PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Midterm Exam #2

Due Date: 11/13/2009 by 11:00 am

Instructions:

- This is an open-book open-note exam, all reference material allowed.
- All reference material allowed, but no DISCUSSING PROBLEMS.
- Any questions are to be directed to the instructor.
- Two problems, equally weighted.

1-Consider a simple harmonic oscillator for a particle with mass m and natural frequency ω governed by the standard Hamiltonian $\mathbf{H_0} = \mathbf{P}^2/2m + m\omega^2\mathbf{X}^2/2$. The particle is prepared in the ground state $|0\rangle$. A constatnt force F_0 is applied for a very short time δt, imparting a momentum impulse $p_0 = F_0\delta t$ to the particle.

(a) What is the additional interaction Hamiltonian $\mathbf{H}_{\text{int}}$ during the time the force is applied?

(b) If the impulse time is sufficiently short, we can neglect the dynamical contribution of $\mathbf{H_0}$ during δt, and take the state after the impulse to be

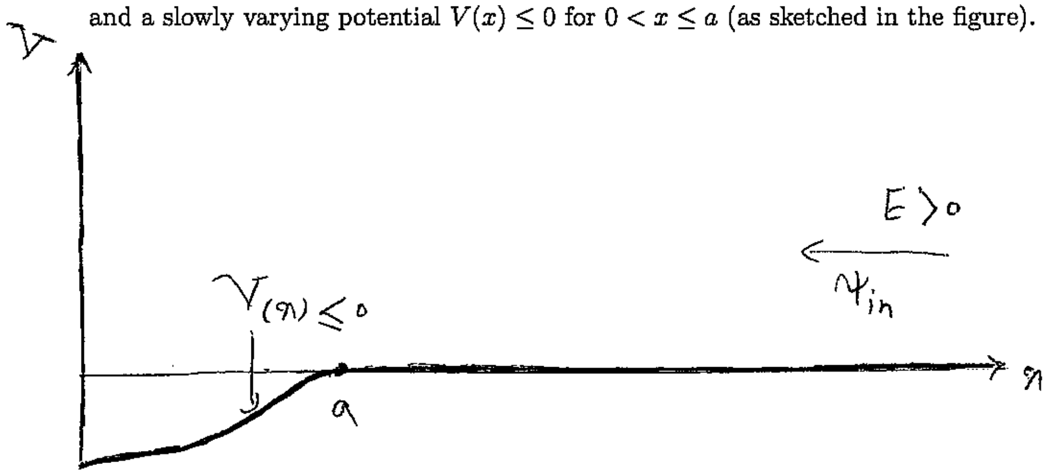
$$|\psi(\delta t)\rangle = \exp\left(\frac{-i\mathbf{H}_{\text{int}}\delta t}{\hbar}\right)|0\rangle\,.$$

What is the condition on δt for this to be a good approximation? Show that $|\psi(\delta t)\rangle$ is a coherent state, i.e. it is an eigenstate of the lowering operator $\mathbf{a}$.

(c) Find $|\psi(t)\rangle$ for all times and show that it remains a coherent state.

(d) Find the mean energy $\langle E\rangle$ and energy uncertainty ΔE as a function of time. What is the restriction on the impulse momentum p_0 such that we are in a truly clasccial regime, i.e. $\Delta E \ll \langle E\rangle$.

2- Consider a one-dimensional square well with an infinite wall at $x = 0$, $V = 0$ at $x > a$, and a slowly varying potential $V(x) \leq 0$ for $0 < x \leq a$ (as sketched in the figure).



(a) An incoming plane wave with energy E that is sent from $+\infty$, $\psi_{\text{in}} \propto e^{-i\sqrt{2mE}x/\hbar}$, scatters off the potential. Use the WKB approximation and write the overall (incoming plus reflected) wavefunction ψ.

(b) The reflected plane wave emerges from the potential with a phase shift, $\psi_{\text{out}}(x) = -Ae^{i\sqrt{2mE}x/\hbar}e^{i\phi}$ (for $x > a$). Find the phase shift ϕ as a function of the potential $V(x)$.

(c) The phase shifted reflected wave is equivalent to that which would arise from an infinite wall but moved to a distance b from the origin. Find b as a function of the phase shift ϕ. What is the sign of b for the potential shown in the above figure? Comment.

(d) Calculate the phase shift ϕ for $V(x) = -V_0(x-a)^2$ where $V_0 > 0$.