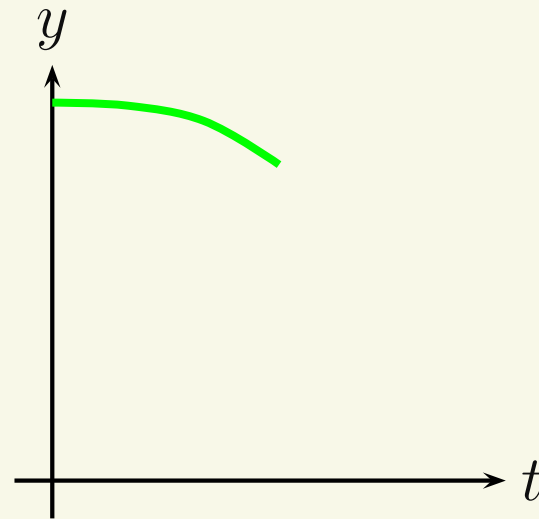
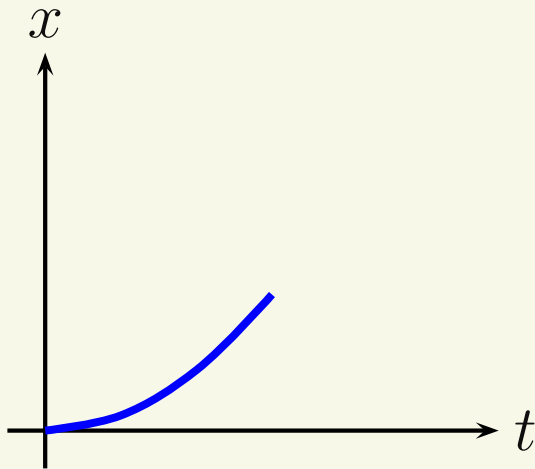
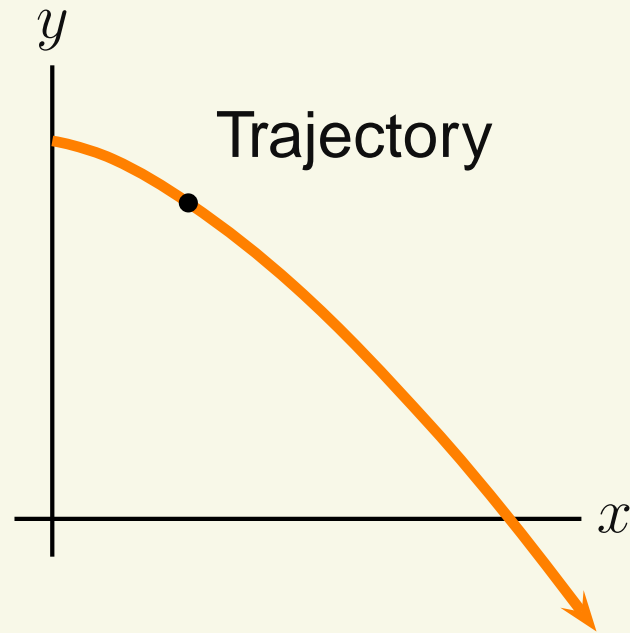
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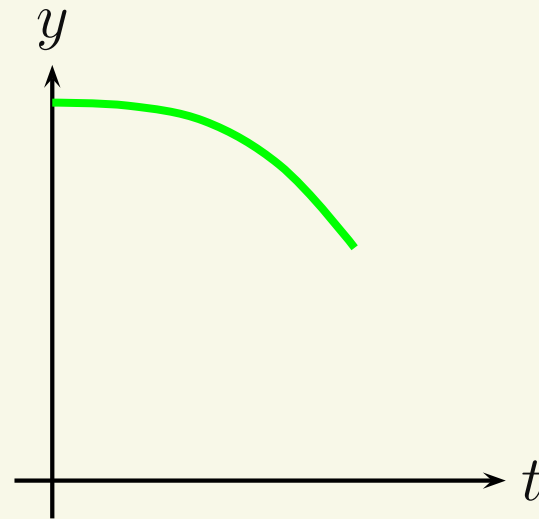
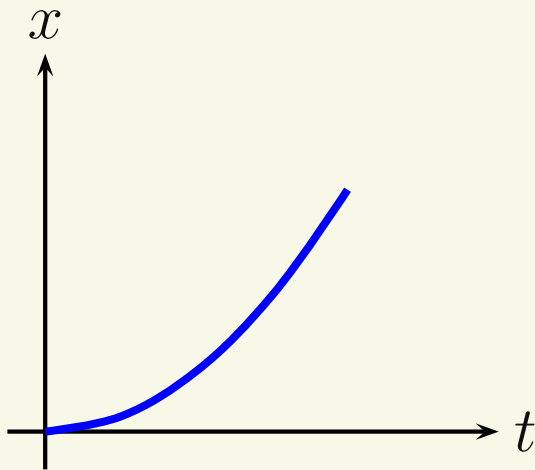
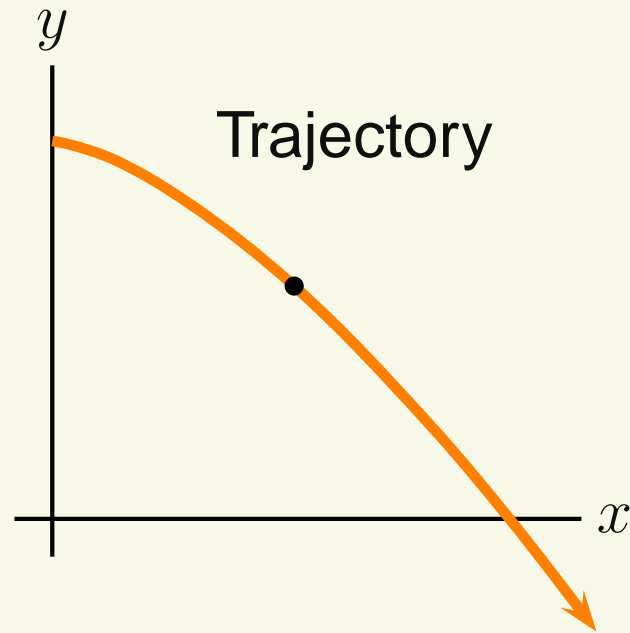
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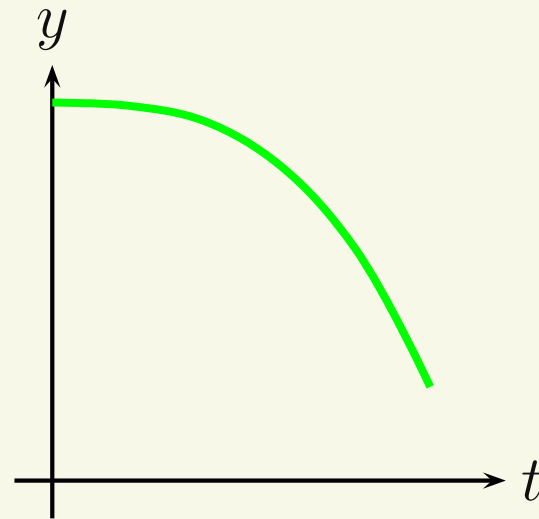
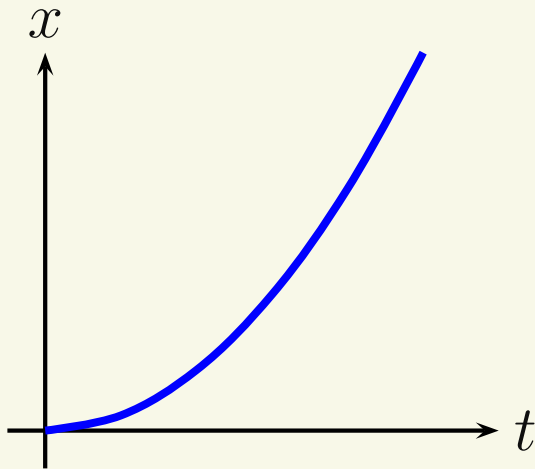
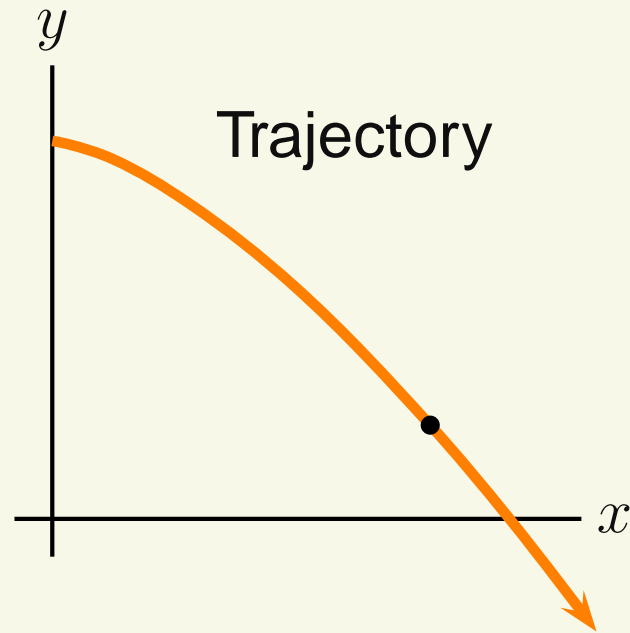
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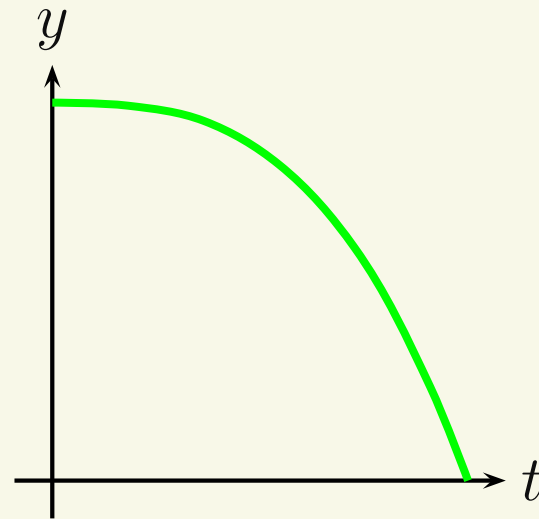
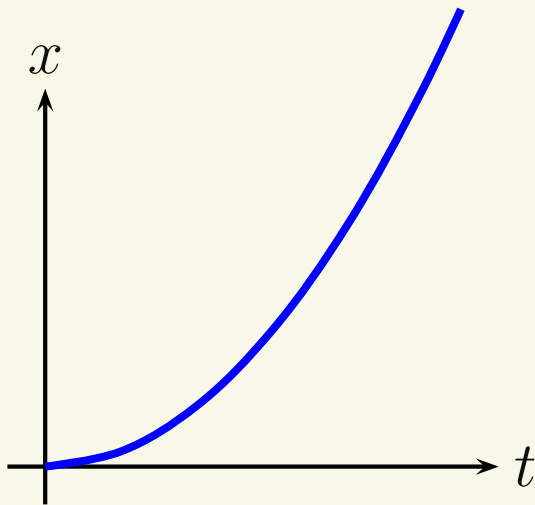
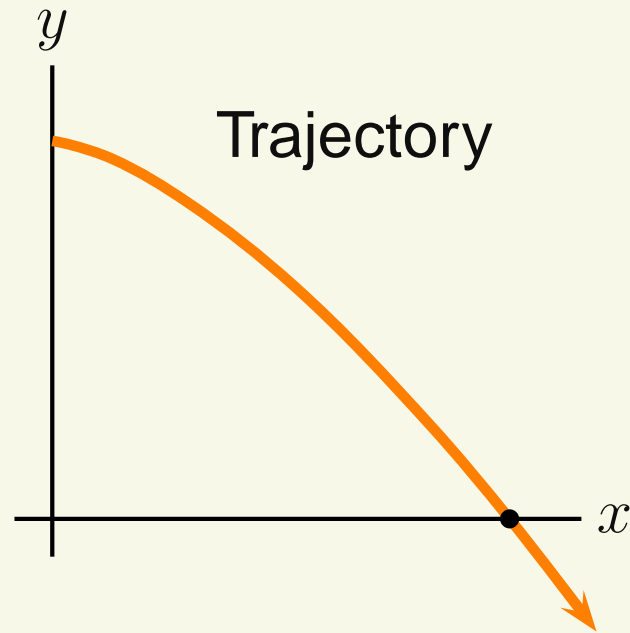
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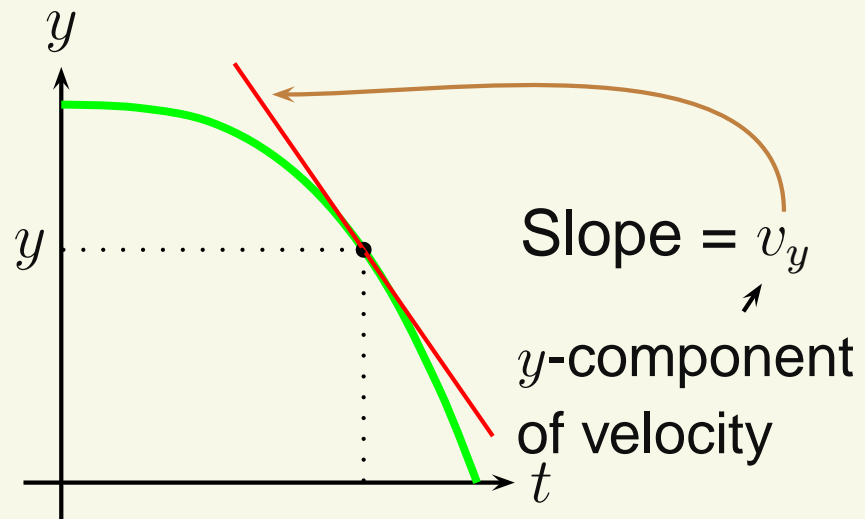
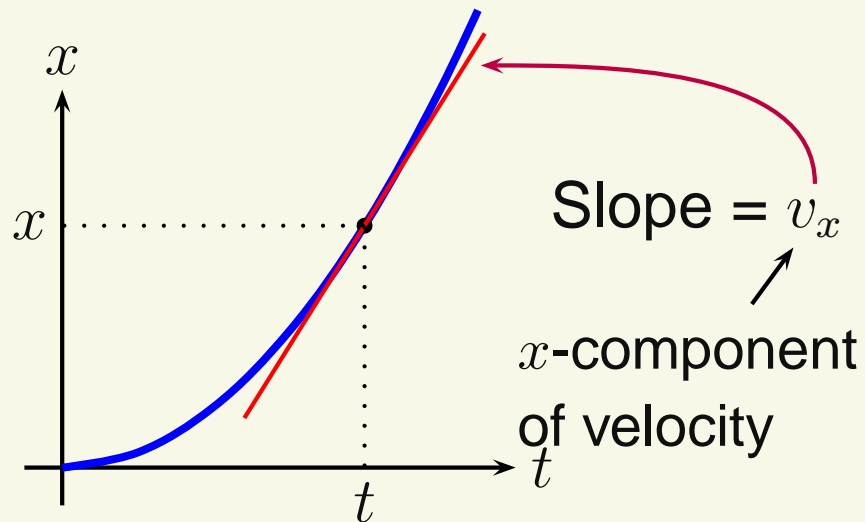
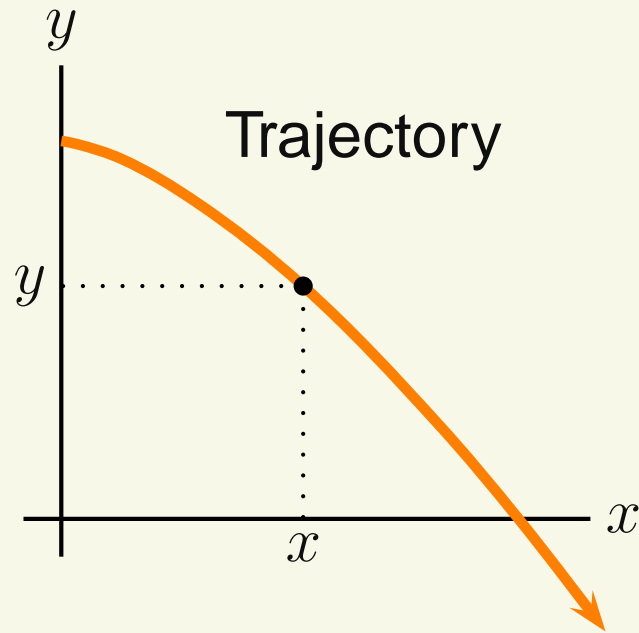
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Velocity Components

