PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Homework Assignment #1

(Due September 4)

1- Ω is a Hermitian operator, and operator U is defined as $U = \exp(-i\Omega t)$ (t is a real parameter).

(a) Show that U is unitary.

(b) The eigenvalues of Ω are given by ω_i. What are the eigenvalues of U?

(c) For an arbitrary vector $|V\rangle$ show that

$$i\frac{d}{dt}|V'\rangle = \Omega|V'\rangle$$

where $|V'\rangle = U|V\rangle$.

(d) For an arbitrary time-independent operator Λ show that

$$i\frac{d}{dt}\Lambda' = [\Lambda', \Omega]$$

where $\Lambda' = U^\dagger \Lambda U$.

2- Consider the Hilbert space of functions $f(x)$ defined in the interval $x \in [0, 2\pi]$. Show that the operator $D^2 \equiv \frac{d^2}{dx^2}$ is Hermitian if $[0, 2\pi]$ represents

(a) A line segment with vanishing boundary condition $f(0) = f(2\pi) = 0$.

(b) A circle of unit radius with periodic boundary condition $f(0) = f(2\pi)$ and $\frac{df}{dx}(0) = \frac{df}{dx}(2\pi)$.

Find the eigenvalues and eigenvectors of D^2 in both cases. What is the number of linearly independent eigenvectors for a given eigenvalue in each case?