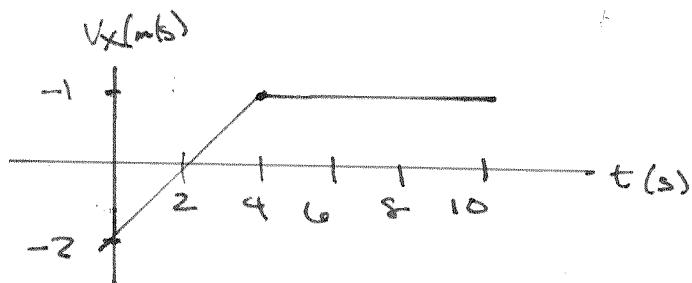


Physics 151

Hw #2

#1



Train starts at ORIGIN
Draw position graph

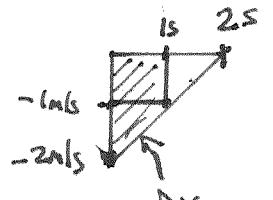
Notice: for first 4s, $V_x = \text{line}$
⇒ Constant Acceleration
So x vs t is a PARABOLA

For $t > 4$, $V_x = \text{horizontal line}$
⇒ Uniform motion ($a_x = 0$) ⇒ so
• x vs t is a straight line.

To get Mastering to correctly draw a curve (like a parabola), we need to give it at least 4 points, so let's find position at $t = 1s, 2s, 3s, 4s$ (and also at $t=0, x=0$).

Easiest to use that area under V_x vs t graph is Δx

For $t = 1s$



$$so -1 \text{ m/s} \times 1 \text{ s}$$

$$\Delta x = -(1 \text{ s})(-1 \text{ m/s}) = +1 \text{ m}$$

$$-(2 \text{ s} - 1 \text{ s}) \times 1 \text{ s} \\ = -1 \text{ m/s}$$

$$\Delta x = \frac{1}{2} (1 \text{ s}) (-1 \text{ m/s}) = -0.5 \text{ m}$$

$$\Rightarrow \Delta x = -1.5 \text{ m}$$

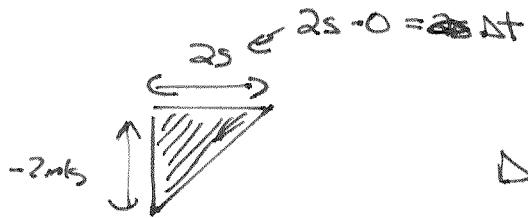
TRAIN Starts from origin \Rightarrow ~~starts from origin~~

$$x_1 = -1.5 \text{ m}$$

$$x_{\text{at } t=1s}$$

To find at $t=2s$, Actually easier to start from O Again

Since then we can just use Area of triangle.



$$\Delta X = \frac{1}{2}(2s)(-2m) = -2m$$

Since this from when train was at origin

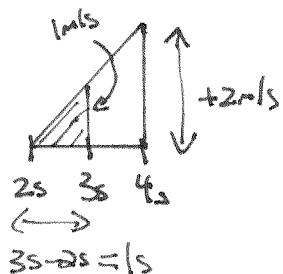
$$X_2 = -2m$$

(By the way, Neither of these Negative values should surprise you.

V_x negative \Rightarrow moving to left. Train starts at 0 , so Negative position to begin with)

$t=3s$, Here its probably easier to find area starting at $t=2s$

So now $\Delta X = X_f - X_i \Rightarrow X_f = X_i + \Delta X$. ~~we'll need to~~ Use $X_i = X_2 = -2m$



$$\Delta X = \frac{1}{2}(1s)(2m) = 0.5m \leftarrow \begin{array}{l} \text{Velocity now positive} \\ \text{So train is moving to right} \end{array}$$

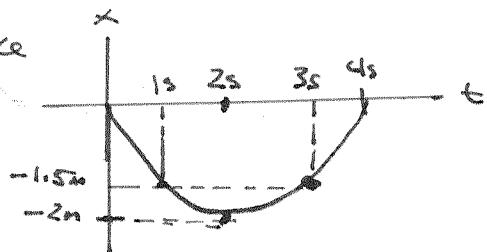
$$X_3 = X_2 + \Delta X = -2m + 0.5m = -1.5m$$