In curved motion, the path taken by a moving object is called its trajectory

There are two separate position plots which give the velocity vector's components



Curved Motion II

On the trajectory plot, the velocity vector is described as being “tangent” to the curve.

Curved Motion II

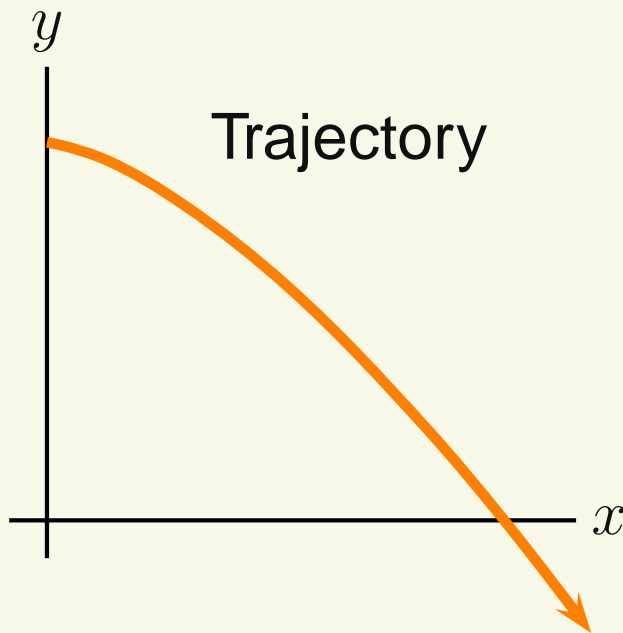
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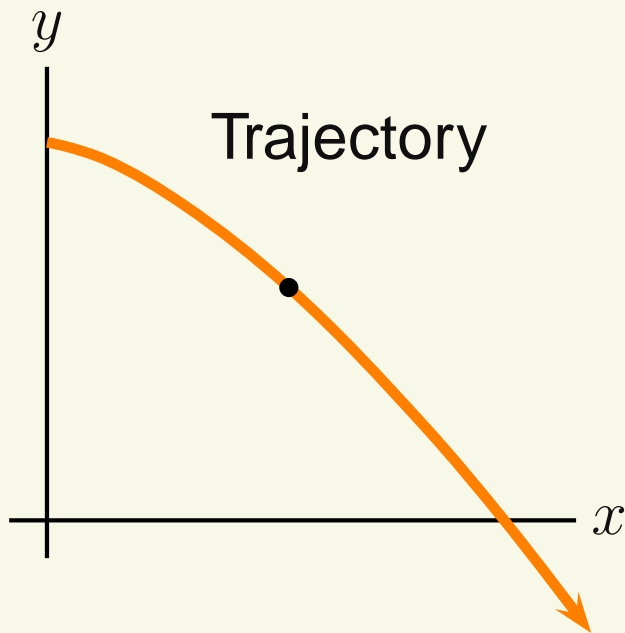
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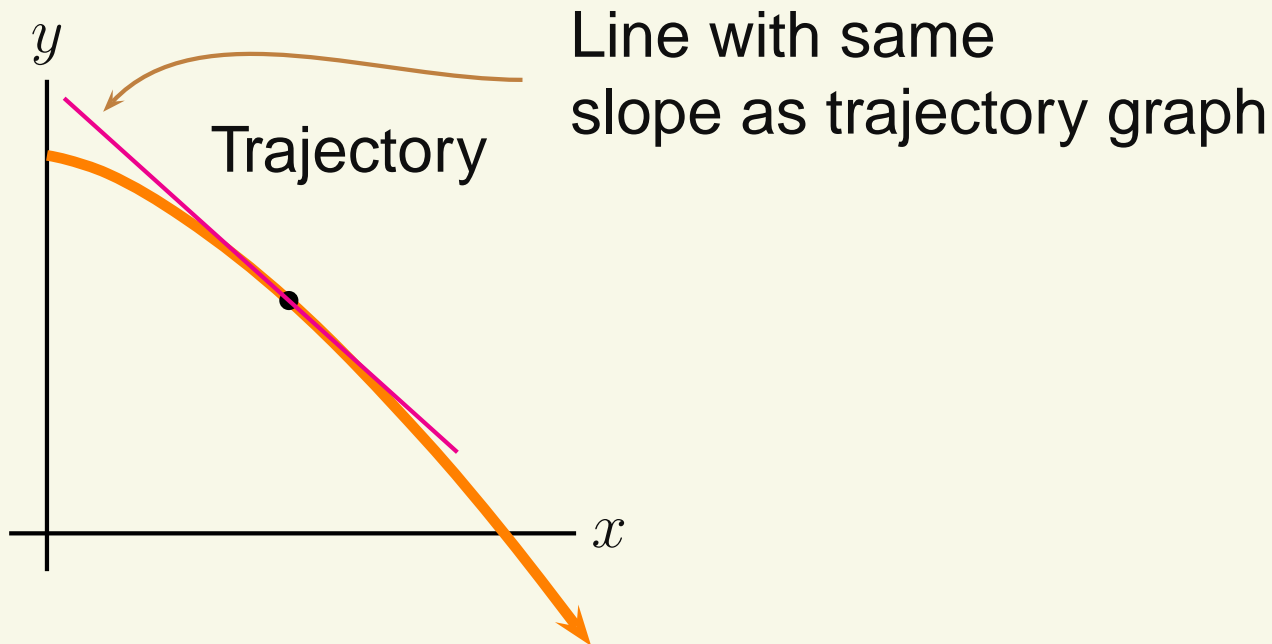
