Physics 160-02 Sp. 2017 Exam #4 Name:_______________

1) A computer hard disk starts from rest. It speeds up with constant angular acceleration until it has an angular speed of 7200 rpm. If it completes 150 revolutions while speeding up, what is its angular acceleration?

This is just a constant angular acceleration problem with $\theta_i = 0rad$, $\theta_f = 150rev \cdot \frac{2\pi rad}{rev} = 942.5rad$, $\omega_i = 0\frac{rad}{s}$, and $\omega_f = 7200\frac{rev}{\min} \cdot \frac{2\pi rad}{rev} \cdot \frac{\min}{60s} = 754\frac{rad}{s}$. Using the constant acceleration equation:

$$\omega_{fx}{}^2 = \omega_{ix}{}^2 + 2\alpha_z(\theta_f - \theta_i) \Rightarrow$$

$$\left(754\frac{rad}{s}\right)^2 = 0 + 2\alpha_z(942.5rad - 0) \Rightarrow$$

$$\alpha_z = 301\frac{rad}{s^2}$$

2) A solid sphere of mass 5 kg is rolling without slipping with total kinetic energy 7 J. With what speed **is a point on the top of the sphere** instantaneously moving?

The energy of the rolling sphere is just given by the sum of the linear and rotational kinetic energy:

$$KE_{Tot} = KE_{lin} + KE_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 7J$$

and given the relationship between the rolling and linear velocities:

$$7J = \frac{1}{2}(5kg)v^2 + \frac{1}{2}\left(\frac{2}{5}(5kg)\cancel{R^2}\right)\frac{v^2}{\cancel{R^2}}$$

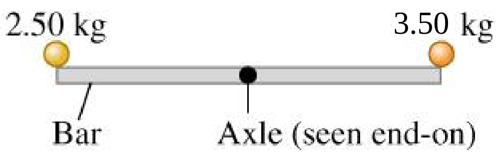
$$= \frac{5kg}{2}v^2 + \frac{2kg}{2}v^2$$

$$7J = \frac{7kg}{2}v^2 \Rightarrow$$

$$v = 1.4\frac{m}{s}$$

is the speed of the center of mass. But the top of the sphere is moving twice this speed, so v=2.8m/s.

3) A thin, uniform 4 kg rod, 1 m long, has two very small (point particle) balls glued at either end, one with mass 2.5 kg and one with mass 3.5 kg. It is supported horizontally by a thin, horizontal, frictionless axle passing through its center and perpendicular to the bar. At the instant shown, what is the angular acceleration of the bar?



This is just a rotational Newton's Second Law problem:

$$\sum \tau_z = I\alpha_z$$

The moment of inertia of the object is the sum of the moment of inertia of the constituents, so

$$I = I_{2.5kg} + I_{3.5kg} + I_{bar}$$

$$= (2.5kg)\cdot(0.5m)^2 + (3.5kg)\cdot(0.5m)^2 + \frac{1}{12}(4.0kg)(1.0m)^2$$

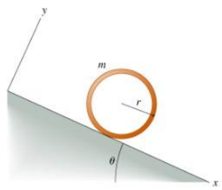
$$= 1.83kgm^2$$

The weight of the bar creates no torque about the axel since the axel passes through the CM of the rod, so:

$$\sum \tau_z = 2.5kg\cdot 9.8\frac{m}{s^2}\cdot 0.5m - 3.5kg\cdot 9.8\frac{m}{s^2}\cdot 0.5m = 1.83kgm^2\alpha_z \Rightarrow$$

$$\alpha_z = -2.67\frac{rad}{s^2}$$

4) A circular hoop of mass m, radius r, and infinitesimal thickness rolls without slipping down a ramp inclined at an angle θ with the horizontal. What is the acceleration a of the center of the hoop?



