

# PHYC 521: Graduate Quantum Mechanics I

Fall 2009

Midterm Exam #1

Due Date: 09/30/2009 by 11:00 am

## **Instructions:**

- This is an open-book open-note exam, all reference material allowed.
- All reference material allowed, but no DISCUSSING PROBLEMS.
- Any questions are to be directed to the instructor.
- Three problems, equally weighted.

1- Consider the potential  $V(x) = -aV_0\delta(x)$ . Show that it admits only one bound state of energy  $E = -ma^2V_0^2/2\hbar^2$ . (Hint: Make proper use of the continuity conditions.)

**2-** Show that an arbitrary state vector  $\psi(x, t)$  belonging to the Hilbert space of a particle in the box of length  $L$  is a periodic function of time  $\psi(x, t + T) = \psi(x, t)$ . What is the largest possible value of the period  $T$ ? (Hint: Expand  $\psi(x, 0)$  in the energy eigenbasis and consider its time evolution).

Compare the evolution of a wavepacket for a particle in the box with that for a free particle. Comment on the difference.

**3-** The potential for a harmonic oscillator in one dimension is given by  $V(X) = m\omega^2 X^2/2$ , where  $\omega$  is natural frequency of the oscillator. For a given state  $\psi(x, t)$  the expectation values of position and momentum at  $t = 0$  are given by:

$$\langle X \rangle(0) = x_0 \quad , \quad \langle P \rangle(0) = p_0 .$$

Find  $\langle X \rangle(t)$  and  $\langle P \rangle(t)$  in terms of  $x_0$ ,  $p_0$ ,  $m$ ,  $\omega$ .