14.1 ⋆ The area of a blueberry is $\sigma = \pi r^2 = \pi d^2/4 = \pi/4\ \text{cm}^2 = 0.79\ \text{cm}^2$. The density of targets (number/area) is $n_{\text{tar}} = N_{\text{tar}}/A = N_{\text{tar}}/(\pi D^2/4) = 0.034\ \text{cm}^{-2}$. The probability of a hit in one try is $n_{\text{tar}}\sigma = 0.027$.

14.3 ⋆ The density is $\varrho = 0.07\ \text{g/cm}^3$, the length of the tank, $L = 50$ cm, and the mass of an H atom is $m_{\text{H}} = 1.66 \times 10^{-27}$ kg. The number density (number/volume) is ϱ/m_{H}, so the target density (number/area) is

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14.7 ⋆ A sphere of radius R at a large distance d subtends a solid angle $\Delta\Omega \approx A/d^2 = \pi R^2/d^2$. For the moon this gives $\Delta\Omega_{\text{moon}} \approx 6.45 \times 10^{-5}$ sr, and for the sun, $\Delta\Omega_{\text{sun}} \approx 6.76 \times 10^{-5}$ sr. Because $\Delta\Omega_{\text{moon}} \approx \Delta\Omega_{\text{sun}}$, the moon and sun *appear* to be about the same size.

14.11 ⋆⋆ The target density is

$$n_{\text{tar}} = \frac{\varrho t}{m_{\text{Ag}}} = \frac{(10.5 \times 10^3\ \text{kg/m}^3) \times (10^{-6}\ \text{m})}{108 \times (1.66 \times 10^{-27}\ \text{kg})} = 5.86 \times 10^{22}\ \text{m}^{-2}$$

and the solid angle subtended by the counter is $\Delta\Omega = (0.1\ \text{mm}^2)/(10\ \text{mm})^2 = 10^{-3}$ sr. Therefore, the number of alphas scattered into $\Delta\Omega$ should be

$$N_{\text{sc}} = N_{\text{inc}} n_{\text{tar}} \frac{d\sigma}{d\Omega}\Delta\Omega = (10^{10}) \times (5.86 \times 10^{22}\ \text{m}^{-2}) \times (0.5 \times 10^{-28}\ \text{m}^2/\text{sr}) \times (10^{-3}\ \text{sr}) \approx 29.$$

14.18 ⋆⋆ The cross section for backward scattering is found by integrating $d\sigma/d\Omega = \sigma_o/\sin^4(\theta/2)$ over the backward hemisphere, $\theta \geq \pi/2$:

$$\sigma(\theta \geq \pi/2) = \int\limits_{\theta \geq \pi/2} d\Omega \, \frac{d\sigma}{d\Omega} = \sigma_o \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} \frac{\sin\theta \, d\theta}{\sin^4(\theta/2)} = 4\pi\sigma_o.$$

[One way to do the integral over θ is to write $\sin\theta$ as $2\sin(\theta/2)\cos(\theta/2)$ and make the substitution $u = \sin(\theta/2)$.]

The number of α's scattered into the backward hemisphere should be $N_{sc}(\theta \geq \pi/2) = N_{inc} n_{tar} \sigma(\theta \geq \pi/2)$. Thus the requested ratio is $N_{sc}(\theta \geq \pi/2)/N_{inc} = n_{tar}\sigma(\theta \geq \pi/2)$. With

$$n_{tar} = \frac{\varrho t}{m_{Pt}} = \frac{(21.4 \times 10^3 \text{ kg/m}^3) \times (3 \times 10^{-6} \text{ m})}{195 \times 1.66 \times 10^{-27} \text{ kg}} = 1.98 \times 10^{23} \text{ m}^{-2}$$

and

$$\sigma_o = \left(\frac{kqQ}{4E}\right)^2 = \left(\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{ C}^2) \times 2 \times 78 \times (1.60 \times 10^{-19} \text{ C})^2}{4 \times (7.8 \times 1.60 \times 10^{-13} \text{ J})}\right)^2 = 5.17 \times 10^{-29} \text{ m}^2$$

the required ratio is $n_{tar}\sigma(\theta \geq \pi/2) = 4\pi n_{tar}\sigma_o = 1.29 \times 10^{-4}$. The reciprocal of this is 7750, so the Rutherford model predicts that 1 particle in 7750 would be "reflected" into the backward hemisphere, in remarkable agreement with Geiger and Marsden's observed "about 1 in 8000."

