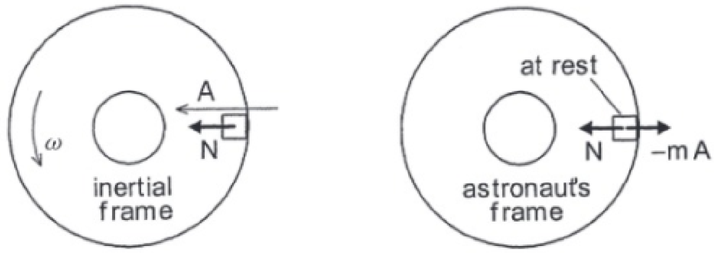
9.2 ⋆ **(a)** As seen by inertial observers outside the station, the (square) astronaut has a centripetal acceleration $A = \omega^2 R$ which is supplied by the normal force $\mathbf{N}$.



(b) As seen by the crew inside the station, the astronaut is at rest under the action of two forces, the normal force $\mathbf{N}$ and the inertial force $-m\mathbf{A}$. To simulate normal gravity, we must have $A = \omega^2 R = g$ or $\omega = \sqrt{g/R} = 0.5$ rad/s $= 4.8$ rpm.

(c) The apparent gravity $g_{\text{app}} = \omega^2 R$ is proportional to R. Thus if we decrease R from 40 m to 38 m, the fractional change in g_{app} is $\delta g_{\text{app}}/g_{\text{app}} = \delta R/R = -5\%$.

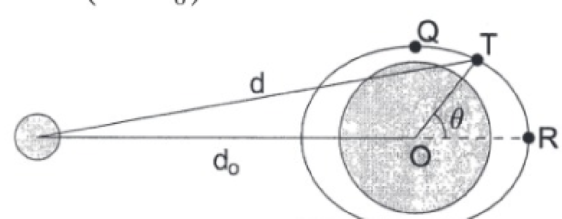
9.6 ⋆⋆⋆ Just as in (9.14), the requirement that the ocean's surface be an equipotential implies that

$$U_{\text{eg}}(T) - U_{\text{eg}}(Q) = U_{\text{tid}}(Q) - U_{\text{tid}}(T), \tag{iv}$$

and the left hand side is just $mgh(\theta)$. The term $U_{\text{tid}}(T)$ on the right is given by (9.13)

$$U_{\text{tid}}(T) = -GM_{\text{m}}m\left(\frac{1}{d} + \frac{x}{d_{\text{o}}^2}\right) \tag{v}$$

where we must find the values of x and d for the point T. Obviously $x = R_{\text{e}}\cos\theta$, but d requires more care. By the law of cosines, $d^2 = d_{\text{o}}^2 + 2d_{\text{o}}R_{\text{e}}\cos\theta + R_{\text{e}}^2$. Thus



$$\frac{1}{d} = \frac{1}{\sqrt{d_{\text{o}}^2 + 2d_{\text{o}}R_{\text{e}}\cos\theta + R_{\text{e}}^2}} = \frac{1}{d_{\text{o}}}\left(1 + 2\frac{R_{\text{e}}}{d_{\text{o}}}\cos\theta + \frac{R_{\text{e}}^2}{d_{\text{o}}^2}\right)^{-1/2}.$$

Since $R_{\text{e}} \ll d_{\text{o}}$, we can approximate the term in parenthesis using the binomial series, $(1+\epsilon)^{-1/2} = 1 - \frac{1}{2}\epsilon + \frac{3}{8}\epsilon^2 + \cdots$. Although we need to keep the term ϵ^2, we can drop from it anything higher than $(R_{\text{e}}/d_{\text{o}})^2$, so we find

$$\begin{aligned}\frac{1}{d} &= \frac{1}{d_{\text{o}}}\left[1 - \frac{1}{2}\left(2\frac{R_{\text{e}}}{d_{\text{o}}}\cos\theta + \frac{R_{\text{e}}^2}{d_{\text{o}}^2}\right) + \frac{3}{8}\left(2\frac{R_{\text{e}}}{d_{\text{o}}}\cos\theta\right)^2\right] \\ &= \frac{1}{d_{\text{o}}}\left[1 - \frac{R_{\text{e}}}{d_{\text{o}}}\cos\theta + \frac{1}{2}\frac{R_{\text{e}}^2}{d_{\text{o}}^2}(3\cos^2\theta - 1)\right].\end{aligned} \tag{vi}$$

