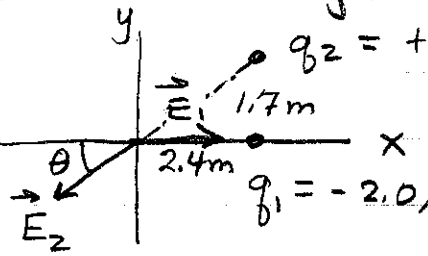
P161 Spring 2009 Test 2 Solutions

(1)



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E_x = E_{1x} + E_{2x}$$

$$E_{1x} = |E_1| = \frac{k|q_1|}{r_1^2} \quad \text{where } r_1 = 2.4\text{m}$$

$$E_{2x} = -E_2 \cos\theta = -\frac{k|q_2|}{r_2} \cos\theta$$

$$\text{where } r_2 = \sqrt{(2.4\text{m})^2 + (1.7\text{m})^2}$$

$$\cos\theta = 2.4\text{m}/r_2$$

Plugging in, we get $\underline{E_x = +1400\ \text{N/C}}$ (B)

(2) (C)

(3) $F = \frac{kq_1q_2}{r^2}$, while new force is $F_{new} = \frac{kq_1q_2}{r_{new}^2}$

$$\frac{F_{new}}{F} = \frac{kq_1q_2/r_{new}^2}{kq_1q_2/r^2} = \left(\frac{r}{r_{new}}\right)^2$$

But $F_{new} = 2F$

so $\left(\frac{r}{r_{new}}\right)^2 = 2$ or $\underline{r_{new} = \frac{r}{\sqrt{2}}}$ (E)

④ From Gauss' Law: $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$

Integrating $\vec{E}$ over the cylinder, we get no contribution from the sides, so

$$q = \epsilon_0 \left[-E_1 A + E_2 A \right] = \epsilon_0 \pi r^2 (E_2 - E_1)$$

Since $r = d/2 = 0.20\,m/2 = 0.10\,m$

$$q = -0.28\,nC \quad \underline{\underline{(E)}}$$

⑤ Since the point charge is $+300\,nC$, the total charge on the inside surface is $-300\,nC$.

$$\sigma = \frac{q_{\text{inside surface}}}{A_{\text{inside Surface}}} = \frac{-300\,nC}{4\pi (0.80\,m)^2}$$

$$\sigma = -37\,\frac{nC}{m^2} = -3.7 \times 10^{-8}\,\frac{C}{m^2} \qquad \underline{\underline{(C)}}$$

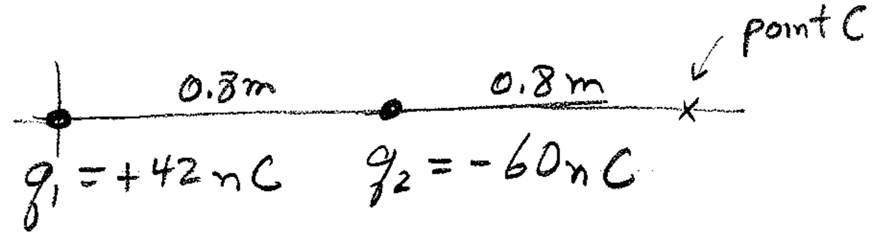
⑥ Since we are always outside the charge distribution, it looks like a point charge. E will be maximum at the surface.

a) at $r_1 = 0.50\,m$, $E_1 = kq/r_1^2$ or $q = \dfrac{E_1 r_1^2}{k}$

b) at surface, $r_2 = 0.30\,m$,

$$E_{max} = \frac{kq}{r_2^2} = \frac{k\left(\frac{E_1 r_1^2}{k}\right)}{r_2^2} = E_1 \left(\frac{r_1}{r_2}\right)^2$$

$$E_{max} = 42{,}000\,N/C \qquad \underline{(A)}$$

(7) 

The electric potential energy of the electron in the presence of the two point charges is changed into kinetic energy when the electron is pushed away to infinity.

$$U_{electron} = k(-e)\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right] \text{ where } \begin{array}{l} r_1 = 1.6\,m \\ r_2 = 0.8\,m \end{array}$$

$$\overset{set}{=} \frac{1}{2} m_{electron} v_{eletron}^2$$

ie $$v_{electron} = \sqrt{\frac{2U_{electron}}{m_{electron}}} = 1.2 \times 10^{7} \frac{m}{s} \quad \underline{(C)}$$

(8) $$\text{Energy gained} = q\Delta V = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

or $$\Delta V = \frac{m}{2q}\left(v_f^2 - v_i^2\right) = 3800V \quad \underline{(A)}$$