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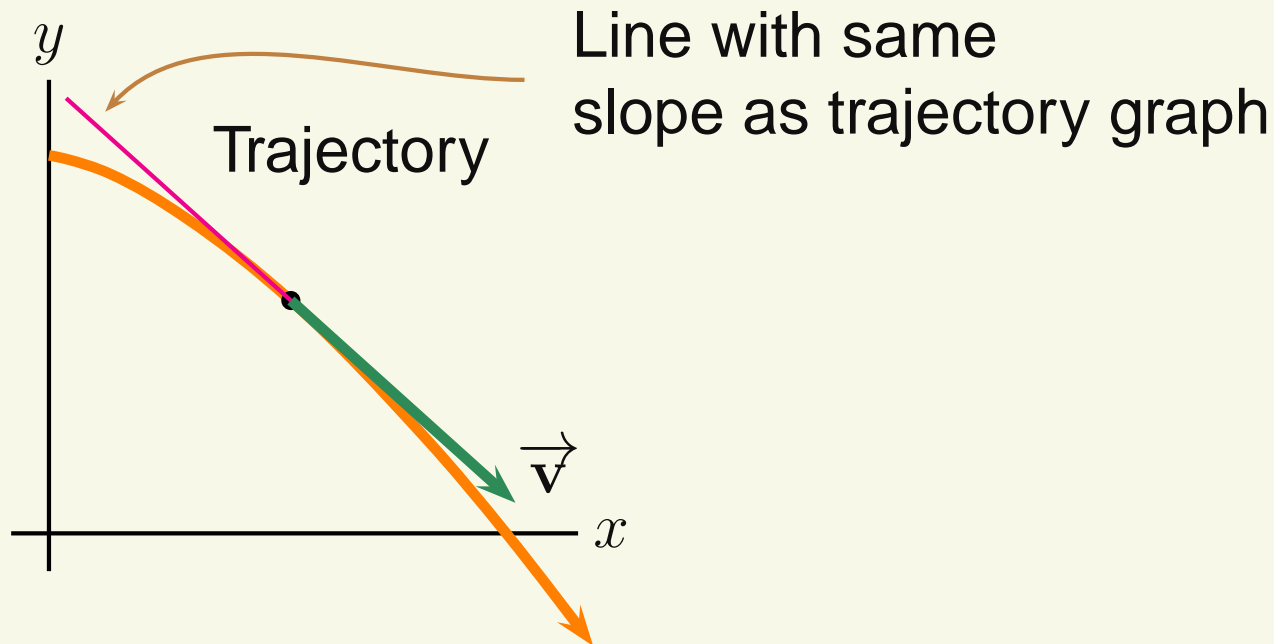
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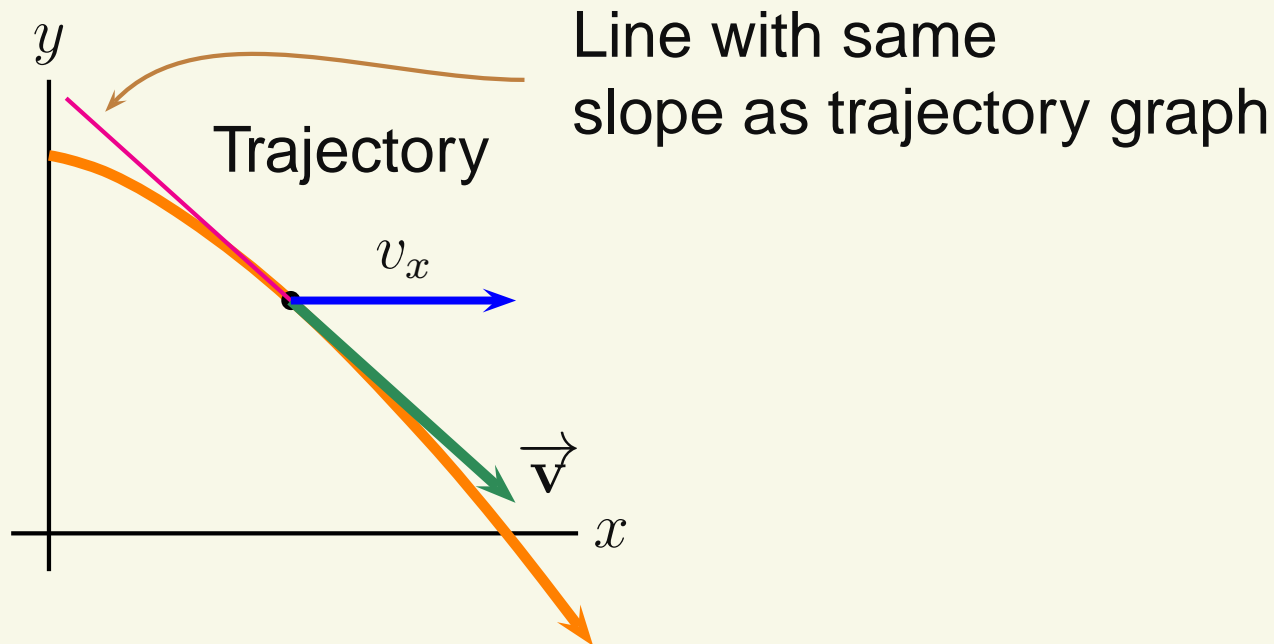
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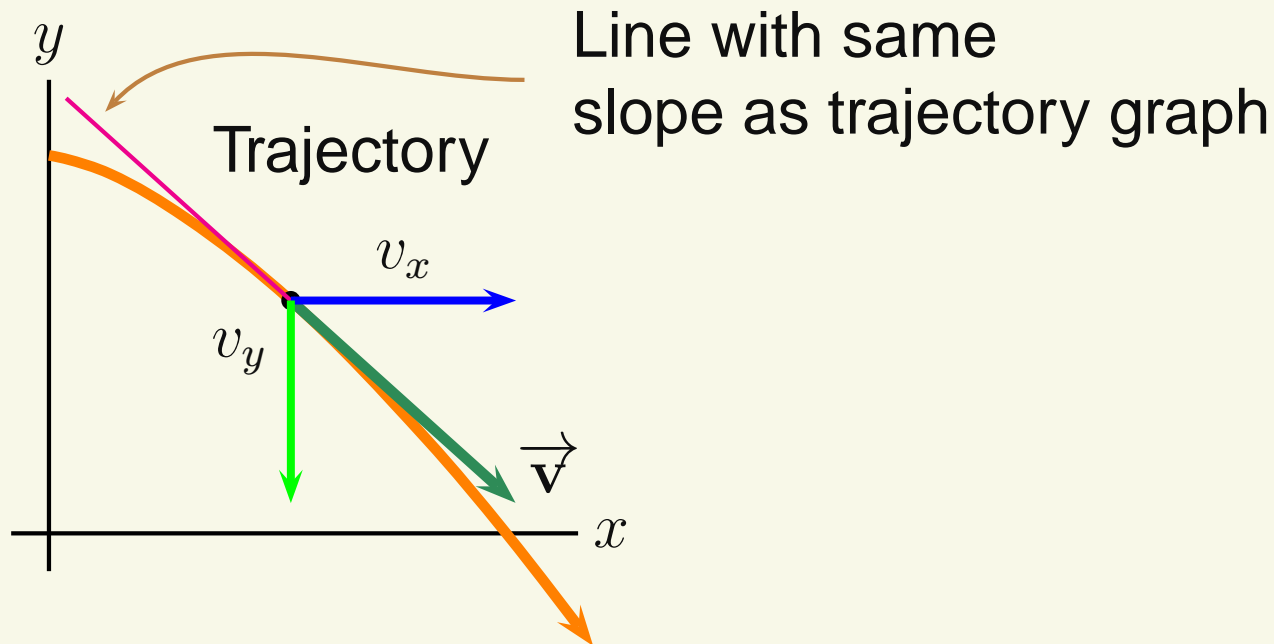
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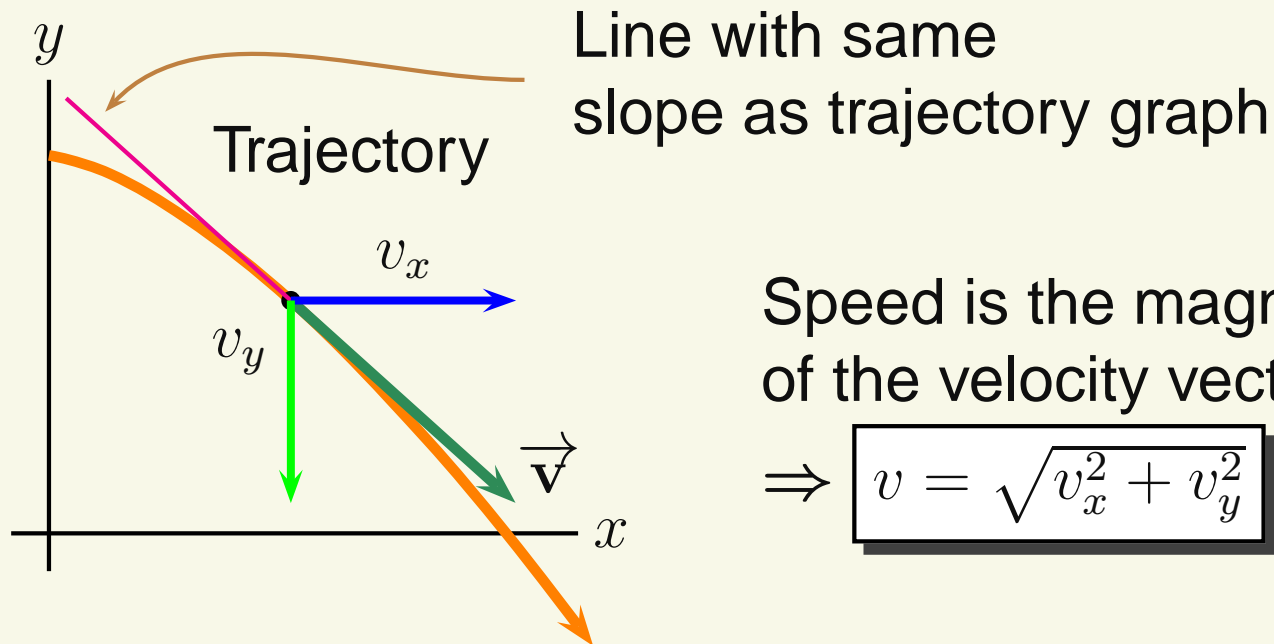
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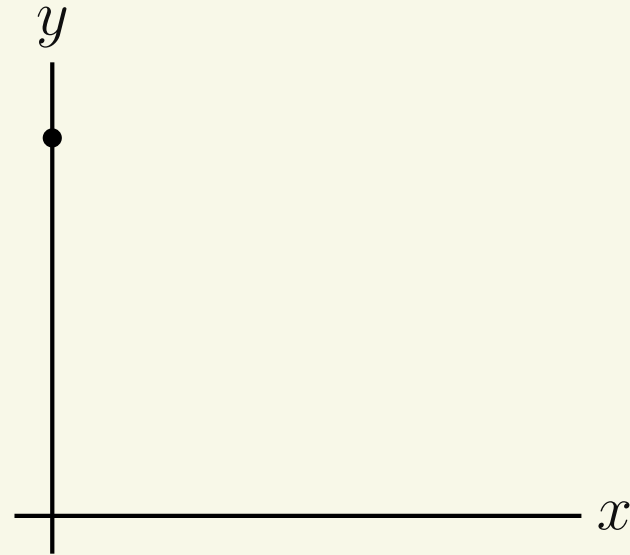