When this is substituted into (v) the term which is linear in $\cos\theta$ exactly cancels the second term on the right of (v) leaving

$$U_{\rm tid}(T) = -\frac{GM_{\rm m}m}{d_{\rm o}}\left[1 + \frac{1}{2}\frac{R_{\rm e}^2}{d_{\rm o}^2}(3\cos^2\theta - 1)\right].$$

The value of $U_{\rm tid}(Q)$ is found by putting $\theta = \pi/2$ (and hence $\cos\theta = 0$), and the difference on the right of (iv) is

$$U_{\rm tid}(Q) - U_{\rm tid}(T) = \frac{3GM_{\rm m}mR_{\rm e}^2}{2d_{\rm o}^3}\cos^2\theta.$$

Since the left side of (iv) is $mgh(\theta)$, we conclude that $h(\theta) = h_{\rm o}\cos^2\theta$, where

$$h_{\rm o} = \frac{3GM_{\rm m}mR_{\rm e}^2}{2d_{\rm o}^3 mg} = \frac{3M_{\rm m}R_{\rm e}^4}{2M_{\rm e}d_{\rm o}^3}$$

since $g = GM_{\rm e}/R_{\rm e}^2$.

The height $h(\theta) = h_{\rm o}\cos^2\theta$ is zero at the point Q where $\theta = \pi/2$ — as it had to be, since it was defined as the height measured up from sea level at Q. It is positive for all other values of θ and symmetrical about $\theta = \pi/2$, rising to a maximum at $\theta = 0$ and π. This produces the oval shape shown in the picture.

9.9 ⋆ $\mathbf{F}_{\rm cor} = 2m\mathbf{v}\times\mathbf{\Omega} = 2mv_{\rm o}\Omega\cos\theta$ due east, and

$$\frac{F_{\rm cor}}{mg} = \frac{2v_{\rm o}\Omega\cos\theta}{g} = \frac{2\times(1000\text{ m/s})\times(7.3\times10^{-5}\text{ rad/s})\times(\cos 40^\circ)}{9.8\text{ m/s}^2} = 0.0114.$$

9.10 ⋆⋆ From Eq.(9.31) to (9.32) the derivation is exactly the same whether $\mathbf{\Omega}$ varies or not. If $\mathbf{\Omega}$ varies, then the first time derivative on the right of (9.32) picks up an extra term involving $\dot{\mathbf{\Omega}}$. Specifically, in place of (9.33) we now have $(d^2\mathbf{r}/dt^2)_{\mathcal{S}_{\rm o}} = \ddot{\mathbf{r}} + 2\mathbf{\Omega}\times\dot{\mathbf{r}} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}) + \dot{\mathbf{\Omega}}\times\mathbf{r}$. If we multiply both sides by m, the left side becomes $\mathbf{F}$, the net "real" force, and we get the equation of motion $m\ddot{\mathbf{r}} = \mathbf{F} + 2m\dot{\mathbf{r}}\times\mathbf{\Omega} + m(\mathbf{\Omega}\times\mathbf{r})\times\mathbf{\Omega} + m\mathbf{r}\times\dot{\mathbf{\Omega}}$.

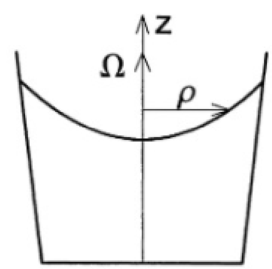
9.12 ⋆ **(a)** All the rules of statics (including those concerned with total torques being zero) are derivable from the requirement that the net force $\mathbf{F}$ on every particle of the system must be zero, $\mathbf{F} = 0$. If we wish a structure to remain static in a rotating frame, then we must use the equation of motion (9.34) for each particle in the rotating frame. Since all of the particles are to be stationary (in the rotating frame), this reduces to $0 = \mathbf{F} + \mathbf{F}_{\rm cf}$. This leads to all of the usual conditions except that where we usually use the net force $\mathbf{F}$ we must include the centrifugal force and use $\mathbf{F} + \mathbf{F}_{\rm cf}$.