(9) $V_0 = q/C_0$. After being moved together, $V = q/C$

So $$\frac{V}{V_0} = \frac{q/C}{q/C_0} = \frac{C_0}{C} = \frac{\epsilon_0 A/d_0}{\epsilon_0 A/(d_0/2)} = \frac{1}{2}$$

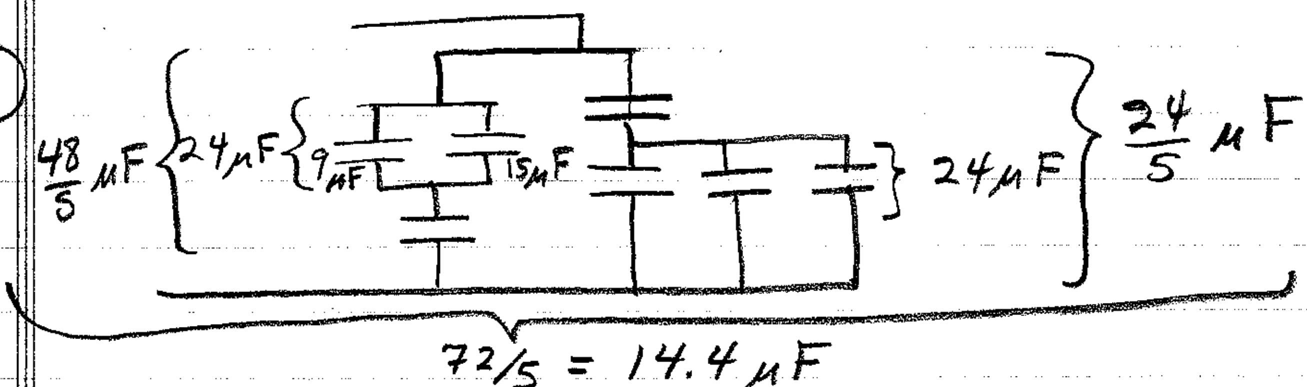
$$V = \frac{V_0}{2} = 10.0V \quad \underline{(A)}$$

(10) Energy density $u = \frac{1}{2}\frac{CV^2}{A \cdot d} = \frac{1}{2}\epsilon_0 E^2$

$$u = \frac{8.854\times10^{-12}\frac{C^2}{Nm^2}}{2}\left(5.3\times10^{6}\frac{V}{m}\right)^2 =$$

$$\underline{u = 124\ J/m^3} \qquad \underline{\underline{(D)}}$$

(11)



Find the equivalent capacitances starting with parallel.
The ~~total~~ charge delivered from the battery is

$$q_{Total} = CV = (14.4\mu F)(100V) = 1440\mu C$$

The $\frac{48}{5}\mu F$ equivalent capacitor on the left has a charge:

$$q_{left} = \left(\frac{48}{5}\mu F\right)(100V) = \frac{4800}{5}\mu C$$

This charge is shared by $C_{9\mu F}$ and $C_{15\mu F}$ which have the same voltage across each:

$$V_{9\,or\,15} = \frac{q_9}{C_9} = \frac{q_{15}}{C_{15}} \quad \text{So} \quad q_9 = \frac{C_9}{C_{15}}q_{15}$$

But $q_{left} = q_9 + q_{15} = \frac{C_9}{C_{15}}q_{15} + q_{15} = \left(\frac{C_9}{C_{15}}+1\right)q_{15}$

or $q_{15} = \dfrac{q_{left}}{\frac{C_9}{C_{15}}+1} = 600\mu C \qquad \underline{\underline{(D)}}$

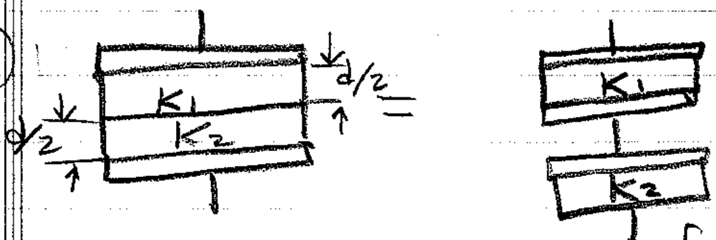
(12) Using the following equation for total energy stored in a capacitor:

$$U = q^2/2C \quad , \text{ take ratio of } U/U_0$$

$$\frac{U}{U_0} = \frac{q^2/2C}{q^2/2C_0} = \frac{C_0}{C} = \frac{C_0}{2C_0}$$

or $U = \dfrac{U_0}{2}$ (c)

(13)



This is 2 capacitors in series

$$\text{So } C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left[K_1 \epsilon_0 A/(d/2)\right]\left[K_2 \epsilon_0 A/(d/2)\right]}{\frac{\epsilon_0 A}{(d/2)}(K_1 + K_2)}$$

$$C_{eq} = \frac{2\epsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right) \quad (A)$$