PHYS302 Fall 2023

Homework 3

Total possible points: 23 out of 20

1. 4.8 from Hecht

3 pts, two for a good approach, one for correctness

2. 4.15 from Hecht

3 pts, one for good set up, one for using Snell's Law, one for correctness

3. 4.23 from Hecht

3 pts, one for identifying a different refraction for red vs violet, one for attempting to find the lateral difference in red and violet beams, and one for correctness.

4. 4.39 from Hecht

3 pts; 2 for good attempt, 1 for correctness

5. 4.54 from Hecht

4 points; 1 for attempt, 1 for identifying half the power as P_perpendicular, 1 for using energy conservation, 1 for correctness

6. 4.84 from Hecht

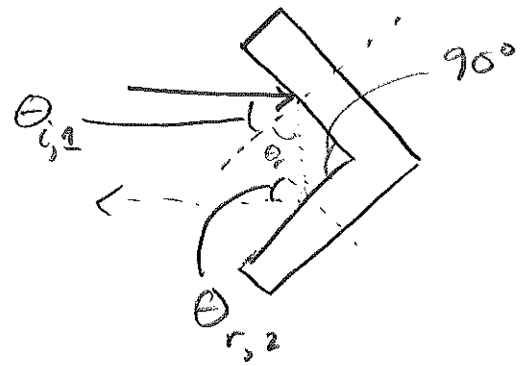
4 points, 2 for a good explanation, 1 for setting up problem well, 1 for correct answer (minus 1 if answer claims this is total internal reflection without specifying that the light is actually refracted—see solutions.)

7. (E.C.) 4.36 from Hecht

   3 pts, 2 points for setting up the problem such that you minimize OPL with respect to the point (or angle) of reflection, 1 point for correctness

HW #3

① (4.8)

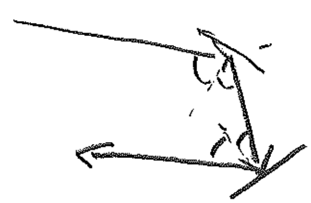


2 reflections:

(1) $\theta_{i,1} = \theta_{r,1}$

(2) $\theta_{i,2} = 90° - \theta_{r,1} = \theta_{r,2}$

Total angle change is the sum of all θ_i & θ_r:

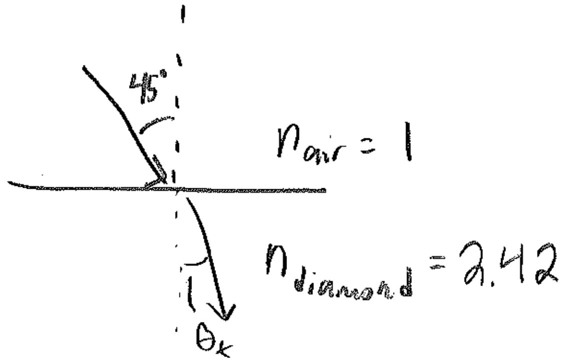


$$\theta_{i,1} + \underset{\downarrow}{\theta_{r,1}} + \theta_{i,2} + \theta_{r,2}$$

$$\Rightarrow \theta_{i,1} + \theta_{i,1} + 90° - \theta_{i,1} + 90° - \theta_{i,1}$$

$= 180°$ // The corner mirror always gives a 180° reflection ("retro-reflection").

(2) (4.15)