(b) For the puck on the rotating horizontal turntable, there are four forces, its weight $m\mathbf{g}$, the normal force $\mathbf{N}$ of the table, the force of friction $\mathbf{f}$, and the centrifugal force. If the puck is not to move on the table these must sum to zero, $m\mathbf{g} + \mathbf{N} + \mathbf{f} + \mathbf{F}_{\rm cf} = 0$. The two vertical forces must balance, so $N = mg$, and the two horizontal forces must also balance, so $F_{\rm cf} = m\Omega^2 r = f \le \mu N = \mu mg$. Therefore $r \le \mu g/\Omega^2$.

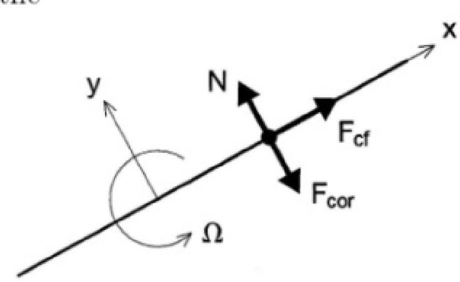
9.14 ⋆⋆ In the rotating frame of the bucket, the water is in equilibrium and its surface is an equipotential surface for the combined gravitational force (PE $= mgz$) and centrifugal force (force $= m\Omega^2\rho$ and hence PE $= -m\Omega^2\rho^2/2$). Therefore, the surface is given by $mgz - m\Omega^2\rho^2/2 =$ const, or

$$z = \frac{\Omega^2\rho^2}{2g} + \text{const},$$

which is a parabola, as claimed.

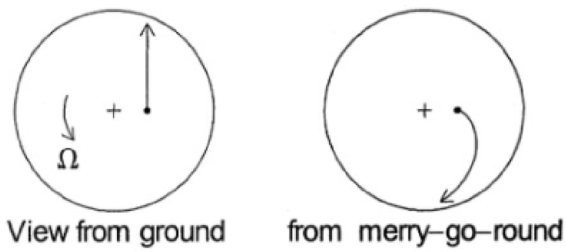


9.16 ⋆ With axes fixed on the rotating rod as shown, the bead stays on the x axis and its velocity is $\mathbf{v} = \dot{x}\hat{\mathbf{x}}$. The three forces on the bead are the normal force $\mathbf{N} = N\hat{\mathbf{y}}$, the centrifugal force $\mathbf{F}_{\rm cf} = m\Omega^2 x\hat{\mathbf{x}}$, and the Coriolis force $\mathbf{F}_{\rm cor} = -2m\Omega\dot{x}\hat{\mathbf{y}}$. The two components of the equation of motion are $m\ddot{x} = F_{\rm cf} = m\Omega^2 x$ and $N = F_{\rm cor}$. The solution is $x(t) = Ae^{\Omega t} + Be^{-\Omega t}$. The centrifugal force drives the bead out along the rod. The normal and Coriolis forces just balance out.

