Lecture 14
PHYC 161 Fall 2016
Q22.3

Two point charges, \( +q \) (in red) and \( -q \) (in blue), are arranged as shown. Through which closed surface(s) is/are the net electric flux equal to zero?

A. surface \( A \)
B. surface \( B \)
C. surface \( C \)
D. surface \( D \)
E. both surface \( C \) and surface \( D \)
Applications of Gauss’s law

• Suppose we construct a Gaussian surface inside a conductor.

• Because \( \vec{E} = 0 \) everywhere on this surface, Gauss’s law requires that the net charge inside the surface is zero.

• Under electrostatic conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor’s surface.
There is a negative surface charge density in a certain region on the surface of a solid conductor. Just beneath the surface of this region, the electric field

A. points outward, toward the surface of the conductor.
B. points inward, away from the surface of the conductor.
C. points parallel to the surface.
D. is zero.
E. Not enough information is given to decide.
Electric Field of Various Charge Distributions

The old way: treat the charge distribution as a bunch of small point charges

\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}} \hat{i}
\]

infinite line, limit: \( x \ll a \)

infinite disk, limit: \( x \ll R \)
Field of a uniform line charge - Gauss’ Law

- Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is $\lambda$ (assumed positive).

- Using Gauss’s law, we can find the electric field:

\[
E = \frac{1}{2\pi \epsilon_0} \frac{\lambda}{r}
\]
Field of an infinite plane sheet of charge

- Gauss’s law can be used to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density \( \sigma \).

\[
E = \frac{\sigma}{2\varepsilon_0}
\]
Ex. 22.5 and 22.9

22.18 Calculating the electric field of a conducting sphere with positive charge \( q \). Outside the sphere, the field is the same as if all of the charge were concentrated at the center of the sphere.

\[
E(R) = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}
\]

Inside the sphere, the electric field is zero:

\[
E = 0.
\]

22.22 The magnitude of the electric field of a uniformly charged insulating sphere. Compare this with the field for a conducting sphere (see Fig. 22.18).

\[
E(R) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}
\]

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3}
\]
• How much charge is on the outer surface of a conductor if the cavity contains a charge +q?

a. zero
b. +2q
c. -q
d. -2q
e. +q
Charges on conductors

• Suppose we place a small body with a charge $q$ inside a cavity within a conductor. The conductor is uncharged and is insulated from the charge $q$.

• According to Gauss’s law the total there must be a charge $-q$ distributed on the surface of the cavity, drawn there by the charge $q$ inside the cavity.

• The total charge on the conductor must remain zero, so a charge $+q$ must appear on its outer surface.

For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$. 
A conducting spherical shell with inner radius $a$ and outer radius $b$ has a positive point charge $Q$ located at its center. The total charge on the shell is $-3Q$, and it is insulated from its surroundings. In the region $a < r < b$,

A. the electric field points radially outward.

B. the electric field points radially inward.

C. the electric field points radially outward in parts of the region and radially inward in other parts of the region.

D. the electric field is zero.

E. Not enough information is given to decide.
Charge Distribution on a Conducting Shell - 1

Description: Conceptual problem. A positive charge sits in the center of a conducting spherical shell. Find the charge distribution on the inside and outside surfaces of the shell.

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell.

Part A

The positive charge is equal to roughly 16 of the smaller charges shown on the surfaces of the spherical shell. Which of the pictures best represents the charge distribution on the inner and outer walls of the shell?
Conductors

- This is the basic idea behind the very first method to store electric charge, the Leyden Jar:

(a) Insulating thread
Charged conducting ball

(b) Metal lid
Charged ball induces charges on the interior and exterior of the container.

(c) Metal lid
Once the ball touches the container, it is part of the interior surface; all the charge moves to the container’s exterior.

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The Van de Graaff generator

• The Van de Graaff generator operates on the same principle as in Faraday’s icepail experiment.

• The electron sink at the bottom draws electrons from the belt, giving it a positive charge.

• At the top the belt attracts electrons away from the conducting shell, giving the shell a positive charge.
Electrostatic shielding

• A conducting box is immersed in a uniform electric field.

• The field of the induced charges on the box combines with the uniform field to give zero total field inside the box.
Electrostatic shielding

• Suppose we have an object that we want to protect from electric fields.

• We surround the object with a conducting box, called a Faraday cage.

• Little to no electric field can penetrate inside the box.

• The person in the photograph is protected from the powerful electric discharge.
Field at the surface of a conductor

- Gauss’s law can be used to show that the direction of the electric field at the surface of any conductor is always perpendicular to the surface.

- The magnitude of the electric field just outside a charged conductor is proportional to the surface charge density \( \sigma \).

\[
E_\perp = \frac{\sigma}{\varepsilon_0}
\]
Conductors

• If we stay very close to the conductor and put a very small “pill box” (just a cylinder) through the surface of the conductor, then using what we’ve just discovered:
  – $E = 0$ inside a conductor.
  – $E$ is perpendicular to the surface immediately outside a conductor.
  – All (free) charges on a conductor lie on it’s surfaces.

$$\Phi_{E,\text{Net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow$$

$$EA_{\text{top cap}} = \frac{\sigma A_{\text{top cap}}}{\varepsilon_0} \Rightarrow$$

$$E = \frac{\sigma (\text{local})}{\varepsilon_0} \quad (\text{Near a conductor})$$
Gravitational Potential Energy

• Once again, the similarities of the gravitational force law and the electric force law bring us to review some physics from last semester.

• Remember that the gravitational potential energy was defined as:

$$\Delta U_g = -W_g = -\int_{1}^{2} \vec{F}_g \cdot d\vec{r}$$

• If you are close to the earth’s surface, you can take the force to be essentially constant (=mg) and the above just gets you

$$\Delta U_g = \int_{1}^{2} (mg) dy = mg(y_2 - y_1) = mgh$$
Electric potential energy in a uniform field

- In the figure, a pair of charged parallel metal plates sets up a uniform, downward electric field.
- The field exerts a downward force on a positive test charge.
- As the charge moves downward from point \( a \) to point \( b \), the work done by the field is independent of the path the particle takes.

\[
\text{The work done by the electric force is the same for any path from } a \text{ to } b: \\
W_{a\rightarrow b} = -\Delta U = q_0Ed
\]
A positive charge moving in a uniform field

- If the positive charge moves in the direction of the field, the field does *positive* work on the charge.
- The potential energy *decreases*.
A negative charge moving in a uniform field

• If the negative charge moves opposite the direction of the field, the field does *positive* work on the charge.

• The potential energy *decreases*. 
A negative charge moving in a uniform field

- If the negative charge moves in the direction of the field, the field does *negative* work on the charge.

- The potential energy *increases*. 

![Diagram of a negative charge moving in a uniform electric field](image)