Preliminary Examination: Thermodynamics and Statistical Mechanics

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Instructions:
• The exam consists of 10 short-answer problems (10 points each).
• Where possible, show all work; partial credit will be given if merited.
• Personal notes on two sides of an 8×11 page are allowed.
• Total time: 3 hours.
**P1:** The application of statistical mechanics results in an expression for the average magnetic moment of an atom in the direction of a uniform applied magnetic field of strength $B$ in certain common physical systems at temperature $T$, which is of the form

$$\mu = \text{const} \cdot f(B, T).$$

where the limit of $f(B, T)$ as $T$ tends to 0 or $B$ tends to $\infty$ is 1. What is the physical meaning of the factor $\text{const}$? Give the explicit form of the function $f(B, T)$ for two different physical systems (indicate which is which and explain symbols) and point out what the expected qualitative behavior of $f(B, T)$ is, in both examples you give, as $T$ and $B$ are varied.

**P2:** Calculate the energy density of states for free noninteracting quantum particles in 2 dimensions and answer from your expression whether it increases, decreases or remains constant as the energy varies.

**P3:** The Hamiltonian for an extreme classical anharmonic oscillator in 1-dimension is given by

$$H = \frac{p^2}{2m} + \frac{1}{10}kx^{10}.$$ 

The oscillator is in contact with a heat reservoir at temperature $T$. The equipartition law states that the average kinetic energy of a particle in thermal equilibrium is $k_BT/2$ per degree of freedom. Using that law together with the virial theorem,

$$\langle p \frac{\partial H}{\partial p} \rangle = \langle x \frac{\partial H}{\partial x} \rangle,$$

determine the average energy of the oscillator.
**P4:** How is the partition function of a system in equilibrium related to the energy state density of the system? How can you calculate the free energy of the system knowing the partition function? A simplified model of a gas at volume $V$ and temperature $T$ results in the following expression for the free energy $A$:

$$A + Nk_B T \ln(V - b) + \frac{a}{V} = 0$$

Work out an expression for the pressure exerted by the gas as a function of $N$, $T$, $V$, and comment on what physical meaning $N$, $a$, $b$ might have in this model.

**P5:** Consider a charmonic oscillator whose energy levels are given by $E_n = n\epsilon$ where $n$ is a non-negative integer smaller or equal to a finite number $N$. Calculate the specific heat of a collection of noninteracting charmonic oscillators and specify clearly the difference relative to a collection of harmonic oscillators, each with the same energy spacing $\epsilon$ but with a nonvanishing zero-point energy and infinite $N$. Make plots of the two specific heats and indicate on the plot the effects of the difference in $N$ and in the zero-point energy.

**P6.** Consider a gas of $N$ atoms which can be considered to be non-interacting bosons the energy spectrum of each comprising only of two states of energies $E_1$ and $E_2$. The gas is in thermal equilibrium at temperature $T$. Calculate the average energy as a function of $N$, $T$, $E_1$ and $E_2$ and sketch as a function of $T$. 


**P7.** Briefly explain in *any three* of the following cases the term and its relevance:

**P8.** A system of $N$ identical noninteracting particles is in equilibrium at temperature $T$. The single-particle energy, i.e., the energy that any of the particles can possess, varies from 0 to $\infty$. Sketch the dependence of the average number of particles in a (single-particle) state of a given energy versus that energy for the three cases when the statistics are: (i) Maxwell-Boltzmann, (ii) Fermi-Dirac, and (iii) Bose-Einstein. Explain under what physical conditions any of these may be approximated by any other(s).

**P9.** Explain the origin of the Boltzmann factor $\exp(-E/k_B T)$ where $E$ is the energy and $T$ the temperature? Is it a law of nature, a postulate of mathematics, a mysterious religious belief or something else?

**P10.** The specific heat of a metal has an important part that varies strongly as $T^n$. So does that of an insulating solid at high $T$ and also at low $T$. The values of $n$ are different in these three cases: 0, 1, and 3, NOT necessarily in that order. Explain what physical characteristics of the systems lead to these three power law dependences and why $n$ has the particular value for each system. Justify your answer on the basis of analytic expressions and clear arguments.