$t=4s$, Again start from $2s$, picture above can be used $\Delta X = \frac{1}{2}(2s)(2m) = 2m$

$$\text{So } X_4 = X_2 + \Delta X = -2m + 2m = 0$$

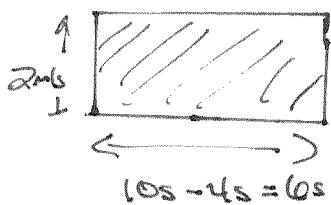
So by $t=4s$, train is back at origin

Our PARAbola Looks like



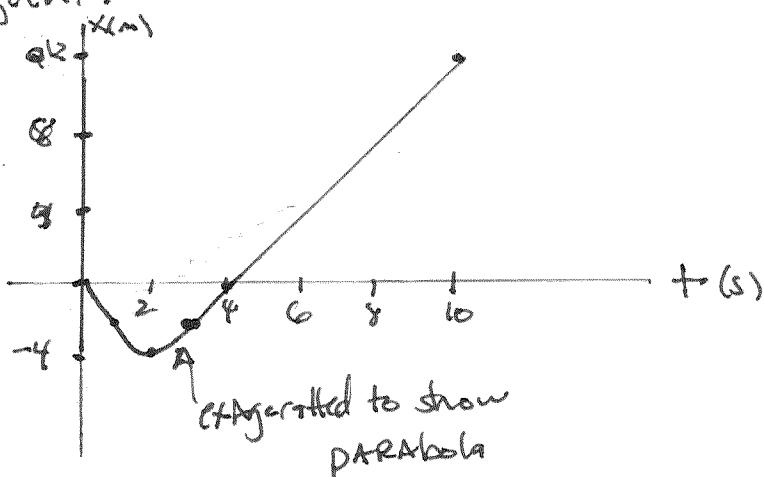
For STRAIGHT Line piece: Pretty Simple since at $t=4s$ $x_4 = 0$

So Area under Rectangle from $4s$ to $10s$ gives $\frac{x_{10} - x_4}{10}$ since $\Delta x = \frac{x_{10} - x_4}{10}$
 $= x_{10} - 0$
 $= x_{10}$



$$\Delta x = (6s)(12mb) = 12m \Rightarrow x_{10} = 12m$$

All together:



b.) Find The Acceleration at $t = 2s$

For All times between 0 and 4s, Acc has the same value. That value is the slope of the line.

It's Easiest to use $t_i = 0, t_f = 4s$

$$\Rightarrow V_{xi} = -2 \text{ m/s}, V_{xf} = +2 \text{ m/s}$$

$$\therefore a_x = \frac{\Delta V_x}{\Delta t} = \frac{+2 \text{ m/s} - (-2 \text{ m/s})}{4s - 0} = \frac{2 \text{ m/s} + 2 \text{ m/s}}{4s} = \frac{4 \text{ m/s}}{4s} = 1 \text{ m/s}^2$$

c.) What is Acc. at $t = 6s$?

For All times Between 4s and 10s, $\Delta V_x = 0$

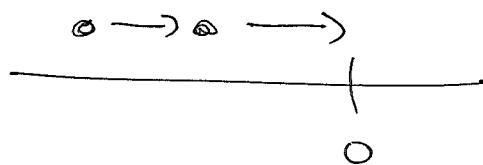
Since V_x is Constant at 2m/s

$$\therefore a_x = 0$$

#2

For EACH MOTION DIAGRAM, Determine
the sign of position, velocity, and Acceleration

(a)



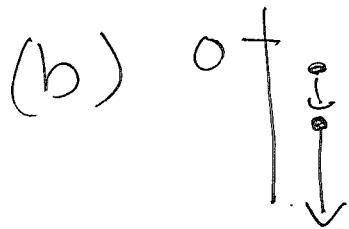
All dots to left of O

\Rightarrow Negative x

Both arrows to Right

\Rightarrow positive v_x

Increasing Length \Rightarrow same
sign as $v \Rightarrow$ positive a_x

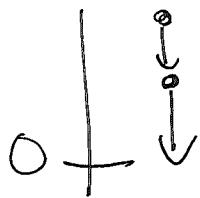


All dots below O \Rightarrow Negative y

Both arrows down \Rightarrow Negative v_y

Arrows increasing in length \Rightarrow Negative a_y
 $\hookrightarrow a_y$ has same sign as v_y

