## Homework 4

## (Due Date: Monday, Feb. 26)

Problem 1.- Write a Matlab (or Mathematica) program to fit a set of four runs of data shown in the figure to a straight line using the method of maximum likelihood estimation (MLE). Obtain the slope " $m$ " and intercept " $b$ " of the line, and their estimated errors $\sigma_{m}$ and $\sigma_{b}$. Note that to perform the fit you need first to calculate the mean (Yave) and standard deviation ( $\sigma_{Y}$ ) for data points " y " corresponding to each position of the independent variable " $x$ ".

The data of the four runs is:

| Position | Run 1 | Run 2 | Run 3 | Run 4 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{x}$ | $\mathbf{y 1}$ | $\mathbf{y 2}$ | $\mathbf{y 3}$ | $\mathbf{y 4}$ | Yave | $\boldsymbol{\sigma}_{\mathbf{Y}}$ |
| 0 | 5.57 | 6.30 | 5.11 | 4.88 |  |  |
| 2 | 6.28 | 8.59 | 5.89 | 10.85 |  |  |
| 4 | 13.50 | 12.73 | 12.87 | 8.48 |  |  |
| 6 | 11.70 | 13.35 | 11.65 | 14.56 |  |  |
| 8 | 19.79 | 18.34 | 13.88 | 16.31 |  |  |
| 10 | 19.41 | 20.85 | 17.00 | 19.43 |  |  |
| 12 | 24.61 | 24.95 | 22.30 | 24.57 |  |  |
| 14 | 24.99 | 27.67 | 27.59 | 27.52 |  |  |
| 16 | 27.49 | 33.79 | 26.62 | 34.16 |  |  |
| 18 | 36.36 | 31.71 | 32.13 | 32.40 |  |  |
| 20 | 38.42 | 39.17 | 32.20 | 36.81 |  |  |
| 22 | 43.26 | 39.47 | 39.66 | 40.00 |  |  |
| 24 | 43.38 | 42.70 | 44.02 | 43.29 |  |  |



In your homework provide
a) The Matlab file with comments describing the command lines in the program.
b) A plot showing the average data with error bars indicating the standard deviations and the fitted line. The plot should show legends and an inset describing the equation of the line with the values of the slope and intercept with the corresponding errors. Include labels in axes.

IMPORTANT: do not use predefined functions "mean" or "std" and do not use automatic fit tools. Instead, use the formulas seen in class for calculating the slope, intersect and their errors. Also you can check the formulas in Bevington CH 6 Sections 6.3-6.4.

Problem 2.- (Bevington 6.5) A student hangs masses on a spring and measures the spring's extension as a function of the applied force in order to find the spring constant " $k$ ". Her measurements are:

| Mass (Kg) | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Extension (cm) | 5.1 | 5.5 | 5.9 | 6.8 | 7.4 | 7.5 | 8.6 | 9.4 |

There is an uncertainty of $\mathbf{0 . 2}$ in each measurement of the extension. The uncertainty in the masses is negligible. For a perfect spring, the extension $\Delta L$ of the spring will be related to the applied force by the relation " $k \Delta L=F$ ", where $F=m g$, and $\Delta L=L-L_{0}$, and $L_{0}$ is the unstretched length of the spring. Use these data and the method of "Maximum Likelihood estimation" to find the spring constant $k$, the unstretched length of the spring $L_{0}$, and their uncertainties.

Repeat the requests from " $a$ " and " $b$ " from the last problem (problem 1) taking the average data as the "Extension (cm)" from the table above.