$$\sin \theta_t = \frac{n_{air}}{n_{diamond}} \sin(45°)$$

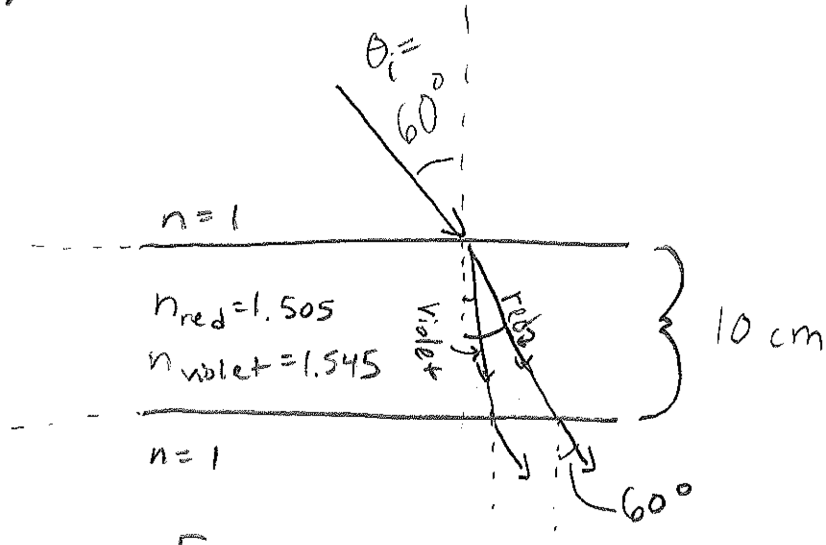
$$\theta_t \approx 17°$$

Angular deviation:

$\Delta\theta = \theta_i - \theta_t \Longrightarrow$

$$\boxed{\Delta\theta \approx 28°}$$

③ (4.23)

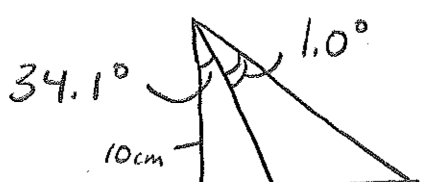


$$\theta_{t,red} = \arcsin\left[\sin(60°)\cdot\frac{1}{1.505}\right]$$

$$\approx 35.1°$$

$$\theta_{t,violet} = \arcsin\left[\sin(60°)\cdot\frac{1}{1.545}\right]$$

$$\approx 34.1°$$



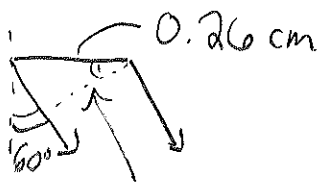
(not to scale!)

$\tan(34.1°)\cdot 10.0\text{cm} = 6.77\text{ cm}$

$\tan(35.1°)\cdot 10.0\text{ cm} = 7.03\text{cm}$

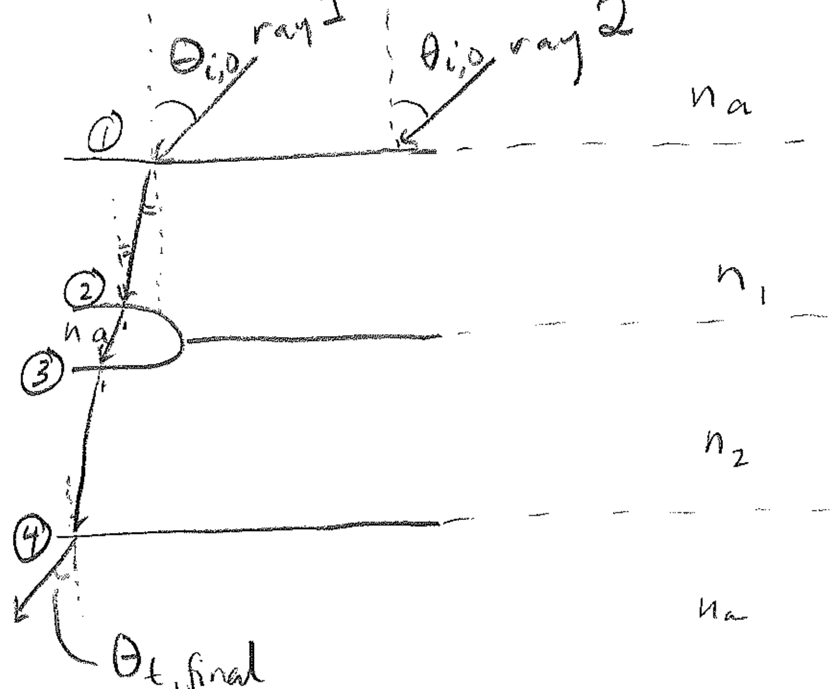
$d = 7.03 - 6.77 = \boxed{0.26\text{ cm}}$

This answer is okay, too.



