## June 12, Week 2

Today: Chapter 3, Vectors

Homework \#2 due tomorrow

No reading quiz for Monday

Please register your clicker. (There are 2 students without registered clickers.)

## Vector-Exercise Followup

If $\overrightarrow{\mathbf{A}}=5 \mathrm{~m} / \mathrm{s}$ at $37^{\circ}$, which of the following drawing correctly shows $\overrightarrow{\mathbf{B}}=5 \mathrm{~m} / \mathrm{s}$ at $135^{\circ}$ and $\overrightarrow{\mathbf{C}}=10 \mathrm{~m} / \mathrm{s}$ at $330^{\circ}$ ?




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$2 \times$ longer than $\overrightarrow{\mathbf{A}}$
$\underset{\overrightarrow{\mathrm{C}}}{\overrightarrow{\mathrm{C}} \quad \text { Equal length to } \overrightarrow{\mathrm{A}} \text {. }}$

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## Scalar Multiplication

Multiplying a vector by a scalar changes the magnitude but not the direction of a vector.
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Of particular interest:
$\overrightarrow{\mathrm{A}}=-\overrightarrow{\mathrm{B}}$
$\Rightarrow$ equal magnitude but opposite direction

- equal but opposite


## Vector Addition

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Graphical Addition - Drawing pictures and placing the vectors, "tip-to-tail" in order to determine the vector sum.

Component Addition - Uses trigonometry to calculate the vector sum.

## Graphical Addition Example

Add the following vectors.


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The vector sum or resultant, $\overrightarrow{\mathbf{R}}$, goes from the remaining tail to tip.

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A carefully drawn picture can give magnitude and direction of $\overrightarrow{\mathbf{R}}$. Simply use a ruler and protractor

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You can add vectors in either order and the answer is the same!

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$\xrightarrow{\vec{B} \xrightarrow[A]{ }} \overrightarrow{ }$
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$1 \cdot \cdot 2$
$0 \bullet$

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The average acceleration vector: $\overrightarrow{\mathbf{a}}_{a v}=\frac{\overrightarrow{\Delta \mathbf{v}}}{\Delta t} \Rightarrow \overrightarrow{\mathbf{a}}_{a v}$ in same direction as $\overrightarrow{\Delta \mathbf{v}}$
 Acceleration can be in a completely different direction than the motion

For point 1: $\quad \overrightarrow{\Delta \mathbf{v}}$ from $\overrightarrow{\mathrm{v}}_{i}$ to $\overrightarrow{\mathrm{v}}_{f}$ but we have to redraw them starting from the same place

## Components

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Mathematically, the components are the horizontal and vertical lengths from tip to tail.
$\overrightarrow{\mathbf{A}}_{x}, \overrightarrow{\mathbf{A}}_{y}$ are the vector components.
The components and the original vector are related by vector addition: $\overrightarrow{\mathbf{A}}_{x}+\overrightarrow{\mathbf{A}}_{y}=\overrightarrow{\mathbf{A}}$.

## Vector and Scalar Components



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$\overrightarrow{\mathbf{A}}_{x}, \overrightarrow{\mathbf{A}}_{y}, \overrightarrow{\mathbf{B}}_{x}, \overrightarrow{\mathbf{B}}_{y}=$ Vector Components

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$\overrightarrow{\mathbf{A}}_{x}, \overrightarrow{\mathbf{A}}_{y}, \overrightarrow{\mathbf{B}}_{x}, \overrightarrow{\mathbf{B}}_{y}=$ Vector Components
$A_{x}, A_{y}, B_{x}, B_{y}$ and their signs $=$ Scalar Components

## Scalar Component Exercise

Which of the following vectors has negative $x$ and positive $y$ scalar components?

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## Scalar Component Followup


$A_{x}$ positive $A_{y}$ positive

$B_{x}$ negative
$\overrightarrow{\mathrm{B}} \quad B_{y}$ negative

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$A_{x}$ positive $A_{y}$ positive

$B_{x}$ negative
$\overrightarrow{\mathrm{B}} \quad B_{y}$ negative


## Scalar Component Followup


$D_{x}$ positive
$D_{y}$ negative

## Scalar Component Followup


$D_{x}$ positive $D_{y}$ negative

## Scalar Component Followup



## $D_{x}$ positive $D_{y}$ negative



## Scalar Component Followup


$D_{x}$ positive
$D_{y}$ negative

$E_{x}$ negative
$E_{y}$ positive

## Trigonometry

The scalar components' numerical values are found using trigonometry since the magnitude and the scalar components always form a right triangle.