Speed is the magnitude of the velocity vector

$$\Rightarrow v = \sqrt{v_x^2 + v_y^2}$$

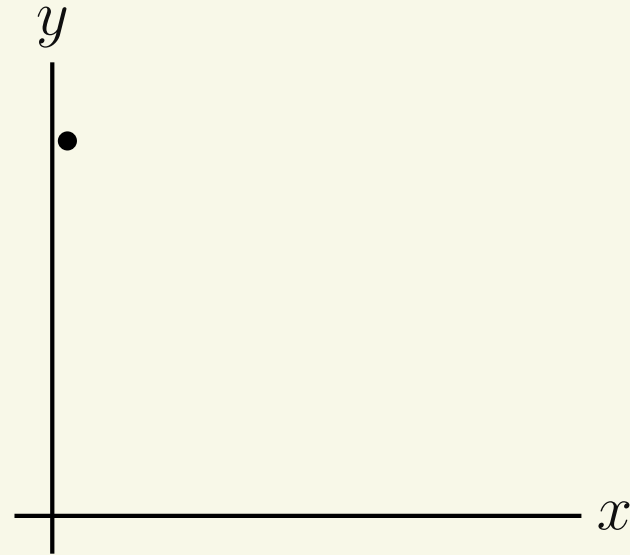
Acceleration Components

We can find the acceleration components in the same way as velocity



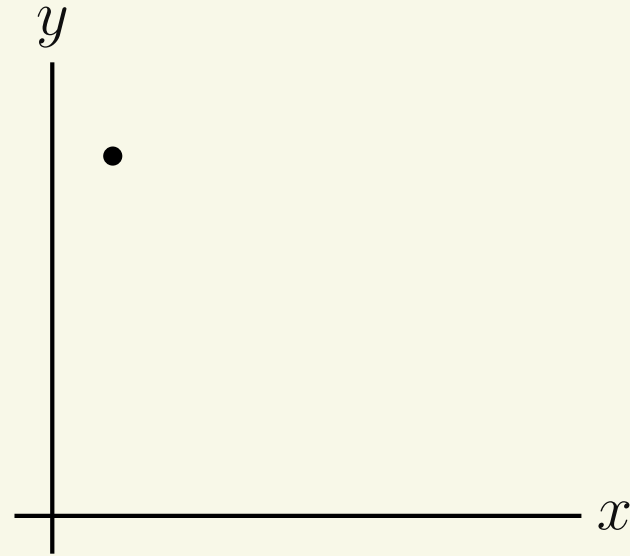
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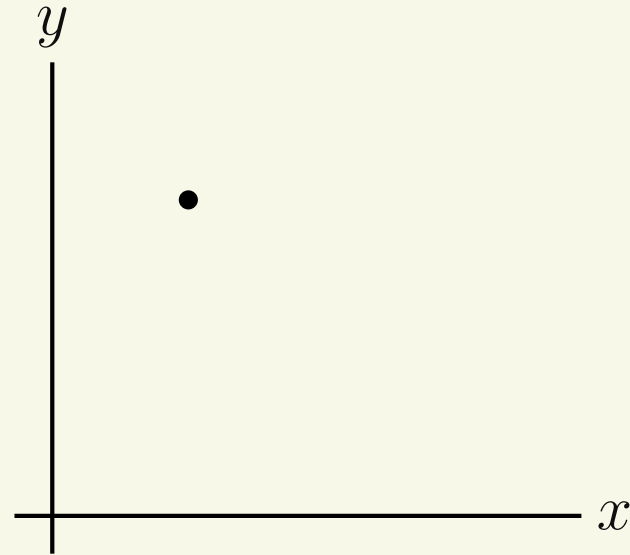
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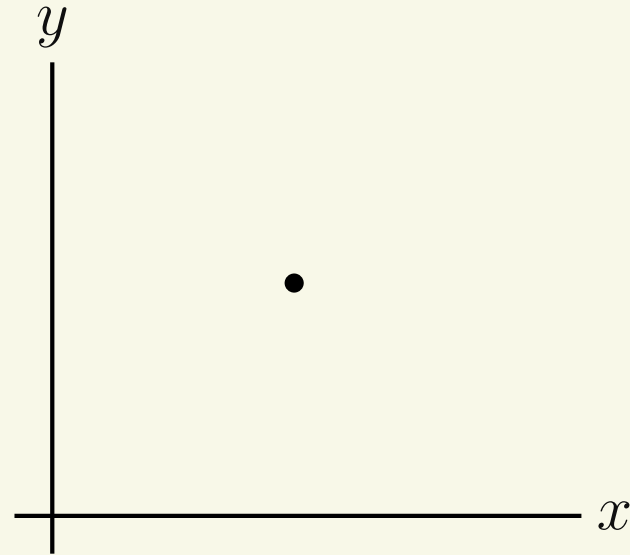
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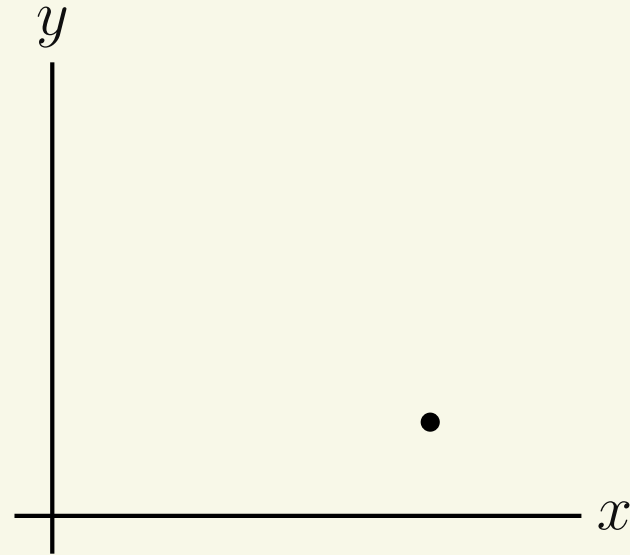