(c)



All dots above O \Rightarrow positive y

Both arrows down \Rightarrow Negative v_y

Arrows increasing in length \Rightarrow Negative a_y

~~#3~~

$t(s)$	$U(\text{mph})$	$V(\text{m/s})$
0	0	0
2	28	13
4	46	21
6	66	27
8	70	31
10	78	35

c) Convert to m/s. Easiest to use $1\text{mph} = .447\text{m/s}$

$$28\text{mph} \times \frac{.447\text{m/s}}{\text{mph}} = 12.516 = 13\text{m/s}$$

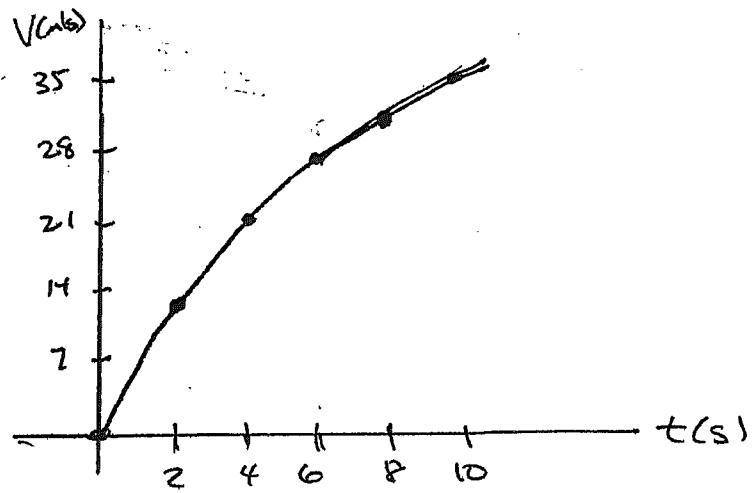
$$46\text{mph} \times \frac{.447\text{m/s}}{\text{mph}} = 20.562 = 21\text{m/s}$$

$$66\text{mph} \times \frac{.447\text{m/s}}{\text{mph}} = 29.82 = 27\text{m/s}$$

$$70\text{mph} \times \frac{.447\text{m/s}}{\text{mph}} = 31.29 = 31\text{m/s}$$

$$78\text{mph} \times \frac{.447\text{m/s}}{\text{mph}} = 34.866 = 35\text{m/s}$$

MAKE GRAPH (Next Page) - Is Acceleration Constant?



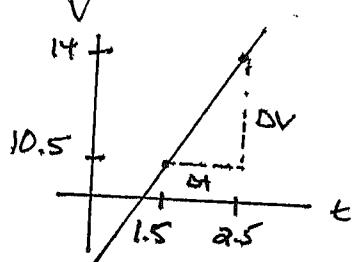
GRAPH DEFINITELY
NOT A STRAIGHT
 LINE. SO
 ACCELERATION NOT
 CONSTANT

- b) DRAW Smooth Curve (See Above) Estimate Acceleration at 2s AND 8s.

For 2s, Look at $t_i = 1.5s$ AND $t_f = 2.5s$

From graph, I estimate that at $t_i = 1.5s$, $V_i = 10.5 \text{ m/s}$ (your # MAY BE DIFFERENT)