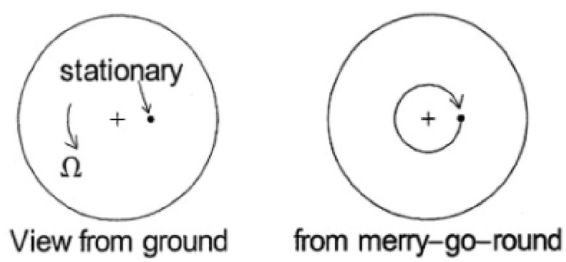


9.18 ⋆⋆ As seen in a frame rotating with the system, there are four forces on the mass: its weight $-mg\hat{\mathbf{y}}$, the centrifugal force $m\Omega^2 x\hat{\mathbf{x}}$, the normal force $\mathbf{N}$ of the confining plane, and the Coriolis force $\mathbf{F}_{\text{cor}}$. The last two both act in the z direction (normal to the confining plane) and must cancel each other, because there is no motion in this direction. The equations of motion in the remaining two directions are $\ddot{y} = -g$ with solution $y = y_{\text{o}} + v_{y\text{o}}t - \frac{1}{2}gt^2$, and $\ddot{x} = \Omega^2 x$ with solution $x = Ae^{\Omega t} + Be^{-\Omega t}$. The vertical motion is the same as that of a body in free fall. Except in the special case that $A = 0$, the x motion may be inward or outward initially, but eventually the particle moves outward at an exponentially increasing rate, caused by the centrifugal force. In the case that $A = 0$, the particle moves inward, slowing down because of the centrifugal force, and approaches the y axis as $t \to \infty$.

9.19 ⋆⋆ **(a)** As seen by a ground-based observer, the puck has initial velocity ΩR in the tangential direction. Since it is subject to zero net force, it travels in a straight line at constant speed (left picture). As seen from the merry-go-round, the puck is subject to the two inertial forces (centrifugal and Coriolis). It is initially at rest, so the Coriolis force is initially zero, and the puck is accelerated outward by the centrifugal force. As it speeds up, the Coriolis force becomes increasingly important and the puck curves to the right, spiralling outward.



(b) As seen from the ground, the puck is initially at rest. Since it is subject to zero net force, it remains at rest indefinitely. This means that, as seen from the merry-go-round, the puck describes a clockwise circle with angular velocity Ω and speed ΩR. This is quite a subtle result in the rotating frame. The centrifugal force is $m\Omega^2 R$ outward, and the Coriolis force is $2m\Omega^2 R$ inward; thus the net force is $m\Omega^2 R$ inward (as seen by observers on the merry-go-round), and this is just the required centripetal force to hold it in the circular orbit!



9.22 ⋆⋆ Let $\mathcal{S}_o$ be the inertial frame in which the charge $-q$ orbits Q in a weak magnetic field $\mathbf{B}$. In this frame the equation of motion is

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{\mathcal{S}_o} = -\frac{kqQ}{r^2}\hat{\mathbf{r}} - q\left(\frac{d\mathbf{r}}{dt}\right)_{\mathcal{S}_o} \times \mathbf{B} \qquad \text{(viii)}$$

where the first term on the right is the Coulomb attraction of Q and the second is the magnetic force $-q\mathbf{v}\times\mathbf{B}$. Let us now move to a frame $\mathcal{S}$ rotating with angular velocity $\mathbf{\Omega}$ relative to $\mathcal{S}_o$. We can rewrite the two derivatives of Eq.(viii) in terms of the corresponding derivatives in $\mathcal{S}$, as in Section 9.5. (I'll call these latter derivatives $\ddot{\mathbf{r}}$ and $\dot{\mathbf{r}}$ as before.) In $\mathcal{S}$ Eq.(viii) becomes

$$m\ddot{\mathbf{r}} - 2m\dot{\mathbf{r}}\times\mathbf{\Omega} - m(\mathbf{\Omega}\times\mathbf{r})\times\mathbf{\Omega} = -\frac{kqQ}{r^2}\hat{\mathbf{r}} - q(\dot{\mathbf{r}} + \mathbf{\Omega}\times\mathbf{r})\times\mathbf{B}.$$

If we choose the angular velocity so that $\mathbf{\Omega} = q\mathbf{B}/(2m)$, then the terms involving $\dot{\mathbf{r}}$ on either side cancel exactly. The terms involving double cross products don't quite cancel, and we're left with

$$m\ddot{\mathbf{r}} = -\frac{kqQ}{r^2}\hat{\mathbf{r}} - \frac{q^2}{4m}(\mathbf{B}\times\mathbf{r})\times\mathbf{B}.$$

If the B field is sufficiently weak, we can drop the second term on the right, and we're left with the equation for a body orbiting in an inverse square force (the Kepler problem). Therefore, in the rotating frame $\mathcal{S}$ the charge q moves in an ellipse (or hyperbola), and in the original frame $\mathcal{S}_o$ (relative to which $\mathcal{S}$ is rotating slowly), the elliptical orbit precesses slowly.