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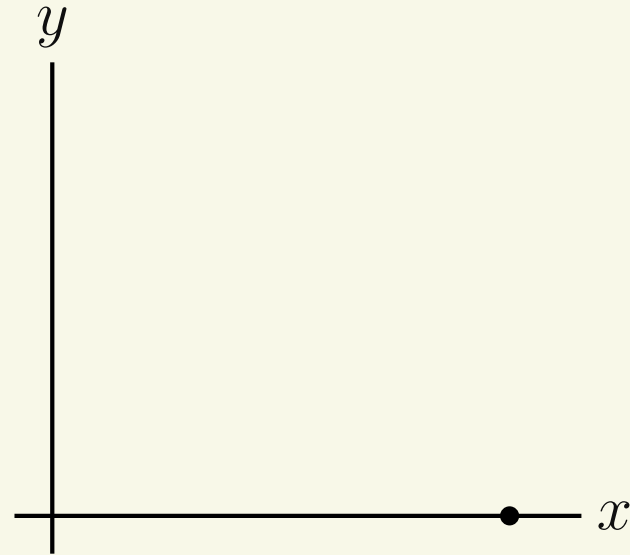
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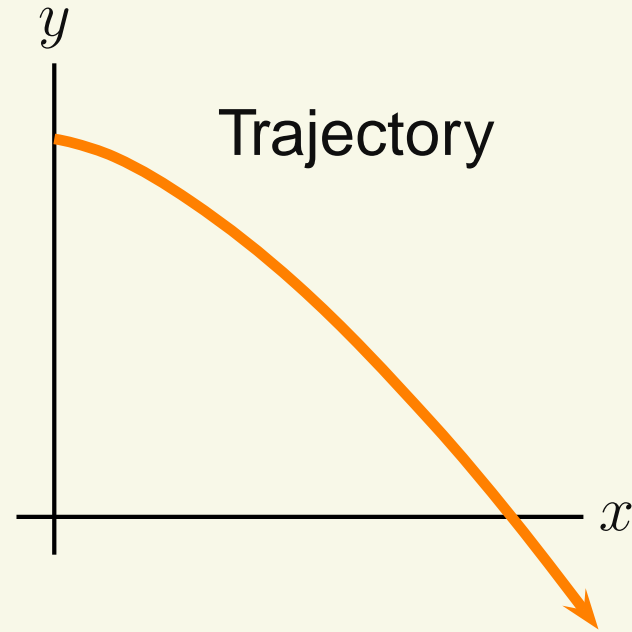
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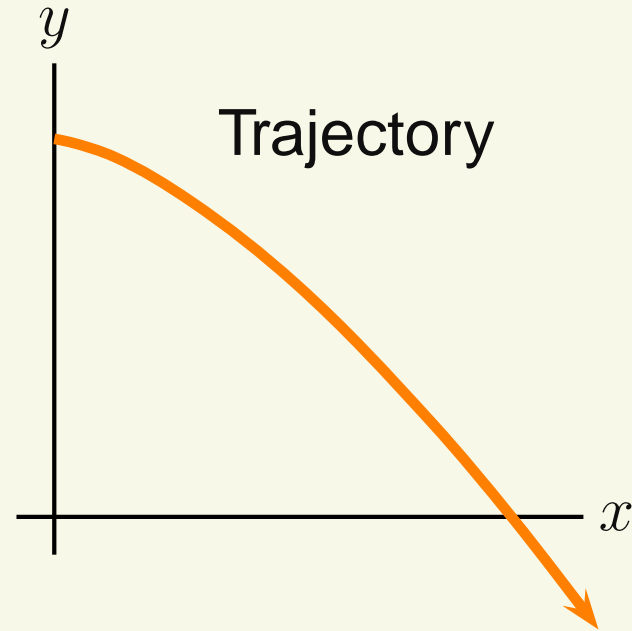
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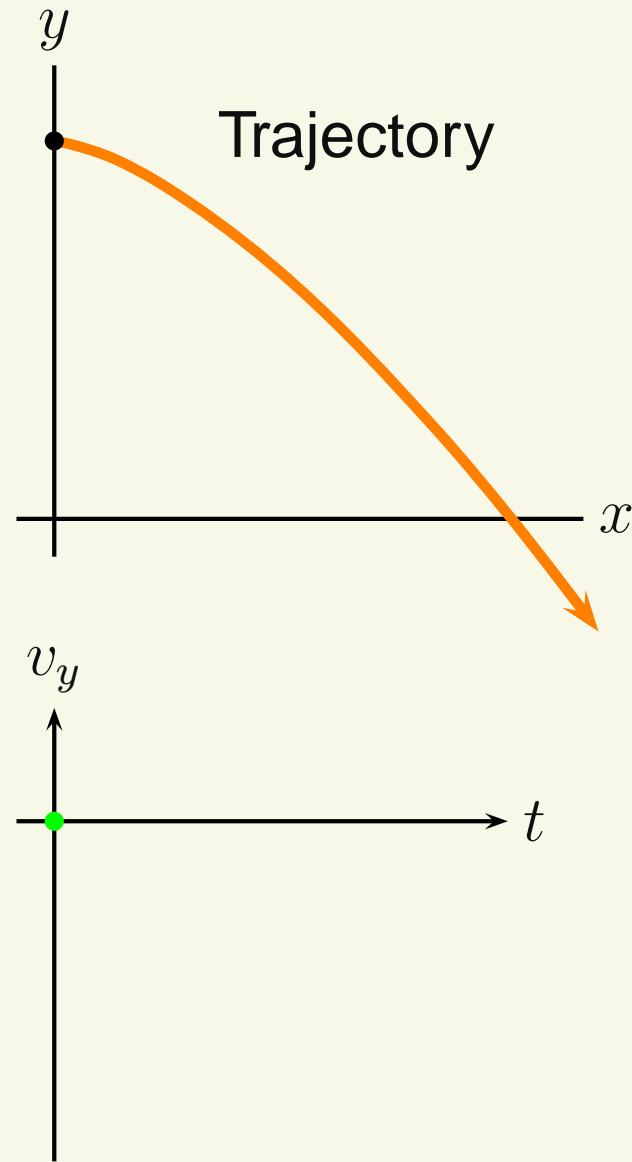
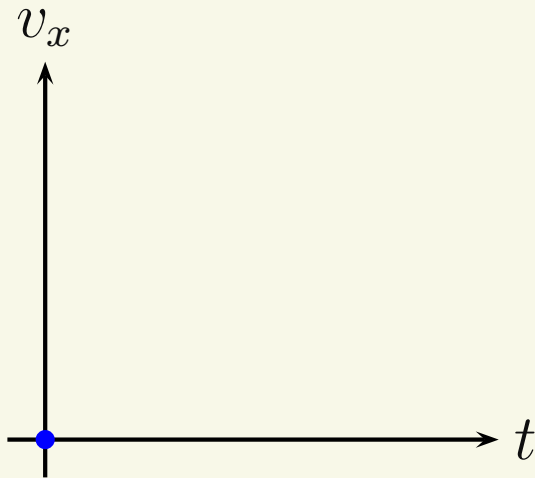
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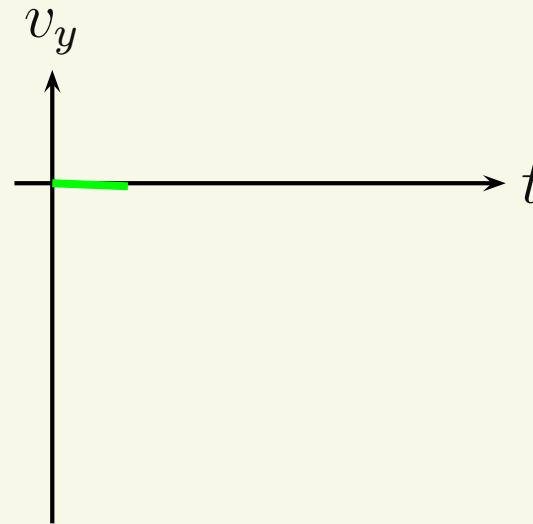