AND $t_f = 2.5s$ $V_f = 14 \text{ m/s}$



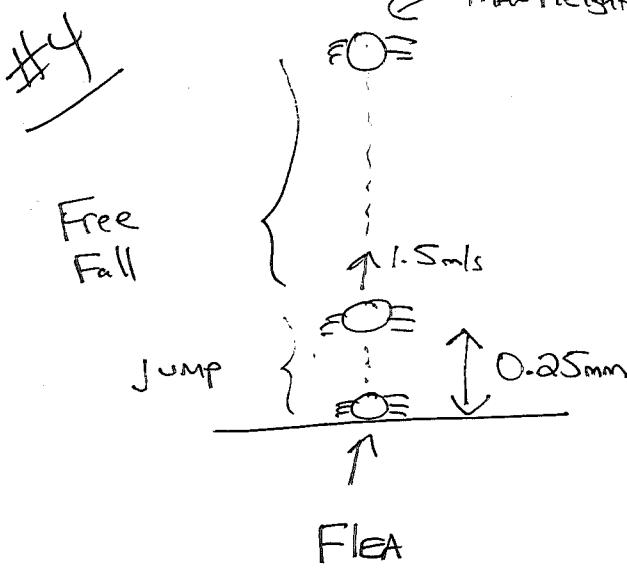
$$a = \text{slope} = \frac{\Delta V}{\Delta t} = \frac{(14 \text{ m/s} - 10.5 \text{ m/s})}{(2.5s - 1.5s)} = \frac{3.5 \text{ m/s}}{1s} = 3.5 \text{ m/s}^2$$

$$\Rightarrow a \approx 3.5 \text{ m/s}^2 \text{ at } 2s$$

For 8s, use $t_i = 7.5s$ and $t_f = 8.5s$

From graph $V_i = 30 \text{ m/s}$, $V_f = 31.5 \text{ m/s}$

$$\Rightarrow a = \frac{\Delta V}{\Delta t} = \frac{(31.5 \text{ m/s} - 30 \text{ m/s})}{(8.5s - 7.5s)} = \frac{1.5 \text{ m/s}}{1s} = 1.5 \text{ m/s}^2$$



Known

Let jump be motion #1 \Rightarrow

$$y_{i,1} = 0 \quad y_{f,1} = 0.25\text{mm}$$

$$= 0.25 \times 10^{-3}\text{m}$$

$$= 2.5 \times 10^{-4}\text{m}$$

$$(V_y)_{i,1} = 0 \quad (V_y)_{f,1} = 1.5\text{m/s}$$

Unknown: $(a_y)_1 = ?$

Let FREE-FALL BE MOTION #2 \Rightarrow

$(V_y)_{i,2} = 1.5\text{m/s}$ \leftarrow THE FINAL velocity of the 1st motion
is the INITIAL Velocity of the 2nd

$$y_{i,2} = 0.25\text{mm} \quad \leftarrow \text{Final position of 1st is initial of 2nd}$$

$$(a_y)_2 = -9.8\text{m/s}^2$$

$$(V_y)_{f,2} = 0 \quad \leftarrow \text{at max height, velocity is ZERO.}$$

a.) what is Acceleration During jump? \leftarrow NO info about time, so

$$(V_y)_{f,1}^2 = (V_y)_{i,1}^2 + 2(a_y)(y_f - y_i) \Rightarrow (1.5\text{m/s})^2 = 0 + 2(a_y)(2.5 \times 10^{-4}\text{m})$$

$$\Rightarrow (a_y)_1 = \frac{(1.5\text{m/s})^2}{2(2.5 \times 10^{-4}\text{m})} = \frac{2.25\text{m}^2/\text{s}^2}{5 \times 10^{-4}\text{m}} = 4500\text{m/s}^2$$

Unit: $\frac{\text{m}^2}{\text{s}^2} \times \frac{1}{\text{m}} = \text{m/s}^2$

b.) How long does acceleration last?

Now THAT WE HAVE ACCELERATION

$(V_y)_{f,1} = (V_y)_{i,1} + (a_y)_{1,1} \Delta t_1$ will work

$$\Rightarrow 1.5 \text{ m/s} = 0 + (4500 \text{ m/s}^2) \Delta t_1$$

$$\Rightarrow \Delta t_1 = \frac{1.5 \text{ m/s}}{4500 \text{ m/s}^2} = 0.00033\ldots \text{ s} \times \frac{1000 \text{ ms}}{\text{s}} = 0.33 \text{ ms}$$

$\frac{1}{3}$ of a millisecond

c.) How high?