9.26 ⋆⋆ The equations of motion are given by (9.53). To zeroth order in Ω these reduce to

$$\ddot{x} = 0, \quad \ddot{y} = 0, \quad \text{and} \quad \ddot{z} = -g,$$

with the familiar solutions

$$x = v_{xo}t, \quad y = v_{yo}t, \quad \text{and} \quad z = v_{zo}t - \tfrac{1}{2}gt^2.$$

If you substitute these into those terms of (9.53) that already contain a factor of Ω (and hence are only small corrections), you will find the equations

$$\left.\begin{aligned} \ddot{x} &= 2\Omega(v_{yo}\cos\theta - v_{zo}\sin\theta) + 2\Omega g t\sin\theta \\ \ddot{y} &= -2\Omega v_{xo}\cos\theta \\ \ddot{z} &= -g + 2\Omega v_{xo}\sin\theta. \end{aligned}\right\}$$

These three equations can be integrated twice to give precisely the requested equations (9.73).

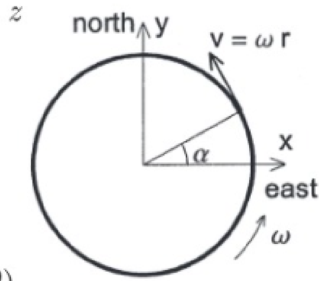
9.28 ⋆⋆ **(a)** If we ignore Ω entirely and set $v_{yo} = 0$, Eqs.(9.73) become $x = v_{xo}t$, $y = 0$, and $z = v_{zo} - \frac{1}{2}gt^2$. Thus the time of flight (time until $z = 0$ again) is $t = 2v_{zo}/g$ and the range R (value of x at landing) is $R = 2v_{xo}v_{zo}/g = 2v_o^2\cos(\alpha)\sin(\alpha)/g$. If $v_o = 500$ m/s and $\alpha = 20°$, these become $t = 34.9$s and $R = 16.4$ km.

(b) According to the second of Eqs. (9.73) (with $v_{yo} = 0$), $y = -\Omega v_o \cos(\alpha)\cos(\theta)t^2$. At latitude 50° north, $\theta = 40°$ and

$$y = -(7.3 \times 10^{-5}\ \mathrm{s}^{-1}) \times (500\ \mathrm{m/s}) \times \cos(20°) \times \cos(40°) \times (34.9\ \mathrm{s})^2 = -32\mathrm{m};$$

that is, the shell lands 32 m to the south of the target. At latitude 50° south, the factor $\cos\theta$ has the opposite sign, and the shell lands 32 m to the north.

9.30 ⋆⋆⋆ I'll choose axes as usual, with x east, y north, and z vertically up. The picture shows the hoop as seen from above. Consider first a small segment of hoop subtending an angle $d\alpha$ with polar angle α. The mass of this segment is $dm = m d\alpha/2\pi$, and the Coriolis force on it is



$$d\mathbf{F}_{\mathrm{cor}} = 2\,dm\,(\mathbf{v} \times \mathbf{\Omega})$$

where

$$\mathbf{v} = \omega r(-\sin\alpha, \cos\alpha, 0) \quad \text{and} \quad \mathbf{\Omega} = \Omega(0, \sin\theta, \cos\theta).$$

The segment's position vector is $\mathbf{r} = r(\cos\alpha, \sin\alpha, 0)$ and the torque on it is

$$d\mathbf{\Gamma}_{\mathrm{cor}} = \mathbf{r} \times d\mathbf{F}_{\mathrm{cor}} = 2\,dm\,\mathbf{r} \times (\mathbf{v} \times \mathbf{\Omega}) = 2\,dm[\mathbf{v}(\mathbf{r}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\mathbf{r}\cdot\mathbf{v})]$$
$$= 2\,dm\,\omega r^2\Omega(-\sin^2\alpha, \sin\alpha\cos\alpha, 0)\sin\theta.$$