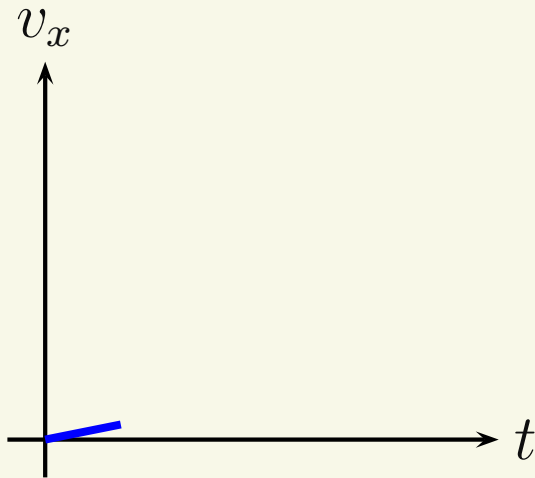
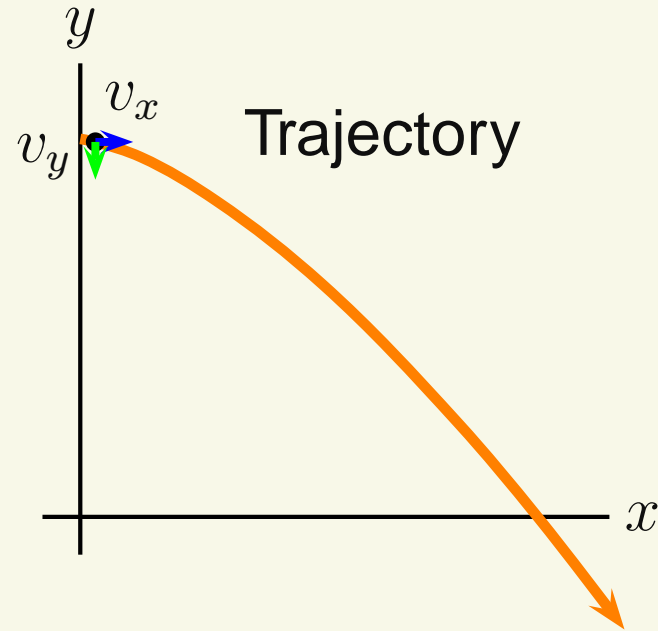
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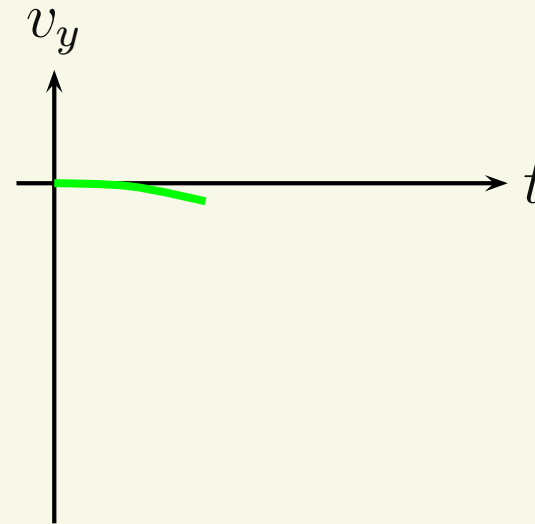
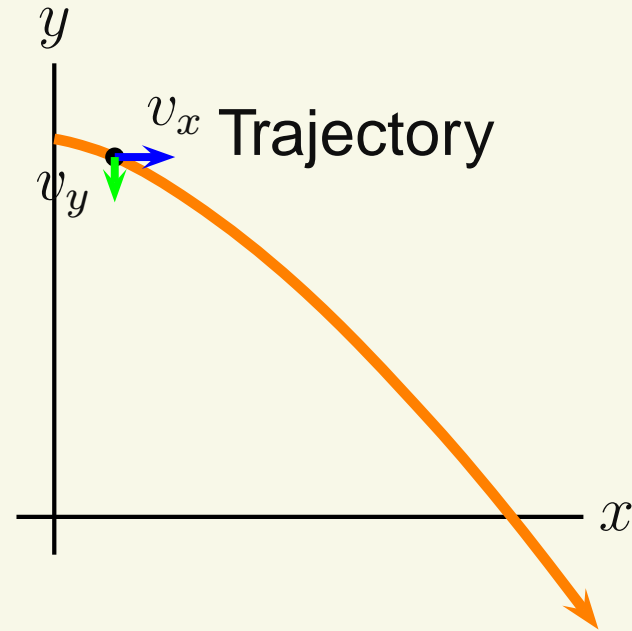
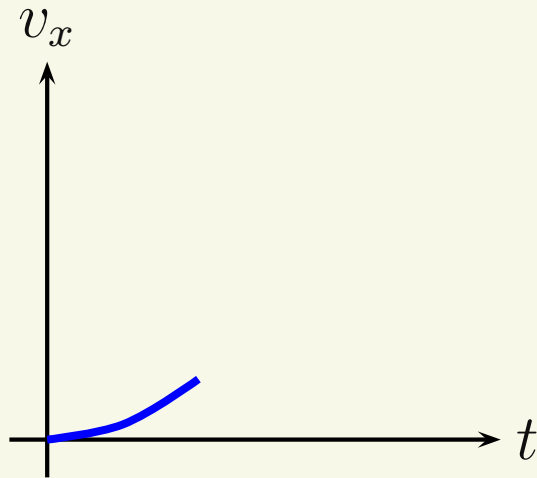
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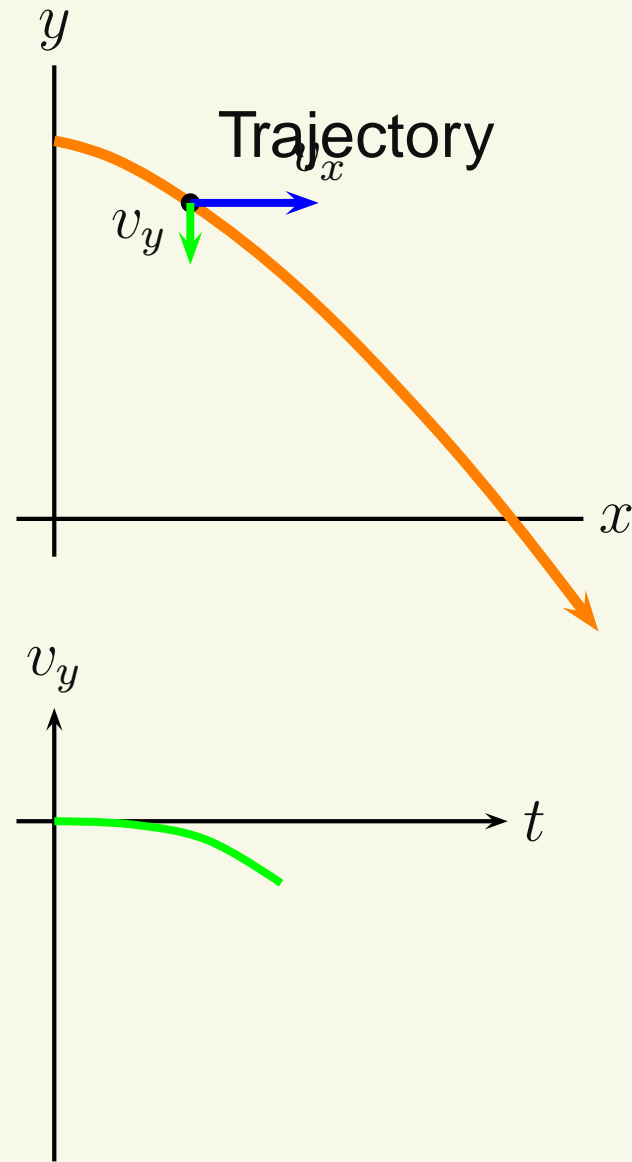
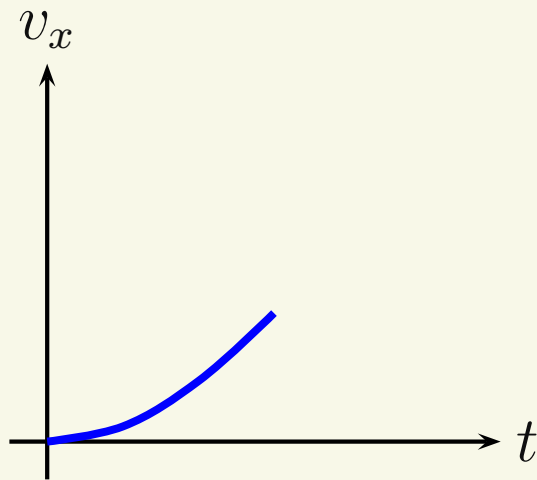
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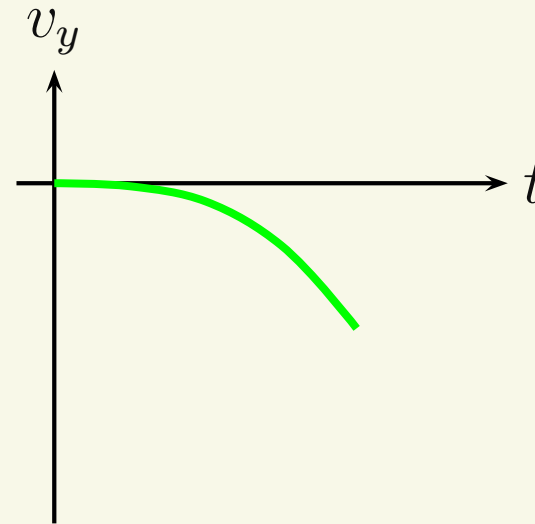
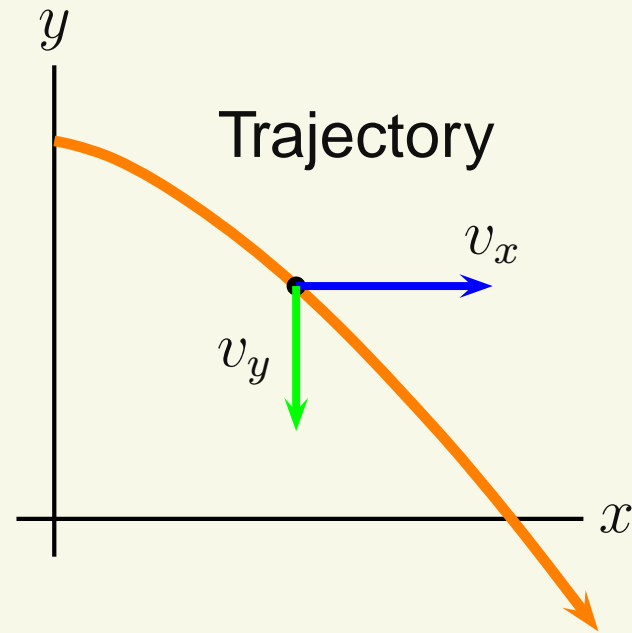
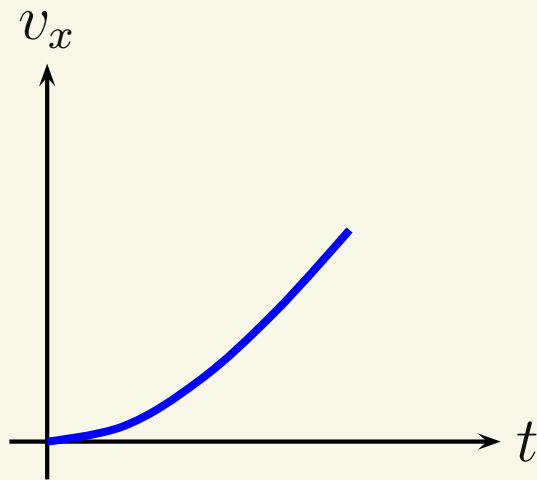