For #2 motion, Again there's no info about time

$$\Rightarrow (V_y)_{f,2}^2 = (V_y)_{i,2}^2 + 2(a_y)_2(y_{f,2} - y_{i,2})$$

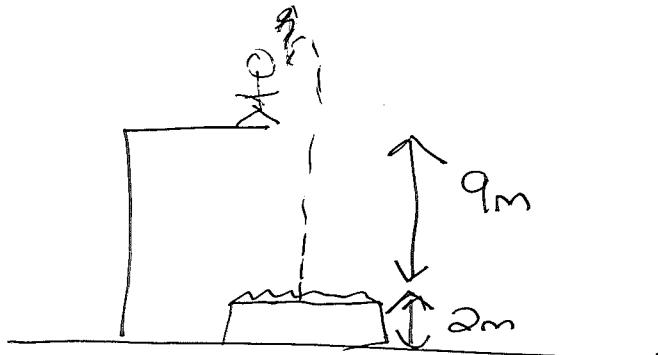
$$\Rightarrow 0^2 = (1.5 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(y_{f,2} - 2.5 \times 10^{-4} \text{ m})$$

$$\Rightarrow 0 = 2.25 \text{ m}^2/\text{s}^2 - 19.6 \text{ m/s}^2 (y_{f,2} - 2.5 \times 10^{-4} \text{ m})$$

$$\Rightarrow y_{f,2} = 2.5 \times 10^{-4} \text{ m} + \frac{2.25 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = 2.5 \times 10^{-4} \text{ m} + 0.115 \text{ m} = \underline{\underline{0.115 \text{ m}}}$$

still

#5



WHAT MINIMUM ACCELERATION
IN THE WATER TO KEEP FROM
HITTING BOTTOM OF POOL?
MAKE DOWN NEGATIVE.

DIVER FREE FALLS FOR 9M THEN DECELERATES ~~BECAUSE~~ TO POOL'S
WATER.

KEY: THE FINAL VELOCITY OF THE FREE FALL IS THE INITIAL
VELOCITY OF THE DECELERATION.

LET FREE FALL BE MOTION #1, DECELERATION #2

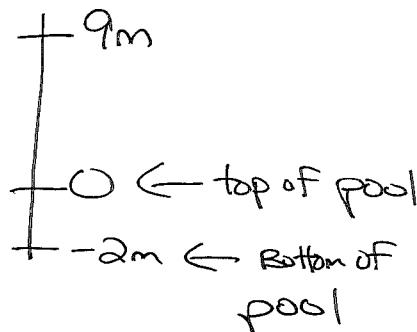
$$\Rightarrow \underline{(V_y)_{i,2}} = \underline{(V_y)_{f,1}}$$

KNOWN FOR #1

$$(V_y)_{i,1} = 0 \leftarrow \text{steps off platform}$$

$$(a_y)_1 = -9.8 \text{ m/s}^2 \leftarrow \text{Free Fall And Down
is Negative}$$

COORDINATE SYSTEM:



$$\Rightarrow (y_i)_1 = 9 \text{ m}$$

$$(y_f)_1 = 0 \text{ m}$$

$$\left\{ (y_i)_2 = 0 \text{ m}, (y_f)_2 = -2 \text{ m} \right\}$$

UNKNOWN for #1 : $(V_y)_{f,1} = ?$, $\Delta t_1 = ?$

KNOWN for #2 : (AS given on Previous page) In the particle model, we don't worry about where the diver "really" is. We just have a dot that we want to have stopped when it gets to the bottom $\Rightarrow (y_f)_2 = -2m$, $(V_y)_{f,2} = 0$

UNKNOWN for #2 : $(V_y)_{i,2} = ?$, $\Delta t_2 = ?$, $(a_y)_2 = ?$

Final Answer

Since NO TIME INFO use $(V_y)_f^2 = (V_y)_i^2 + 2a_y(y_f - y_i)$

This CAN give us $(V_y)_{f,1}$ and since that EQUALS $(V_y)_{i,2}$ we can then use it again to find $(a_y)_2$

$$\text{So } (V_y)_{f,1}^2 = 0^2 + 2(-9.8 \text{ m/s}^2)(0 - 9 \text{ m}) = 0 + 2(-9.8 \text{ m/s}^2)(-9 \text{ m})$$

$$\Rightarrow (V_y)_{f,1}^2 = +176.4 \text{ m}^2/\text{s}^2. \text{ To Be Careful we should}$$