To find the total torque, we must replace dm by $m\,d\alpha/2\pi$ and integrate over α from 0 to 2π. The integral of $\sin^2\alpha$ gives π, while that of $\sin\alpha\cos\alpha$ is zero. Thus, the total torque on the hoop is

$$\mathbf{\Gamma}_{\mathrm{cor}} = -(m\omega r^2\Omega\sin\theta)\hat{\mathbf{x}},$$

which points west with magnitude $m\omega r^2\Omega\sin\theta$.

9.32 ⋆⋆⋆ The enemy ship is due east of the gun, which is aimed in that direction. That is, $v_{y\text{o}} = 0$, and Eqs.(9.73) simplify to

$$\left.\begin{array}{l} x = v_{x\text{o}}t - (\Omega v_{z\text{o}}\sin\theta)t^2 + \frac{1}{3}(\Omega g\sin\theta)t^3 \\ y = -(\Omega v_{x\text{o}}\cos\theta)t^2 \\ z = v_{z\text{o}}t - \frac{1}{2}gt^2 + (\Omega v_{x\text{o}}\sin\theta)t^2. \end{array}\right\} \qquad \text{(ix)}$$

(a) If we ignore Ω entirely, we get the same answers as in part (a) of Problem 9.28. In particular, the range is $R_\text{o} = 2v_{x\text{o}}v_{z\text{o}}/g$. (I've called it R_o to emphasize that it's the range ignoring Ω.)

(b) We now wish to work to first order in Ω, and we must first use the third of Eqs.(ix) to find the time at which the shell lands. Solving that equation for t when $z = 0$, we find

$$v_{z\text{o}} = \tfrac{1}{2}g\left(1 - \frac{2\Omega v_{x\text{o}}\sin\theta}{g}\right)t \quad \text{whence} \quad t \approx \frac{2v_{z\text{o}}}{g}\left(1 + \frac{2\Omega v_{x\text{o}}\sin\theta}{g}\right)$$

to first order in Ω. (I used the binomial approximation in solving for t.) This gives t as the sum of two terms. The first is the answer of part (a) (ignoring Ω entirely) and the second is the first order correction to t. To find by how much the shell misses the target, we must substitute this corrected time into the expressions for x and y in Eqs.(ix). The expression for y already contains a factor of Ω, so, to first order, we can just use the zeroth order time, to give

$$y = -(\Omega v_{x\text{o}}\cos\theta)\left(\frac{2v_{z\text{o}}}{g}\right)^2 = -32 \text{ m}$$

(at latitude 50° north). This is the same answer as in Problem 9.28. The east-west position x requires more care. The first term in the expression for x in Eqs.(ix) does not involve Ω at all. Thus to get x correct to first order in Ω we must include the first order correction to t in this term. (In the other two terms we don't need to do this, because they already contain one factor of Ω.) Thus from (ix) we get

$$\begin{aligned} x &= v_{x\text{o}}\frac{2v_{z\text{o}}}{g}\left(1 + \frac{2\Omega v_{x\text{o}}\sin\theta}{g}\right) - (\Omega v_{z\text{o}}\sin\theta)\left(\frac{2v_{z\text{o}}}{g}\right)^2 + \tfrac{1}{3}(\Omega g\sin\theta)\left(\frac{2v_{z\text{o}}}{g}\right)^3 \\ &= R_\text{o} + \frac{4\Omega v_{z\text{o}}\sin\theta}{g^2}\left(v_{x\text{o}}^{\;2} - v_{z\text{o}}^{\;2} + \frac{2}{3}v_{z\text{o}}^{\;2}\right) = R_\text{o} + \frac{4\Omega v_\text{o}^{\;3}\sin\alpha\sin\theta}{g^2}\left(\cos^2\!\alpha - \frac{1}{3}\sin^2\!\alpha\right). \end{aligned}$$

Because R_o is the actual distance of the target, the second term is the distance (east-west) by which the shell misses. Putting in the given numbers this gives +70 m. That is, the shell overshoots by 70 m to the east (in addition to being 32 m to the south).