width of beam is $\cos(60°)\cdot 0.26\text{cm} = \boxed{0.13\text{ cm}}$

④ (4.39)



Ray 1: 4 refractions

① $\sin \theta_{t,1} = \frac{n_a}{n_1} \sin \theta_{i,0}$

② $\theta_{t,1} = \theta_{i,2}$:

$\sin \theta_{t,2} = \frac{n_1}{n_a} \sin(\theta_{t,1})$

③ Again, $\theta_{t,2} = \theta_{i,3}$:

$\sin \theta_{t,3} = \frac{n_a}{n_2} \sin \theta_{t,2}$

④ $\theta_{t,3} = \theta_{i,4}$:

$\sin \theta_{t,final} = \frac{n_2}{n_a} \sin \theta_{t,3}$

$$\sin \theta_{t,final} = \frac{n_2}{n_a}\left(\frac{n_a}{n_2}\left(\frac{n_1}{n_a}\left(\frac{n_a}{n_1} \sin \theta_{i,0}\right)\right)\right)$$

$$\sin \theta_{t,final} = \sin \theta_{i,0}$$

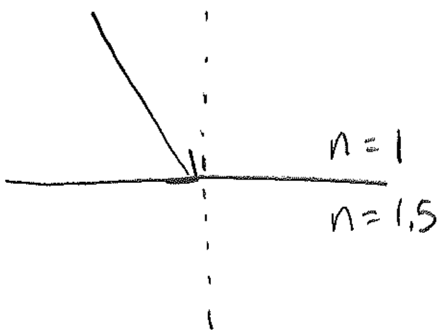
$$\boxed{\theta_{t final} = \theta_{i0}}$$

→ Same can be shown for ray 2 : $\theta_{tfinal\_ray2} = \theta_{i0\_r2}$

→ If rays 1 & 2 are parallel to start, then $\theta_{i0\_ray1} = \theta_{i0\_ray2}$, and

thus $\boxed{\theta_{t,final\_ray1} = \theta_{t,final\_ray2}}$

(5) (4.54)