Use the fact that $(V_y)_{f,1} = \pm \sqrt{176.4 \text{m/s}^2}$

We'll see in a bit that it's not that important^{Algebraically} to this problem, but we should use

$$(V_y)_{f,1} = -\sqrt{176.4 \text{m/s}^2} = -13.28 \text{m/s} \leftarrow \text{Downward}$$

So $(V_y)_{i,2} = -\sqrt{176.4 \text{m/s}^2}$ too

$$(V_y)_{f,2}^2 = (V_y)_{i,2}^2 + 2(a_y)_2 [(y_f)_2 - (y_i)_2]$$

$$\Rightarrow 0^2 = [-\sqrt{176.4 \text{m/s}^2}]^2 + 2(a_y)_2 (-2m - 0)$$

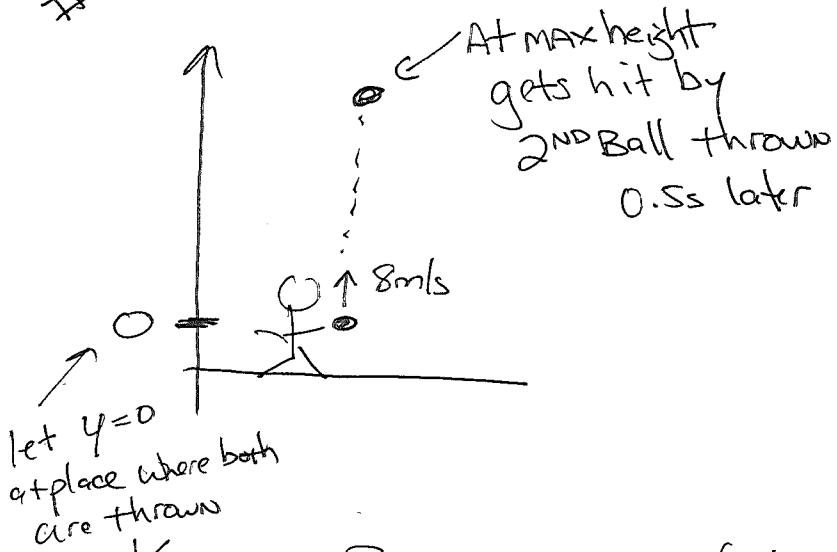
\curvearrowleft when SQUARED becomes positive,
so ~~negative~~ wasn't that important to the algebra.

$$\Rightarrow 0 = 176.4 \text{m/s}^2 - 4m(a_y)_2$$

$$\Rightarrow (a_y)_2 = +\frac{176.4 \text{m/s}^2}{4m} = \underline{\underline{+44.1 \text{m/s}^2}}$$

Positive Acceleration is Correct. Negative velocity, Decreasing Speed $\Rightarrow V_y$ and a_y have opposite signs

#6



WHAT SPEED NEEDED
FOR 2ND BALL?

KNOWN FOR Ball #1 : $(V_y)_{i,1} = 8 \text{ m/s}$ $(a_y)_1 = -9.8 \text{ m/s}^2$

$(V_y)_{f,1} = 0$ ← max height, $(y_i)_1 = 0$

UNKNOWN : $(y_f)_2 = ?$, $\Delta t_2 = ?$

KNOWN FOR Ball #2 : $(y_i)_2 = 0$, $(a_y)_2 = -9.8 \text{ m/s}^2$

We also know that #2 at the same place as #1

$$\therefore (y_f)_2 = (y_f)_1$$

And that #2 gets there quicker! It has 0.5s less ~~less~~ time $\Rightarrow \Delta t_2 = \Delta t_1 - 0.5 \text{ s}$