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Trigonometry - The mathematics of right $\left(90^{\circ}\right)$ triangles. Uses the fact that the ratio of the lengths of the sides of right triangle is always the same for the same angle.

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$$
H Y P=\text { Length of the Hypotenuse }
$$

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$H Y P=$ Length of the Hypotenuse
$O P P=$ Length of Opposite Side
$A D J=$ Length of Adjacent Side
Sine Function: $\sin \theta=\frac{O P P}{H Y P}$

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Sine Function: $\sin \theta=\frac{O P P}{H Y P}$
Cosine Function: $\cos \theta=\frac{A D J}{H Y P}$

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$A D J$
Cosine Function: $\cos \theta=\frac{A D J}{H Y P}$
Sine Function: $\sin \theta=\frac{O P P}{H Y P}$
Tangent Function: $\tan \theta=\frac{O P P}{A D J}$

## Scalar Components

In physics, the sides of the right triangle are the scalar components.

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Only guaranteed to work for the standard angle

## Component Exercise

Find the components of the vector $\overrightarrow{\mathbf{r}}=5 \mathrm{~m}$ at $110^{\circ}$.

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Find the components of the vector $\overrightarrow{\mathbf{r}}=5 \mathrm{~m}$ at $110^{\circ}$.
(a) $x=1.71 \mathrm{~m}, y=4.7 \mathrm{~m}$

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(a) $x=1.71 \mathrm{~m}, y=4.7 \mathrm{~m}$
(b) $x=1.71 m, y=-4.7 m$

## Component Exercise

Find the components of the vector $\overrightarrow{\mathbf{r}}=5 \mathrm{~m}$ at $110^{\circ}$.
(a) $x=1.71 \mathrm{~m}, y=4.7 \mathrm{~m}$
(b) $x=1.71 m, y=-4.7 m$
(c) $x=-1.71 m, y=4.7 m$

## Component Exercise

Find the components of the vector $\overrightarrow{\mathbf{r}}=5 \mathrm{~m}$ at $110^{\circ}$.
(a) $x=1.71 \mathrm{~m}, y=4.7 \mathrm{~m}$
(b) $x=1.71 m, y=-4.7 m$
(c) $x=-1.71 m, y=4.7 m$
(d) $x=-1.71 m, y=-4.7 m$

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(c) $x=-1.71 m, y=4.7 m$
(d) $x=-1.71 m, y=-4.7 m$
(e) Intentionally left blank.

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Find the components of the vector $\overrightarrow{\mathbf{r}}=5 \mathrm{~m}$ at $110^{\circ}$.
(a) $x=1.71 m, y=4.7 \mathrm{~m}$
(b) $x=1.71 m, y=-4.7 m$
(c) $\quad x=-1.71 m, y=4.7 m$
(d) $x=-1.71 m, y=-4.7 m$
(e) Intentionally left blank.

## Component Exercise

Find the components of the vector $\overrightarrow{\mathbf{r}}=5 \mathrm{~m}$ at $110^{\circ}$.

$$
\text { (c) } \quad x=-1.71 m, y=4.7 m
$$



Using the standard angle automatically gives correct signs:

$$
\begin{aligned}
& x=(5 m) \cos 110^{\circ}=-1.71 m \\
& y=(5 m) \sin 110^{\circ}=4.7 m
\end{aligned}
$$

## Scalar Components II

To find the magnitude and the angle from the components:


## Scalar Components II

To find the magnitude and the angle from the components:


$$
\tan \theta=\frac{A_{y}}{A_{x}}
$$

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## Scalar Components II

To find the magnitude and the angle from the components:


## Scalar Components II

To find the magnitude and the angle from the components:


Arctangent

$$
A^{2}=A_{x}^{2}+A_{y}^{2}
$$

## Scalar Components II

To find the magnitude and the angle from the components:


## Scalar Components II

To find the magnitude and the angle from the components:


Example: Find the magnitude and direction for the vector with components $A_{x}=1 \mathrm{~m}$ and $A_{y}=1 \mathrm{~m}$.

