Lec 6: 09/04/2009

Quantum Mechanics:

Postulates:

1- The state of a particle is represented by a vector $|\psi(t)\rangle$ in the Hilbert space.

2- Variable x, p of classical mechanics are represented by Hermitia operator X, P ($P = -i\hbar D$). In the eigenbasis of X;

$$\langle x|X|x'\rangle = \delta(x-x') \qquad \langle x|P|x'\rangle = -i\hbar\,\delta'(x-x')$$

Also;

$$\omega(x,p) \rightarrow \Omega(X,P)$$

3- Observables are represented by Hermitian operators. Only eigenvalues ω of a Hermitian operator corresponding to an observable Ω are obtained as a result of measurement in the lab.

If the particle is in state $|\psi\rangle$, the probability to find ω in a measurement is;

$$P(\omega) \propto |\langle\omega|\psi\rangle|^2$$

where $|\omega\rangle$ is the eigenvector corresponding to ω. The state will change to $|\omega\rangle$ right after the measurement.

4- The state vector $|\psi(t)\rangle$ obeys Schrodinger equation;

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

where $H(X,P)$ is the Hamiltonian operator.

Example: Consider a fictitious Hilbert space $\mathbb{V}^3(C)$. Hermitian operator Ω represents an observable. Eigenvectors of Ω form an orthonormal basis (upon normalization) $\{|1\rangle, |2\rangle, |3\rangle\}$. The state vector can be written as a linear combination of these vectors;

$|\psi\rangle = a|1\rangle + b|2\rangle + c|3\rangle$

Now assume $\omega_1, \omega_2, \omega_3$ are eigenvalues corresponding to $|1\rangle, |2\rangle, |3\rangle$ respectively.

A measurement of this observable yields one of $\omega_1, \omega_2, \omega_3$ only.

overall factor

$P(\omega_1) \propto |\langle 1|\psi\rangle|^2 = |a|^2 \Rightarrow P(\omega_1) = A|a|^2$

$P(\omega_2) \propto |\langle 2|\psi\rangle|^2 = |b|^2 \Rightarrow P(\omega_2) = A|b|^2$

$P(\omega_3) \propto |\langle 3|\psi\rangle|^2 = |c|^2 \Rightarrow P(\omega_3) = A|c|^2$

The total probability must be one, thus:

$P(\omega_1) + P(\omega_2) + P(\omega_3) = 1 \Rightarrow A\left(|a|^2 + |b|^2 + |c|^2\right) = 1 \Rightarrow$

$$A = \frac{1}{\sqrt{|a|^2 + |b|^2 + |c|^2}}$$

Therefore if we choose the normal vector:

$$|\psi\rangle = \frac{1}{\sqrt{|a|^2 + |b|^2 + |c|^2}}\left(a|1\rangle + b|2\rangle + c|3\rangle\right)$$

as the state vector, we will have:

$P(\omega_1) = |\langle 1|\psi\rangle|^2$, $P(\omega_2) = |\langle 2|\psi\rangle|^2$, $P(\omega_3) = |\langle 3|\psi\rangle|^2$

Now, if a measurment yields value ω_2, the state vector will change to $|2\rangle$ immediately after the measurement.

One comment is in order regarding postulate 2. If x and p dependent terms in ω are separate, there will be no ambiguity in replacements $x \to X$, $p \to P$. For example, consider the Hamiltonian:

$$H = \frac{p^2}{2m} + V(x) \Rightarrow H = \frac{P^2}{2m} + V(X)$$

However, mixed terms can be ambiguous:

$$xp \to XP \quad \text{or} \quad xp = px \to PX \quad ?$$

Note that $XP \neq PX$. In such cases we use the symmetric sum:

$\omega = xp = \frac{1}{2}(xp + px) \Rightarrow \Omega = \frac{1}{2}(XP + PX)$

This also ensures that Ω is Hermitian.

A state vector $|\psi\rangle$ is defined up to an overall phase.

Consider multiplication by a pure phase:

$|\psi\rangle \rightarrow |\psi'\rangle = e^{i\alpha}|\psi\rangle$

The probability that measurement of observable Ω in the new state yields value ω (such that $\Omega|\omega\rangle = \omega|\omega\rangle$) is:

$P_\omega = |\langle\omega|\psi'\rangle|^2 = |e^{i\alpha}\langle\omega|\psi\rangle|^2 = |\langle\omega|\psi\rangle|^2$

This holds for all observable. Therefore there is no way to distinct between $|\psi\rangle$ and $|\psi'\rangle$ based on measuring physical observables. As a result, $|\psi\rangle$ and $|\psi'\rangle$ are equivalent.