UNKNOWN : $(y_f)_2 = ?$, $(V_f)_2 = ?$, $\Delta t_2 = ?$, $(V_i)_2 = ?$

Not guaranteed to beat its
MAX height

FINAL ANSWER

We have enough information to solve for

$(V_y)_f$, and Δt_1 !

$$(V_y)_f = (V_y)_i + a_y \Delta t \Rightarrow 0 = 8 \text{ m/s} - 9.8 \text{ m/s}^2 (\Delta t_1)$$

$$\Rightarrow \Delta t_1 = \frac{8 \text{ m/s}}{9.8 \text{ m/s}^2} = 0.8163 \text{ s}$$

$$\text{So } \Delta t_2 = 0.8163 \text{ s} - 0.5 \text{ s} = 0.3163 \text{ s}$$

$$(V_y)_f^2 = (V_y)_i^2 + 2a_y(y_f - y_i) \Rightarrow 0^2 = (8 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(y_f) - 0$$

$$\Rightarrow 0 = 64 \text{ m}^2/\text{s}^2 - 19.6 \text{ m/s}^2 (y_f), \Rightarrow (y_f)_1 = \frac{64 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = 3.2653 \text{ m}$$

$$\text{So } (y_f)_2 = 3.2653 \text{ m}$$

$$\text{Finally: } y_f = y_i + (V_y)_i \Delta t + \frac{1}{2} a_y \Delta t^2$$

$$\Rightarrow 3.2653 \text{ m} = 0 + (V_y)_{i,2} (0.3163 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2) (0.3163 \text{ s})^2$$

$$\Rightarrow 3.2653 \text{ m} = (V_y)_{i,2} (0.3163 \text{ s}) - 0.49 \text{ m}$$

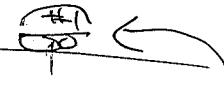
$$\Rightarrow (V_y)_{i,2} = \frac{3.2653 \text{ m} + 0.49 \text{ m}}{0.3163 \text{ s}} = \frac{3.75552 \text{ m}}{0.3163 \text{ s}} = 11.873 \text{ m/s} = \underline{\underline{12 \text{ m/s}}}$$

#7

#1



#1's position after 2s
when #2 starts



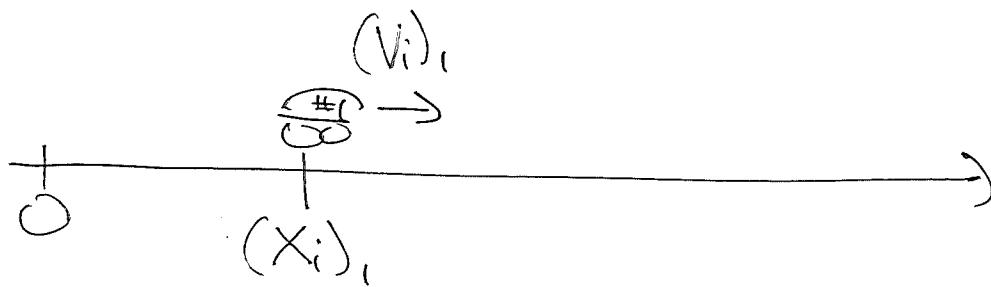
#2 catches up with #1

~~(a)~~ $(a_x)_1 = 2.5 \text{ m/s}^2$ $(a_x)_2 = 6.4 \text{ m/s}^2$

$(x_i)_1 = 0$ $(x_i)_2 = 0$

How long after #2 is launched does it catch up?

So actually we should start problem here:



So figure out what #1 is doing after 2s:

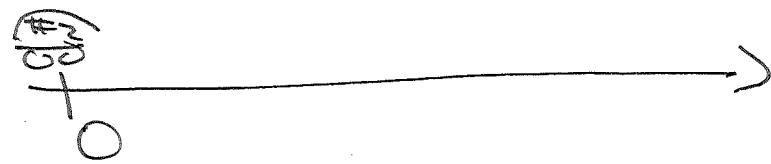
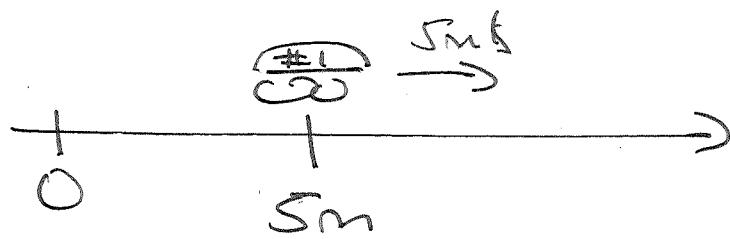
~~(a)~~ $V_f = V_i + a\Delta t \Rightarrow V_f = 0 + (2.5 \text{ m/s}^2)(2\text{s}) = 5 \text{ m/s}$