The forces on the hoop are the weight and the force of friction, so applying Newton's second law: $\sum F_x = mg\sin\theta - F_{fs} = ma_x$. Neither the force of friction nor the acceleration is known, so we must look at rotational motion:

$$\sum \tau_z = -F_{fs} \cdot r = I\alpha_z = -mr^2 \frac{a_x}{r} \Rightarrow$$

$$F_{fs} = ma_x$$

Putting this into the first equation yields:

$$mg\sin\theta - ma_x = ma_x \Rightarrow$$

$$a_x = \frac{1}{2} g\sin\theta$$

5) What is the minimum coefficient of friction μ_{min} needed for the hoop to roll without slipping?

In the case of the minimum coefficient of static friction, $F_{fs} = \mu_s F_N$, so we must analyze the y-component of the forces:

$$\sum F_y = F_N - mg\cos\theta = 0 \Rightarrow F_N = mg\cos\theta .$$

So,

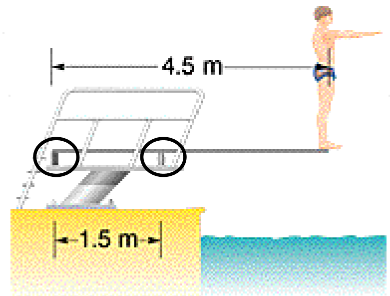
$$F_{fs} = \mu_s F_N = \mu_s mg\cos\theta \Rightarrow$$

$$\mu_s = \frac{F_{fs}}{mg\cos\theta}$$

but from the first problem,

$$\mu_s = \frac{F_{fs}}{mg\cos\theta} = \frac{\frac{1}{2} mg\sin\theta}{mg\cos\theta} = \frac{1}{2}\tan\theta$$

A diver of weight 775 N stands at the end of a 4.5 m diving board of negligible mass. The board is attached to two pedestals 1.5 m apart (circled below).



6) What are the magnitude and direction of the force on the board from the left pedestal?

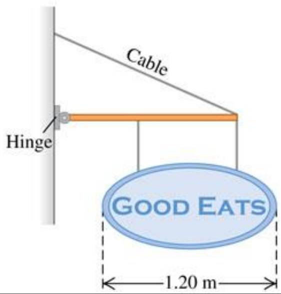
Newton's second law for rotation states that the sum of all the torques will be zero for a static equilibrium situation. Setting the pivot point at the right support, we have F_l *1.5m– 775N*3m = 0 or F_l = 1550N downward.

7) What are the magnitude and direction of the force on the board from the right pedestal?

Newton's second law for a static equilibrium situation says that the sum of all forces must be zero or -775N – 1550N + F_r = 0 or F_r = 2325N upward.

You open a restaurant and hope to entice customers by hanging out a sign. The uniform horizontal beam supporting the sign is 1.60 m long, has a mass of 16.0 kg , and is hinged to the wall. The sign itself is uniform with a mass of 32.0 kg and overall length of 1.20 m . The two wires supporting the sign are each 37.0 cm long, are 90.0 cm apart, and are equally spaced from the middle of the sign. The cable supporting the beam is 2.10 m long.

8) What minimum tension must your cable be able to support without having your sign come crashing down?



Another equilibrium problem, so the sum of all torques and forces must be zero. Start with the torques about the pivot point of the hinge (since we know nothing about those forces):

$$\sum \tau = -0.8m \cdot 16kg \cdot 9.8\frac{m}{s^2} - 1.6m \cdot 16kg \cdot 9.8\frac{m}{s^2} - (1.6m - 0.9m) \cdot 16kg \cdot 9.8\frac{m}{s^2} + 1.6m \cdot F_{Cable} \cdot \sin\left(\cos^{-1}\left(\frac{1.6m}{2.1m}\right)\right) = 0$$

$$= -125.44 - 250.88Nm - 109.76Nm + 1.036m \cdot F_{Cable} = 0 \Rightarrow$$

$$F_{Cable} = 469.2N$$

9) A merry-go-round with a radius of 2m and a moment of inertia 100kgm^2 is rotating at 0.2 revolutions per second. A child of mass 50kg runs at the merry-go-round in a line tangent to its edge, and grabs onto it. After the child is on board, it rotates at 0.3 revolutions per second. How fast was the child running?

