Physics 161

HW #7
From RHR, (Thumb to left in direction of To), B0 is X

Long wire \( \Rightarrow B_0 = \frac{u_0 I_0}{\text{om}} \)

\( \Rightarrow \) Directly prop to To

1) Increasing To, increases B0 which increases flux through loop. Lenz's Law \( \Rightarrow \) induced field will try to cancel by being O. RHR for loop (Thumb = B0, Fingers = current)

\( \Rightarrow \) Counter clockwise current

2) Decreasing To, decreases B0 which decreases flux through loop. Lenz's Law \( \Rightarrow \) induced field will try to maintain by being X

Loop RHR \( \Rightarrow \) clockwise induced current

3) Constant To \( \Rightarrow \) Constant B0 \( \Rightarrow \) No change in flux \( \Rightarrow \) No induced current.

4) To switches direction. In order for To to switch, it must first decrease \( \Rightarrow \) clockwise induced current. Then, it must increase but in the opposite direction

\( \Rightarrow \)

Wire RHR \( \Rightarrow \) B0 now O and increasing.

Increasing flux \( \Rightarrow \) B0 tries to cancel by being X \( \Rightarrow \) B0 is X the entire time \( \Rightarrow \) clockwise induced current the entire time.
### In Summary

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<th></th>
<th>Bu</th>
<th>B_n0</th>
<th>Jino</th>
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<tr>
<td>To increase</td>
<td>✗</td>
<td>✗</td>
<td>CCW</td>
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<tr>
<td>To decrease</td>
<td>✗</td>
<td>✗</td>
<td>CW</td>
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<tr>
<td>To constant</td>
<td>✗</td>
<td>None</td>
<td>None</td>
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<tr>
<td>To switches</td>
<td>✗ then ✗</td>
<td>✗</td>
<td>CW</td>
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a) As loop is entering field, the flux through it is increasing ⇒ Bino tries to cancel by being 0. RHR ⇒ CCW Current

b) Motional emf: A positive charge moving upwards through a magnetic field. RHR ⇒ Force to left

⇒ on the upper end ⇒ An excess of positive charge will build up on the left side and negative charge on the right side. ⇒ battery with terminals

⇒ the right and left sides are too thin to have a noticeable separation of charge. The bottom is not inside field yet ⇒ no magnetic force on its charges ⇒ current flow from + to - terminal.
C.

Loop all the way in field.

Now there's no more change in flux through the loop \(\Rightarrow\) zero induced current.

d.) Motional emf: Now that top and bottom are both inside field, the charges in both feel magnetic force \(\Rightarrow\) they both become batteries with \(-\) terminals.

\[ \#1\] tries to make counterclockwise current.

\[ \#2\] tries to make clockwise current.

\[ E_{\text{ind}} = VB \cdot L \Rightarrow E_1 = E_2 \Rightarrow \text{they cancel each other out} \Rightarrow \text{no induced current} \]
What is direction and amount of induced current at \( t = 0 \), 1s, and very long time.

**a.) Direction?**

RHR for current loop \( \Rightarrow \mathbf{B}_0 \) is (\( \leftarrow \))

At the center of the outer loop (where the inner loop is located) for ccw \( I_0 \).

Current loop \( \Rightarrow \mathbf{B}_0 = \frac{\mu_0 I_0}{2\pi a} \)

\( \Rightarrow \mathbf{B}_0 \) is directly prop. to \( I_0 \). In turn, Omm's law

\( \Rightarrow E = I_0 R_0 \).

\( E = 9(1-e^{-t}) \) Starts @ with zero value and increases to 9V \( \Rightarrow I_0 = \frac{E}{R_0} \) is also increasing \( \Rightarrow \)

\( \mathbf{B}_0 \) increasing \( \Rightarrow \) Flux through inner loop is increasing. Lenz's law \( \Rightarrow \mathbf{B}_0 \text{ tries to cancel by being } (\bigcirc) \). Current loop RHR \( \Rightarrow I_{\text{ind}} \) will be clockwise* (\( \bigcirc \) & \( \bigcirc \)) \( I_{\text{ind}} \)

* After a "long" time \( E \) will stop changing since \( e^{-t} \rightarrow 0 \) \( \Rightarrow \) the flux will stop changing \( \Rightarrow I_{\text{ind}} \) will stop and have no direction.
b. Assume $B_0$ is uniform over inner loops area

$$\Rightarrow B_0 = \frac{\mu_0 I_0}{2b} \text{ over all of inner loops area}$$

$$\Rightarrow \Phi = B_0 A = \frac{\mu_0 I_0 \pi a^2}{2b} = \frac{\mu_0 \pi I_0}{2b}$$