$X_f = X_i + V_i \Delta t + \frac{1}{2} a \Delta t^2 \Rightarrow X_f = 0 + 0 + \frac{1}{2} (2.5 \text{ m/s}^2)(2\text{s})^2 = 5 \text{ m}$

#1 at x=0
with $V_i = 0$

2s until problem starts

So now:



Known #1: $X_{i,1} = 5m$

$$V_{i,1} = 5 \text{ m/s}$$

$$a_1 = 2.5 \text{ m/s}^2$$

#2: $X_{i,2} = 0$

$$V_{i,2} = 0$$

$$a_2 = 6.4 \text{ m/s}^2$$

When #2 catches #1, they will be at the same position

$$\Rightarrow X_{f,2} = X_{f,1}$$

$$X_f = X_i + V_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Rightarrow X_{f,2} = 0 + 0 + \frac{1}{2} (6.4 \text{ m/s}^2) \Delta t^2 = 3.2 \text{ m/s}^2 \Delta t^2$$

$$X_{f,1} = 5m + 5 \text{ m/s} \Delta t + \frac{1}{2} (2.5 \text{ m/s}^2) \Delta t^2$$

$$\Rightarrow X_{f,1} = 5m + 5 \text{ m/s} \Delta t + 1.25 \text{ m/s}^2 \Delta t^2$$

$$\text{So } (3.2 \text{ m/s}^2) \Delta t^2 = 5 \text{ m} + (5 \text{ m/s}) \Delta t + (1.25 \text{ m/s}^2) \Delta t^3$$

$$\Rightarrow (3.2 \text{ m/s}^2) \Delta t^2 - (1.25 \text{ m/s}^2) \Delta t^2 - (5 \text{ m/s}) \Delta t - 5 \text{ m} = 0$$

$$\Rightarrow [(3.2 \text{ m/s}^2 - 1.25 \text{ m/s}^2)] \Delta t^2 - (5 \text{ m/s}) \Delta t - 5 \text{ m} = 0$$

$$\Rightarrow (1.95 \text{ m/s}^2) \Delta t^2 - (5 \text{ m/s}) \Delta t - 5 \text{ m} = 0$$

QUADRATIC EQUATION $\Rightarrow \Delta t = +5 \text{ m/s} \pm \frac{\sqrt{(-5 \text{ m/s})^2 - 4(1.95 \text{ m/s}^2)(-5 \text{ m})}}{2(1.95 \text{ m/s}^2)}$

$$\Rightarrow \Delta t = +5 \text{ m/s} \pm \frac{\sqrt{+25 \text{ m}^2/\text{s}^2 + 39 \text{ m}^2/\text{s}^2}}{3.9 \text{ m/s}^2} = \frac{5 \text{ m/s} \pm \sqrt{64 \text{ m}^2/\text{s}^2}}{3.9 \text{ m/s}^2}$$

$$\Rightarrow \Delta t = \frac{5 \text{ m/s} \pm 8 \text{ m/s}}{3.9 \text{ m/s}^2} = \underline{\underline{3.333 \text{ s}}} \text{ or } -\underline{\underline{0.76923 \text{ s}}}$$

b. So $X_{f,2} = (3.2 \text{ m/s}^2)(3.333 \text{ s})^2 = 14.23 \text{ m} \underline{\underline{35.6 \text{ m}}}$

$$X_{f,1} = 5 \text{ m} + (5 \text{ m/s})(3.333 \text{ s}) + (1.25 \text{ m/s}^2)(3.333 \text{ s})^2 = 35.6 \text{ m}$$