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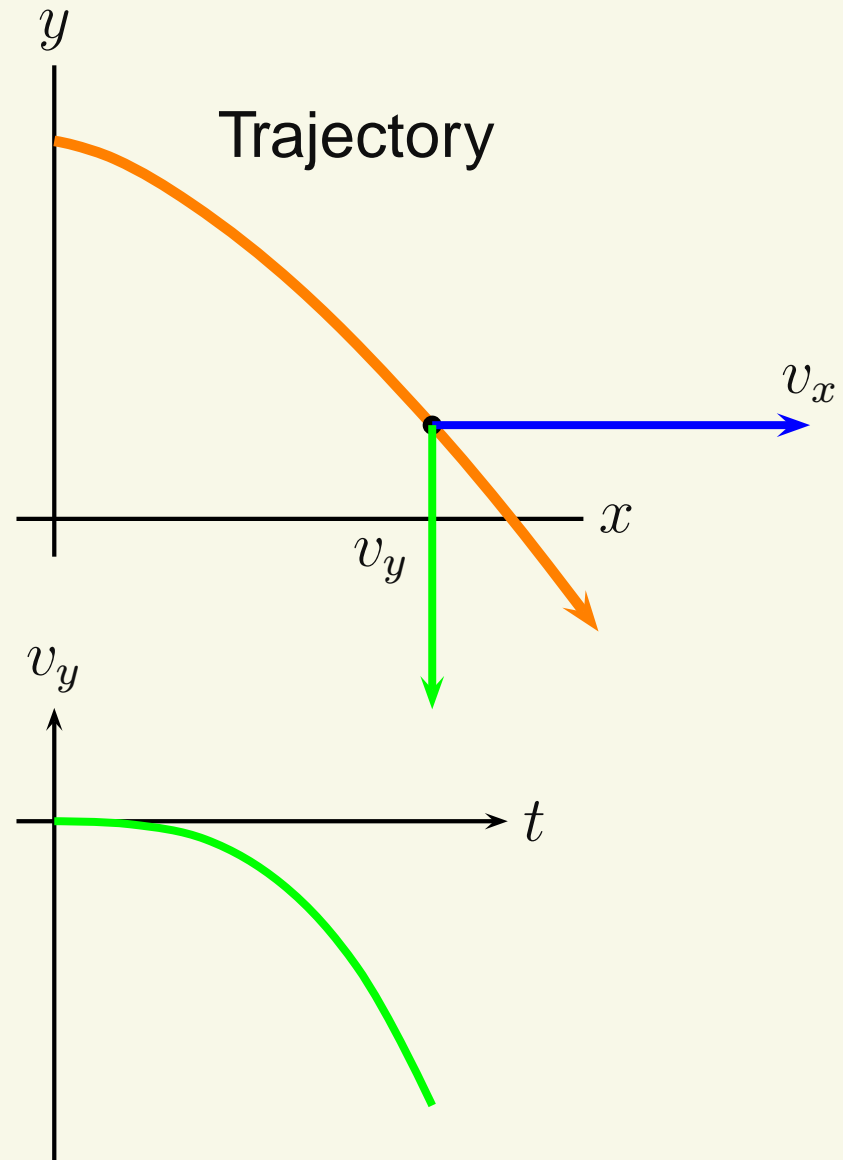
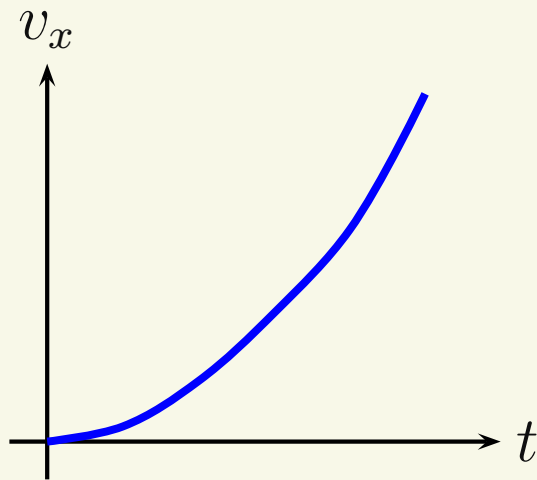
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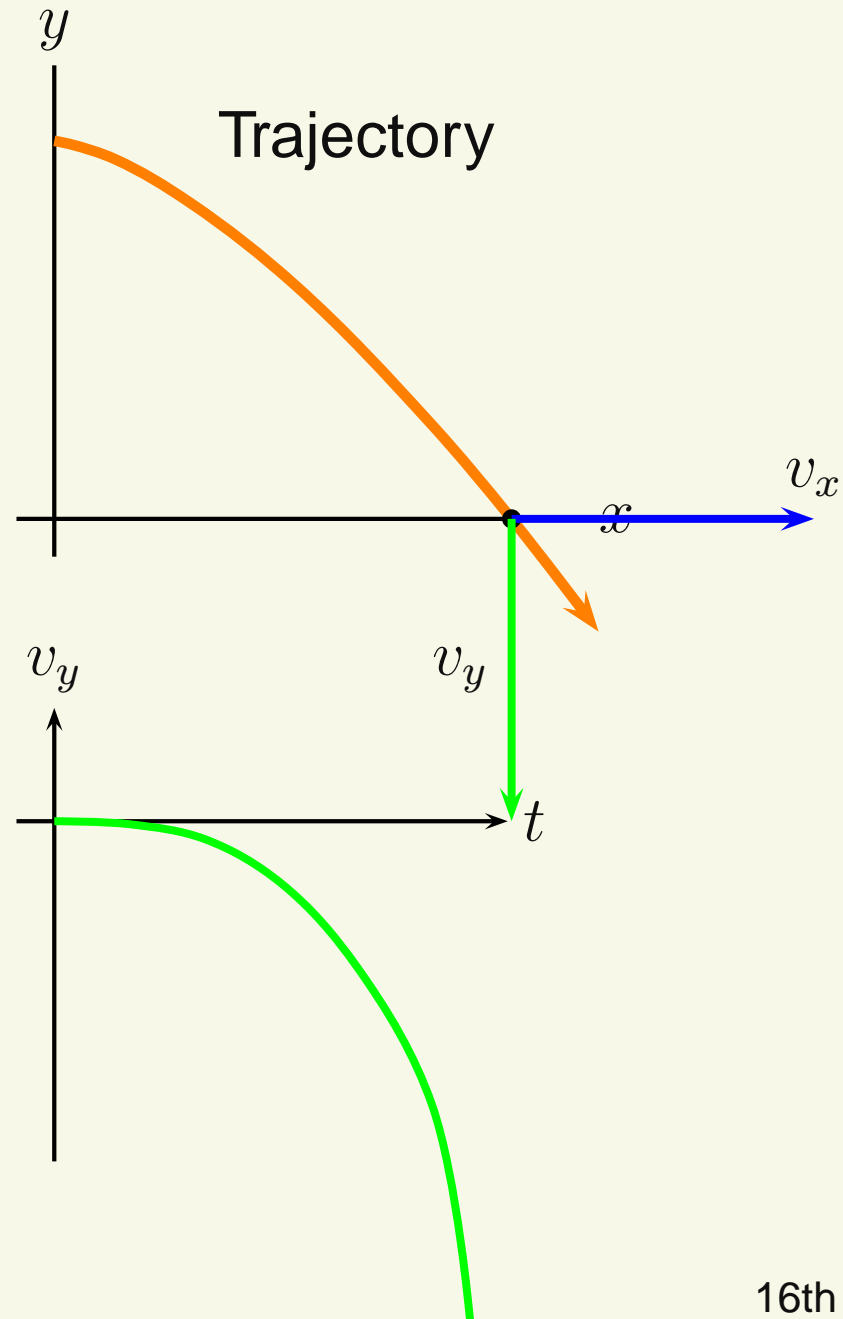
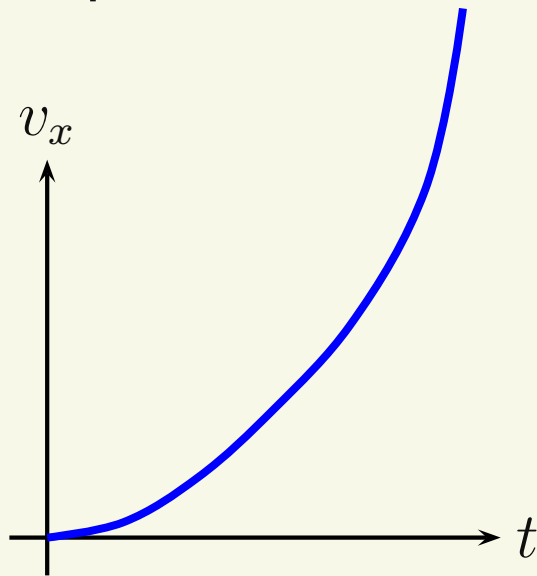
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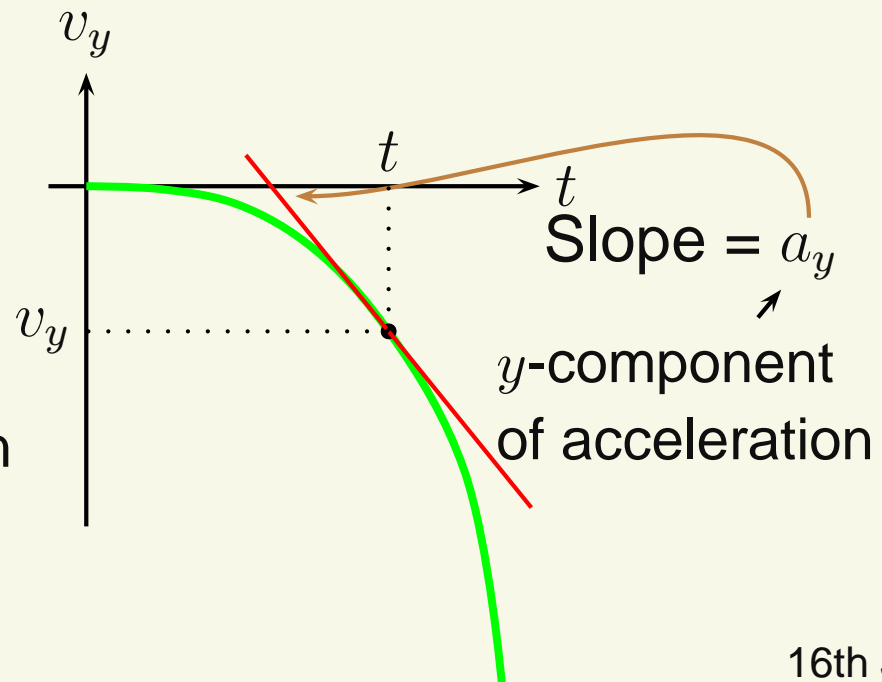
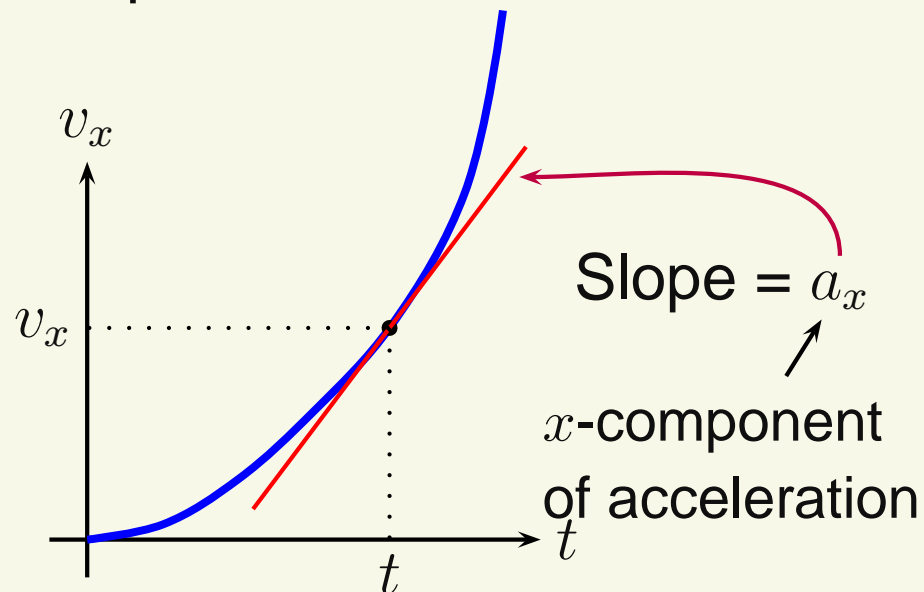
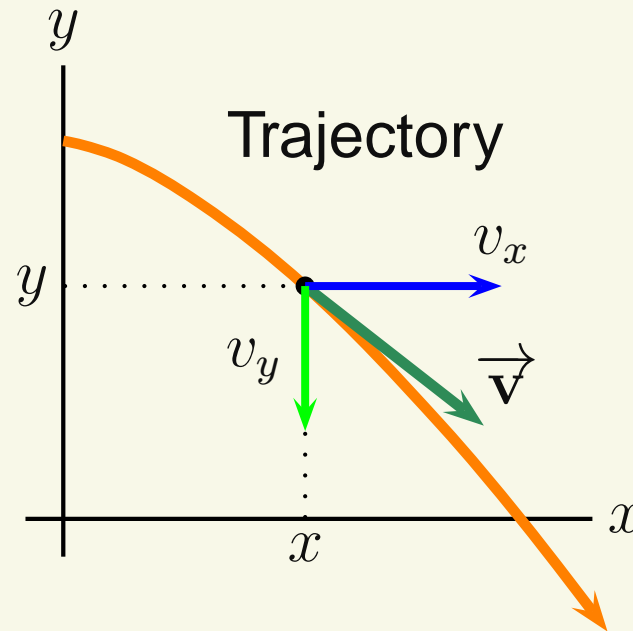
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Projectile Motion