9.34 ⋆⋆⋆ As suggested, I'll write the puck's position as $\mathbf{R}+\mathbf{r}$, where $\mathbf{R}$ points from the earth's center to P and $\mathbf{r}$ from P to the puck. Notice that $\mathbf{R}$ and $\mathbf{r}$ are almost exactly perpendicular and it is certainly true that $r \ll R$. The equation of motion is

$$\ddot{\mathbf{r}} = \mathbf{g}_o(\mathbf{r}) + 2\dot{\mathbf{r}} \times \boldsymbol{\Omega} + [\boldsymbol{\Omega} \times (\mathbf{r}+\mathbf{R})] \times \boldsymbol{\Omega} + \mathbf{N}/m \tag{x}$$

where $\mathbf{g}_o(\mathbf{r})$ is the "true" acceleration of gravity at the position of the puck,

$$\mathbf{g}_o(\mathbf{r}) = -GM\frac{\mathbf{R}+\mathbf{r}}{|\mathbf{R}+\mathbf{r}|^3} = -GM\frac{\mathbf{R}+\mathbf{r}}{R^3}(1+r^2/R^2)^{-3/2} \approx -GM\frac{\mathbf{R}+\mathbf{r}}{R^3} = \mathbf{g}_o(0) - g_o(0)\frac{\mathbf{r}}{R},$$

where in the approximation I dropped terms of order $(r/R)^2$. Returning to the equation of motion, Eq.(x), we note that the centrifugal term consists of two terms. The one involving $\mathbf{r}$ can be ignored (I'll justify this later) and the one involving $\mathbf{R}$ combines with $\mathbf{g}_o(0)$ to give $\mathbf{g}(0)$, the observed free-fall acceleration at P. Therefore

$$\ddot{\mathbf{r}} = \mathbf{g}(0) - g\mathbf{r}/R + 2\dot{\mathbf{r}} \times \boldsymbol{\Omega} + \mathbf{N}/m.$$

[In the second term on the right, I have replaced $g_o(0)$ by $g = g(0)$, because we can ignore their tiny difference in this term, which is already small.] Bearing in mind that $\mathbf{r}$ lies in the xy plane and that $\mathbf{g}(0)$ is perpendicular to that plane, we can write down the x and y components of this equation as [the components of $\dot{\mathbf{r}} \times \boldsymbol{\Omega}$ are given in Eq.(9.52) if you don't want to work them out]

$$\ddot{x} = -gx/R + 2\dot{y}\Omega\cos\theta \quad \text{and} \quad \ddot{y} = -gy/R - 2\dot{x}\Omega\cos\theta$$

These two equations have exactly the form of the Foucault equation (9.61) except that the length of the pendulum L has been replaced by the radius of the earth.

The frequency of the puck's oscillations is $\omega_o = \sqrt{g/R} = 1.24 \times 10^{-3}\ \mathrm{s}^{-1}$, giving a period of $\tau_o = 2\pi/\omega_o \approx 5000$ s or an hour and a bit. This frequency is at least an order of magnitude greater than the frequency of precession, $\Omega_z = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$, so it makes sense to say that the puck oscillates with frequency ω_o and precesses with frequency Ω_z.

If the amplitude of oscillations is A, then the puck's speed v is of order $v \sim A\omega_o$. The three forces to be compared are

gravitational restoring force $= mgr/R \sim mgA/R = mA\omega_o^2$

Coriolis force $= 2m|\mathbf{v} \times \boldsymbol{\Omega}| \sim 2mA\omega_o\Omega$

$|m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}| \sim mA\Omega^2.$

Since $\omega_o \gg \Omega$, this confirms that the gravitational restoring force is much bigger than the Coriolis force and that the term $m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}$ in the centrifugal force can, indeed, be neglected.