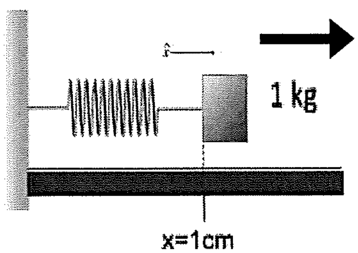
Physics 2310-01 Fall 2023 Exam 1

Name Solutions SHOW ALL WORK!

1) A 1 kg mass is attached to a spring and rests on a frictionless surface. At t=0, the mass is 1cm to the right of the equilibrium point, the mass is moving to the right, and the acceleration of the mass is -100.0 cm/s². What is the value of the spring constant?



$x(t) = A\cos(\omega t + \phi) \Rightarrow .01\,m = A\cos(\phi)$

$\dot{x}(t) = -A\omega\sin(\omega t + \phi)$

$\ddot{x}(t) = -A\omega^2\cos(\omega t + \phi) \Rightarrow -1\,m/s^2 = -A\omega^2\cos(\phi) \Rightarrow$

$1\frac{m}{s^2} = \omega^2 \cdot 0.01\,m \Rightarrow$

$\omega^2 = 100 s^{-2} = \frac{k}{m} \Rightarrow$

$k = 100\frac{kg}{s^2} = 100\frac{N}{m}$

Easier: $F = -kx = ma \Rightarrow k = -\frac{ma}{x} = \frac{1\,kg \cdot -1.\,m/s^2}{0.01\,m} = \boxed{100\frac{N}{m} = k}$

2) If the amplitude of oscillations is 2 cm, what is the velocity (vector) of the mass when it next passes x=0.0 cm?

Could solve above equations for ϕ, and then t.

Better and easier;

$E_T = \frac{1}{2}kA^2 = \frac{1}{2}(100\frac{N}{m})(0.02m)^2$

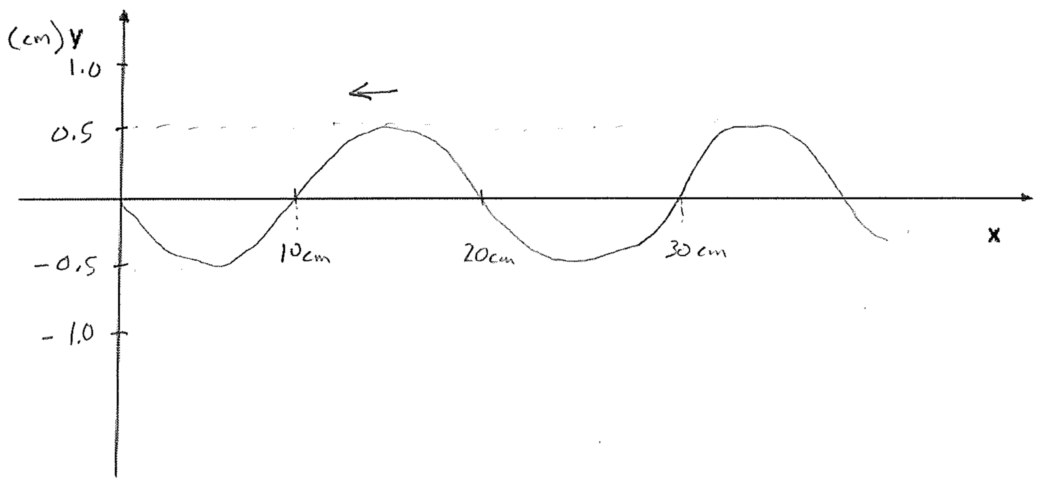
$= 0.02\ N\cdot m$

at x=0, all energy is kinetic!

@ x=0: $E_T = \frac{1}{2}mv^2 \Rightarrow 0.02\,N\cdot m = \frac{1}{2}(1kg)v^2 \Rightarrow$

$\boxed{\vec{V} = -0.2\frac{m}{s}\hat{i}}$

3) A wave is described by the equation: $y(x,t) = 0.5cm \times \cos[2\pi(0.05cm^{-1}x + 4s^{-1}t)]$. Make a sketch of this wave at t=1/16 s (be sure to scale the axes!) and indicate the direction the wave is traveling.