15.6 ⋆ Clearly $\gamma = 1/\sqrt{1-\beta^2} = 3$, so $\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{8/9} = 0.94$, and $v = 0.94c = 2.8 \times 10^8$ m/s.

15.7 ⋆⋆ With $v = 0.99c$, $\gamma = 7.09$. The half-life (measured in the muons' rest frame) is $t_{1/2}(\text{proper}) = 1.5\ \mu\text{s}$, and that measured in the earth frame is $t_{1/2}(\text{earth}) = \gamma t_{1/2}(\text{proper})$. The time of flight measured in the earth frame is $T(\text{earth}) = h/v$, so the number of half-lives that have elapsed is

$$n = \frac{T(\text{earth})}{t_{1/2}(\text{earth})} = \frac{h}{v\gamma t_{1/2}(\text{proper})} = 0.63. \qquad \text{(iv)}$$

Therefore, the number that survive to the ground should be about $N = N_o/2^n = 650/2^{0.63} = 420$.

To find the classical answer, we must delete the factor of γ in the expression (iv) for n, to give $n(\text{clas}) = \gamma n(\text{rel}) = 4.49$ half-lives, and $N(\text{clas}) = 650/2^{4.49} \approx 29$.

15.12 ⋆⋆ The half-life in the pions' rest frame is the proper half-life, $t_{1/2}(\text{proper}) = 1.8 \times 10^{-8}$ s. The length of the pipe as "seen" by the pions is given by the length-contraction formula

$$(\text{length of pipe in pions' frame}) = d/\gamma = 21.6 \text{ m},$$

where $d = 36$ m is the length measured in the lab. The time for the pipe to pass the pions is therefore $T(\pi \text{ frame}) = (d/\gamma)/v = d/(\gamma v)$, and the number of half-lives that elapse is

$$n = \frac{T(\pi \text{ frame})}{t_{1/2}(\pi \text{ frame})} = \frac{d}{\gamma v t_{1/2}(\text{proper})} = 5.00. \qquad \text{(vii)}$$

Therefore the number of pions that survive is $N = N_o/2^n = 32,000/32 = 1000$, the same answer as in Problem 15.8.

The two arguments, in this problem and Problem 15.8, lead to the same formula, (vii) here and (v) in the solution to Problem 15.8. Here the factor of γ comes from the length contraction of the tube, there the same factor came from the time dilation of the time of flight.

15.37 ⋆ If x and y are four-vectors, then under the standard boost $x'_1 = \gamma(x_1 - \beta x_4)$, $x'_2 = x_2$, and so on. Therefore

$$\begin{aligned} x' \cdot y' &= x'_1 y'_1 + x'_2 y'_2 + x'_3 y'_3 - x'_4 y'_4 \\ &= \gamma(x_1 - \beta x_4)\gamma(y_1 - \beta y_4) + x_2 y_2 + x_3 y_3 - \gamma(x_4 - \beta x_1)\gamma(y_4 - \beta y_1) \\ &= \gamma^2(1 - \beta^2)x_1 y_1 + x_2 y_2 + x_3 y_3 - \gamma^2(1 - \beta^2)x_4 y_4 \end{aligned}$$

since the cross terms involving $x_1 y_4$ and $x_4 y_1$ in the second line all cancelled. Finally, notice that $\gamma^2(1 - \beta^2) = 1$, so the last line is just $x \cdot y$, and we've proved that $x' \cdot y' = x \cdot y$.

15.56 ⋆ **(a)** Since $M_i c^2 + T_i = M_f c^2 + T_f$, we see that $\Delta M c^2 = -\Delta T = -5$ eV. Thus

$$\Delta M = -5 \text{ eV}/c^2 = -5.4 \times 10^{-9} \text{ u}.$$

(b) Since the initial mass of two H_2 and one O_2 molecules is 36 u, the fractional change in mass is $\Delta M/M = -(5.4 \times 10^{-9})/36 = -1.5 \times 10^{-10}$.

(c) Whatever the initial mass, the fractional change will be the same, so, with 10 grams initially, the change will be $\Delta M = -1.5 \times 10^{-9}$ gram. Pretty small!

15.67 ⋆ With $v = 0.8c$, $\gamma = 5/3$. Since the initial velocities are equal and opposite, the same is true of the momenta. Therefore the total momentum is zero, and the final body is at rest. Thus $M = E_{\text{fin}}/c^2 = E_{\text{in}}/c^2 = 2\gamma m = 3.33m$.