This involves conservation of angular momentum, $L_i = L_f$.

$$L_i = \left(I\omega_i\right)_{merry-go-round} + \left(\vec{r} \times \vec{p}\right)_{child}$$

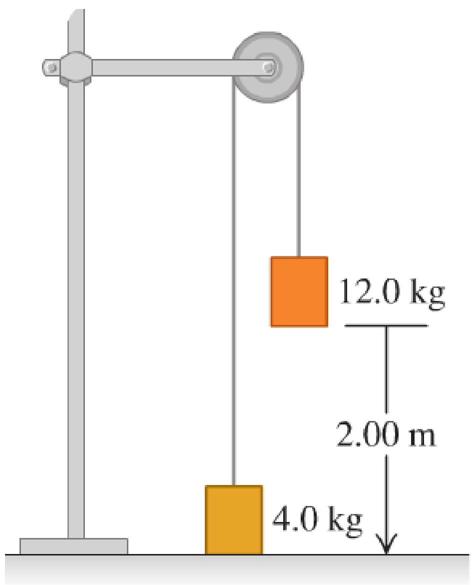
$$= \left(100kgm^2\right)\left(0.2\frac{rev}{s}\right)\left(\frac{2\pi rad}{rev}\right) + (2m)(50kg)v$$

$$L_f = \left[I_{merry-go-round} + I_{child}\right]\omega_f$$

$$= \left[\left(100kgm^2\right) + (50kg)(2m)^2\right]\left(0.3\frac{rev}{s}\right)\left(\frac{2\pi rad}{rev}\right)$$

Solving for v, we get v=4.4m/s

10) A system of two paint buckets connected by a lightweight rope is released from rest with the 12.0 kg bucket 2.00 m above the floor. The pulley has radius 0.160 m and a moment of inertia 0.380 kg·m^2. The rope does not slip on the pulley rim. During the time that the 12.0 kg bucket drops to the floor, friction in the pulley removes 10.0 J of energy from the system. What is the speed of the bucket when it hits the floor?



From conservation of energy, the net change in mechanical energy is the work done by friction, so

$$\Delta E_{mech} = W_{f_k} = \Delta K + \Delta U_g + \Delta U_{el}$$

$$-10J = \left(1/2\,mv_f^2 - 1/2\,m\cancel{v_0^2}\right)^{4kg} + \left(1/2\,mv_f^2 - 1/2\,m\cancel{v_0^2}\right)^{12kg} + \left(1/2\,I\omega_f^2 - 1/2\,I\cancel{\omega_0^2}\right)^{Pulley} + \left(mgy_f - mg\,\cancel{y_0}\right)^{4kg} + \left(mg\,\cancel{y}\right.$$

$$-10J = \left(1/2\,mv^2\right)^{4kg} + \left(1/2\,mv^2\right)^{12kg} + \left(1/2\,I\omega_f^2\right)^{Pulley} + \left(mgy\right)^{4kg} + \left(mgy\right)^{12kg} \Rightarrow$$

$$-10J = \left(1/2\,mv^2\right)^{4kg} + \left(1/2\,mv^2\right)^{12kg} + \left(1/2\,I\frac{v^2}{r^2}\right)^{Pulley} + \left(mgy\right)^{4kg} + \left(mgy\right)^{12kg} \Rightarrow$$

$$-10J = 1/2\left(12kg + 4kg\right)v^2 + \left(1/2\cdot 0.380kgm^2\frac{v^2}{\left(0.16m\right)^2}\right) + \left(4kg\right)g\left(2m\right) - \left(12kg\right)g\left(2m\right) \Rightarrow$$

$$v^2 = \frac{-10J}{15.42kg} - \left(\frac{8kg\cdot m}{15.42kg}\right)\left(9.8\frac{m}{s^2}\right) + \left(\frac{24kg\cdot m}{15.42kg}\right)\left(9.8\frac{m}{s^2}\right)$$

$$v^2 = 9.52\frac{m^2}{s^2} \Rightarrow$$

$$v = 3.08\frac{m}{s}$$