de Broglie Matter Waves

If any of you wonder if you can do something spectacularly brilliant while still in school, I will point out that the following hypothesis was presented as Louis de Broglie's PhD thesis, for which he won a Nobel Prize 5 years later.

In our discussion of Compton scattering, we derived the relationship $p=h/\lambda$ for photons. This led to our particle-like view of photons. Let's review this, and expand a little on Liboff's explanation. From the relativistic equation for total energy:

$$E^2 = c^2 p^2 + \left(m_0^2 c^2\right)^2$$

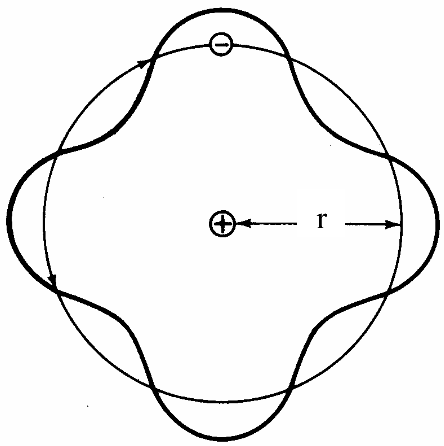
with zero photon mass, we get that $E^2 = c^2p^2$, or $p = E/c$ which gives $p = h\nu/c$ for photons. However, since photons travel at the speed of light, c, we know that the wavelength λ and frequency ν are related by the equation $\lambda\nu = c$, giving us that $p = h/\lambda$. When dealing with sinusoidal functions of space and time [$\sin(kx-\omega t)$], we usually use $\omega = 2\pi\nu$ and $k = 2\pi/\lambda$, the angular frequency and wave number. This gives us $p = \hbar k$, $E = \hbar\omega$ and $\omega = ck$, where $\hbar \equiv h/2\pi$.

So far, we have dealt only with the wave nature of the photons. Based partly on Bohr's presumption of quantized angular momentum of electrons in orbit around a nucleus, de Broglie performed the following brilliant mathematical feat: he turned the equation relating the wavelength and the momentum of a photon upside-down, literally: $\lambda = h/p$, and postulated that matter behaved like waves with wavelengths given by this equation. His reasoning was fairly simple: quantization of the angular momentum can be written as $\vec{r} \times \vec{p} = n\hbar$, or for circular orbits,

$$rp = n\hbar = \frac{nh}{2\pi} \Rightarrow$$

$$2\pi r = \frac{nh}{p}$$

but, recognizing from above that for photons, $h/p = \lambda$, this leads to $2\pi r = n\lambda$, the requirement one would expect for stable path length for a standing wave. One such wave superimposed on a circular orbit is shown in Liboff, and copied below. Another showing more orbits (taken from Eisberg/Resnick) is also shown below. What n is the Liboff picture? How did you determine this? Both quantizing the orbital angular momentum, and requiring standing waves with wavelengths $\lambda = h/p$ lead to the same quantized energy states for electrons in an atom.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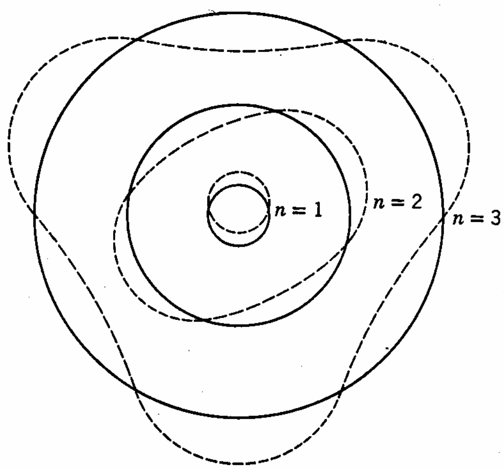
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n = 3