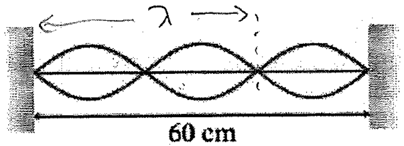
$y(x,t) = A\cos(kx + \omega t)$ moving in $-x$ direction

Amplitude $= 0.5\ cm$

$k = 2\pi \cdot 0.05\ cm^{-1} = 2\pi/\lambda \Rightarrow \lambda = 20cm$

@ $t = \frac{1}{16}s$: $\omega t = 2\pi \cdot 4s^{-1} \cdot \frac{1}{16}s = \frac{2\pi}{4} = \frac{\pi}{2}$, so shifted to left by $\frac{1}{4}\lambda$

4) A standing wave is oscillating at 440 Hz on a string as shown below. The string is tensioned to 500 N. What is the mass of the string?



$f = 440 \text{ Hz}$

$\lambda = \frac{2}{3} \cdot 60\text{cm} = 40\text{cm}$

and $v = \lambda f = 40\text{cm} \cdot 440 s^{-1}$

$v = 176 \frac{m}{s}$

but $v = \sqrt{\frac{T}{\mu}} \Rightarrow 176 \frac{m}{s} = \sqrt{\frac{500 N}{\mu}} \Rightarrow$

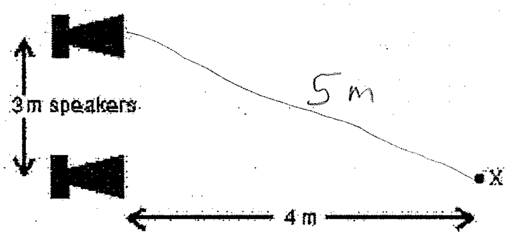
$$\mu = \frac{500 N}{\left(176 \frac{m}{s}\right)^2} = \frac{500 \frac{kg\, \cancel{m}}{\cancel{s^2}}}{30{,}976 \frac{m^{\cancel{2}}}{\cancel{s^2}}} = 0.016 \frac{kg}{m}$$

but the string is 0.6 m long, so the total mass is:

$m = \mu \cdot L = 0.016 \frac{kg}{\cancel{m}} \cdot 0.6 \cancel{m} = 9.68 \times 10^{-3} kg$

$\boxed{m = 9.68 g}$

5) Two small identical speakers are connected in phase to the same source. The speakers are 3 m apart and are both at ear level. An observer stands at X a distance 4 m directly in front of one of the speakers (as shown). The frequency of the sound can be adjusted from 200 Hz to 600 Hz. For what frequency is the sound most intense?



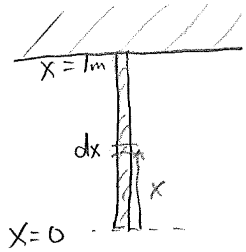
For a maximum sound intensity, the difference in path lengths should be λ, 2λ, 3λ . . .

$\Delta L = 5m - 4m = 1m$

if $\lambda = 1m$, with $V_{sound} = 344\ m/s \Rightarrow$

$V = \lambda f \Rightarrow f = \frac{V}{\lambda} = \frac{344\frac{m}{s}}{1m} = 344\ Hz$

6) A uniform rope of mass 1 kg and length 1 m hangs from a ceiling. The free lower end of the rope is struck. How much time does the resulting pulse take to travel to the ceiling?



At position x, the wave velocity is given by $v = \sqrt{\frac{T}{\mu}}$. The tension at that point is just the weight of the rope below x $\Rightarrow$

$$T(x) = (\mu \cdot x) g$$

$$\therefore \quad v(x) = \sqrt{\frac{T(x)}{\mu}} = \sqrt{\frac{\cancel{\mu} \cdot x \cdot g}{\cancel{\mu}}} = \sqrt{xg}$$

the time it takes for the pulse to pass a distance dx at x is just

$$dt = \frac{dx}{v(x)} = \frac{dx}{\sqrt{x \cdot g}}$$

So, the total time is:

$$\int_0^T dt = T = \int_0^L \frac{dx}{\sqrt{x \cdot g}} = \frac{1}{\sqrt{g}} \int_0^L x^{-1/2} dx$$

$$= \frac{1}{\sqrt{g}} 2x^{1/2} \Big|_0^L$$

$$= \frac{1}{\sqrt{9.8 \, m/s^2}} \cdot 2 \cdot (1m)^{1/2}$$

$$= 0.32s \cdot 2 = \boxed{0.64s = T}$$