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Gravity pulls straight down, so it causes acceleration in the y -direction only.

$$a_x = 0, a_y = -g \quad (\text{Down is negative})$$

Projectile Equations

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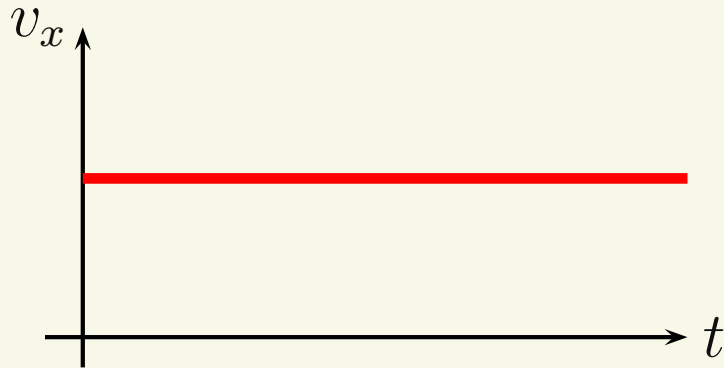
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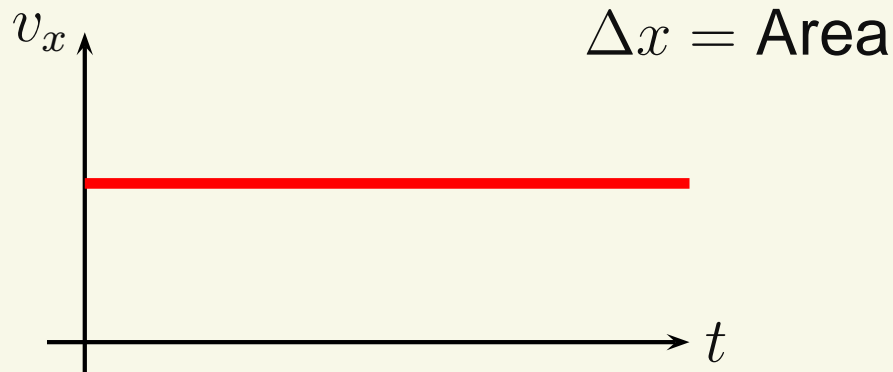
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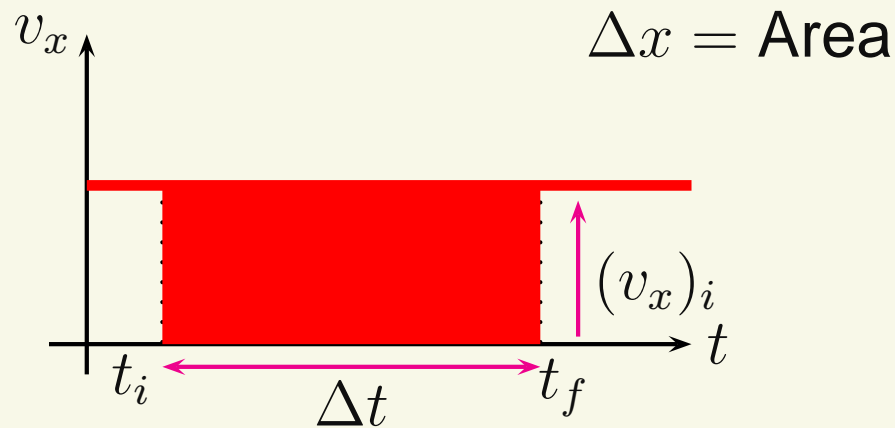
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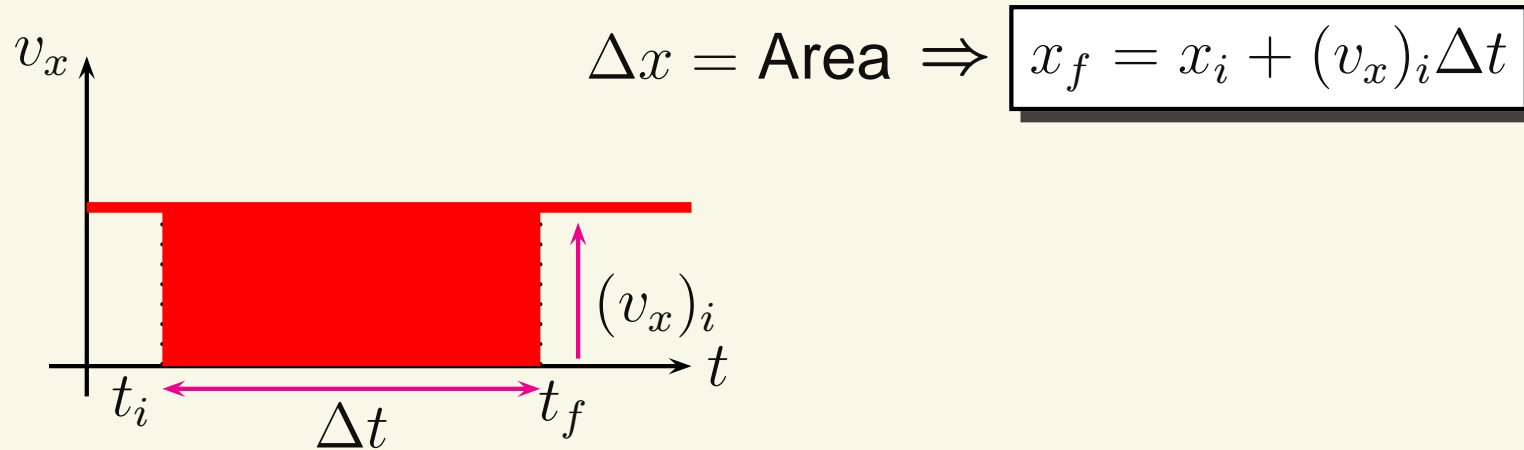
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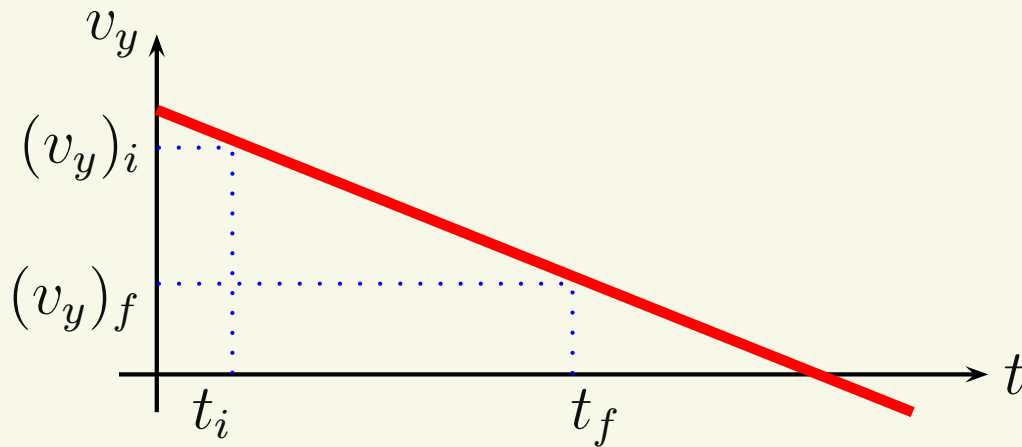
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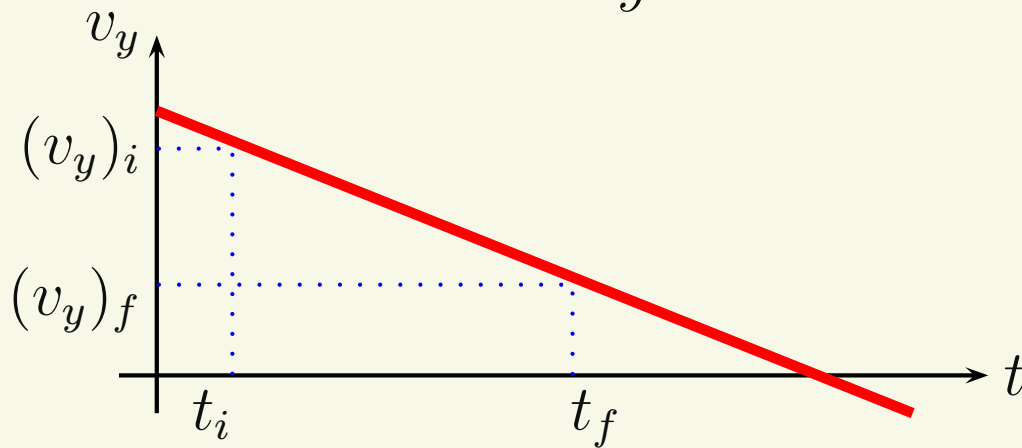


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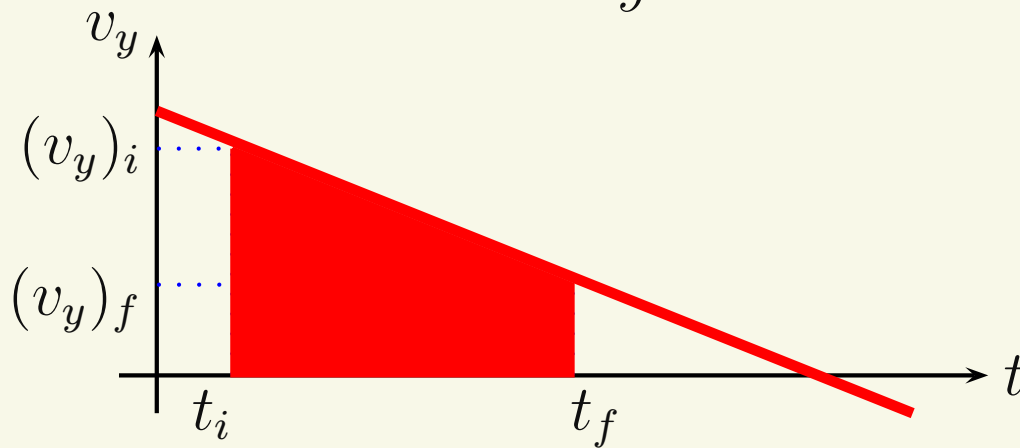


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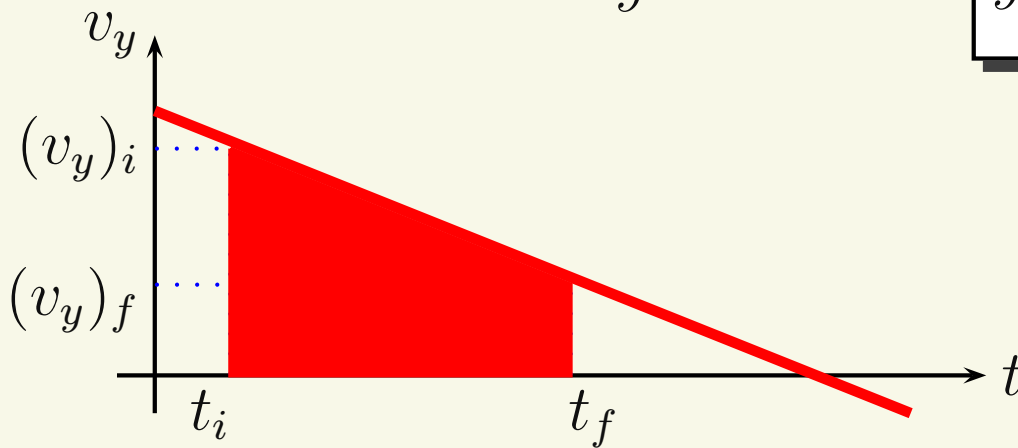


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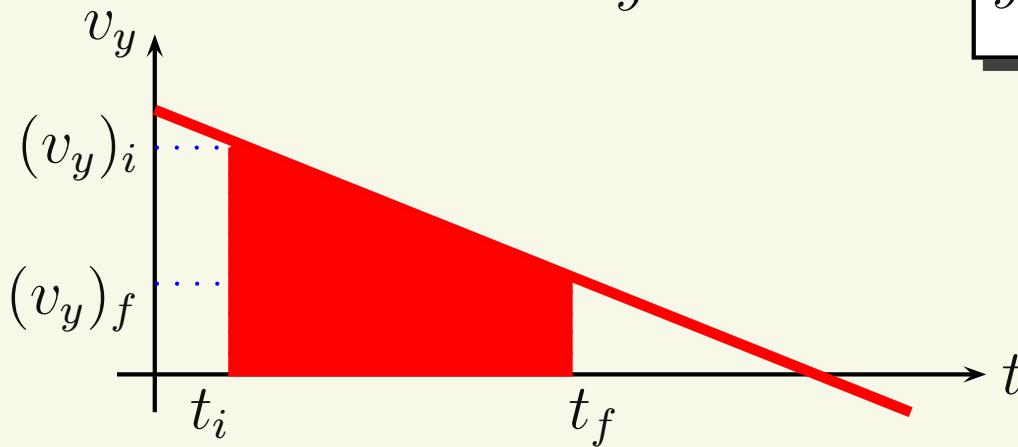


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In both of these equations
 $g = +9.8 \text{ m/s}^2$.