1000 W/m² of unpolarized light

≈ 500 W/m² each of $P_{\parallel}$ & $P_{\perp}$

$$T_{\perp} = 0.80 = 1 - R_{\perp}$$

$$R_{\perp} = 0.2$$

Since area doesn't change in reflection,

$$I_{\text{reflected}, \perp} = 0.2 \cdot I_{0,\perp}$$

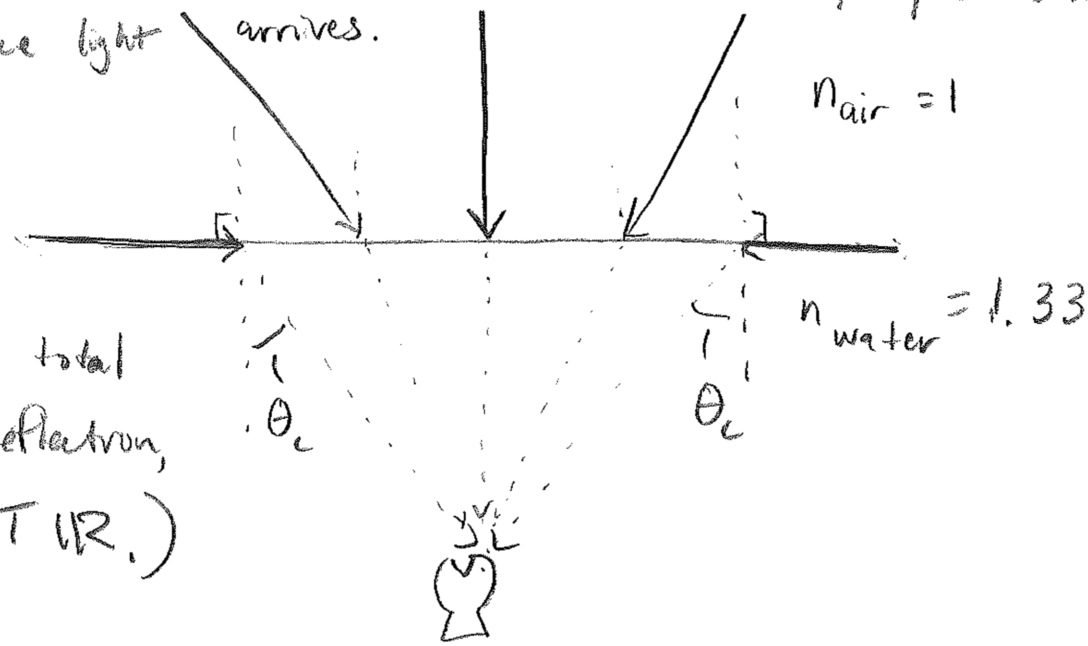
$$= 0.2 \cdot 500 \text{ W/m}^2$$

$$= \boxed{100 \text{ W/m}^2}$$

<u>Note</u>: In the glass, be careful when calculating T. While the <u>power</u> is 80% of the incident $P_{\perp}$, the <u>intensity</u> will <u>not</u> be 400 W/m², as the beam width changes!

⑥ (4.84)

For all light entering the water with incident angle θ_i, there is a corresponding angle θ_t that the fish views the light. Since θ_i can be at most $90°$ and $\theta_t < \theta_i$, there will be a maximum θ_t the fish sees, beyond which no surface light arrives.

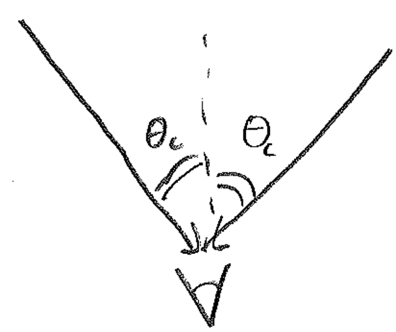


(Related to total internal reflection, but not TIR.)

$$n_{water} \sin \theta_c = n_{air} \sin (\pi/2)$$

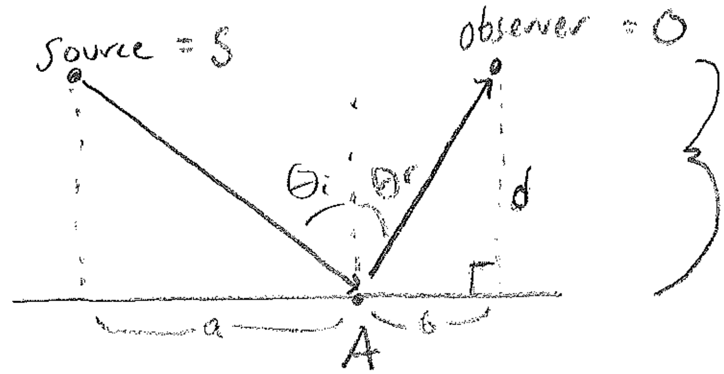
$$\theta_c \approx 49°$$

Cone angle is $2\theta_c = \boxed{98°}$



Extra credit (4.36):

Derive $\theta_i = \theta_r$ by minimizing transit time (i.e. Fermat's Principle).



Both S & O are distance d above interface

Total path length: $\overline{SA} + \overline{AO}$

Path time: $t = \dfrac{\overline{SA}}{v} + \dfrac{\overline{AO}}{v}$

→ Let distance $\overline{SO} = x = a + b$

$$t = \frac{\sqrt{d^2 + a^2}}{v} + \frac{\sqrt{d^2 + b^2}}{v}$$

$$= \frac{\sqrt{d^2 + a^2}}{v} + \frac{\sqrt{d^2 + (x-a)^2}}{v}$$

We want to know which a gives shortest t:

$$\frac{dt}{da} = \frac{1}{2}\frac{(d^2 + a^2)^{-1/2}}{v} \cdot 2a + \frac{1}{2}\frac{(d^2 + (x-a)^2)^{-1/2}}{v} \cdot 2(x-a)(-1)$$

$$0 = \frac{a}{v\sqrt{d^2 + a^2}} + \frac{a - x}{v\sqrt{d^2 + (x-a)^2}} \Leftarrow \boxed{\text{True if } a = \tfrac{1}{2}x \quad \text{Therefore } \theta_i = \theta_r}$$