Finally, Ohm's Law \( \Rightarrow I_0 = \frac{E_0}{R_0} = \frac{9(1-e^{-t})}{R_0} \Rightarrow \Phi = \frac{\mu_0 \pi I_0}{2b} \frac{9(1-e^{-t})}{R_0}$$

If you prefer to add 's here:

$$\Phi = \frac{(4\pi \times 10^{-7} \text{ Tm/A}) \pi (0.01 \text{ m})^2 9 (1-e^{-t})}{2 (0.01 \text{ m}) 3500}$$

$$\Rightarrow \Phi = 1.269 \times 10^{-11} (1-e^{-t})$$

Faraday's Law: \( \{E_{\text{ind}}\} = -\frac{d\Phi}{dt} = -\frac{\mu_0 \pi a^2 q}{2b R_0} (0-e^{-t}(-1)) \)

$$E_{\text{ind}} = -\frac{\mu_0 \pi a^2 q}{2b R_0} e^{-t} \Rightarrow \quad \frac{I_{\text{ind}}}{R_0} = \frac{\{E_{\text{ind}}\}}{R_0} = \frac{\mu_0 \pi a^2 q}{2b R_0} \frac{e^{-t}}{R_0}$$

Just the amount of current

$$\Rightarrow I_{\text{ind}} = 2.26598 \times 10^{-12} e^{-t} \Rightarrow \quad I_{\text{ind}}(t=0) = 2.27 \times 10^{-12} \text{ A} = 2.3 \mu \text{ A}$$

$$I_{\text{ind}}(t=1) = 2.27 \times 10^{-12} e^{-1} = 0.33 \times 10^{-12} \text{ A}$$

$$= 0.83 \mu \text{ A}$$

Very long time, $e^{-t} \to 0 \Rightarrow I_{\text{ind}} = 0$
a.) Find $\mathcal{E}_{\text{ind}}$.

**Faraday's Law:**

$$\mathcal{E}_{\text{ind}} = \frac{d\Phi}{dt}$$

The magnetic field decreases with distance $\Rightarrow$ have to integrate to find flux.

Wire's R+R $\Rightarrow$ $B_0$ is $\bigcirc$ at every point on loop

$\Rightarrow$ use $dA$ that also points into page (i.e. the side that faces away from us) $\Rightarrow d\Phi = B_0dA \cos \theta$

$\Rightarrow d\Phi = B_0dA$

**Using** $r \cos \theta = dA = rdrd\theta$

$$0 \leq r \leq a$$

$$0 \leq \theta \leq 2\pi$$

at each point on circle we only need the horizontal distance, $x$, from the wire to find $B_0$ since $B_0 = \frac{\mu_0 I}{2\pi x}$

As shown here $x = b + r \cos \theta$

$$\therefore \quad d\Phi = \frac{\mu_0 I}{2\pi} \frac{rdrd\theta}{(b + r \cos \theta)}$$

Let's first integrate over $\theta$ since that integral was given as a hint.
\[
\int_{0}^{2\pi} \frac{d\theta}{\sqrt{b^2 - r^2}} = \frac{\pi b}{\sqrt{b^2 - r^2}} \Rightarrow d\Phi = \frac{1}{2\pi} \frac{\mu_0 I}{r} \frac{d\Phi}{dt} \left( \frac{2\pi r}{\sqrt{b^2 - r^2}} \right)
\]

\[
\Rightarrow d\Phi = \frac{\mu_0 I}{\sqrt{b^2 - r^2}} r \, dr
\]

\[
\Phi = \int_{0}^{a} \frac{\mu_0 I}{\sqrt{b^2 - r^2}} r \, dr = \mu_0 I \int_{0}^{a} r \, dr = \mu_0 I \int_{0}^{a} \frac{r}{\sqrt{b^2 - r^2}} \, dr
\]

We can do this with a "u" substitution \( U = b^2 - r^2 \)

\[
\Rightarrow du = -2r \, dr, \quad \Rightarrow r \, dr = -\frac{1}{2} du
\]

\[
\Rightarrow \Phi = \mu_0 I \int_{b^2}^{b^2 - a^2} \frac{1}{2} du = \mu_0 I \int_{b^2}^{b^2 - a^2} \frac{-1}{2} U^{-\frac{1}{2}} du
\]

\[
= -\mu_0 I \left[ U^{\frac{1}{2}} \right]_{b^2}^{b^2 - a^2} = -\mu_0 I \left( \sqrt{b^2 - a^2} - \sqrt{b^2} \right)
\]

\[
= +\mu_0 I \left( -\sqrt{b^2 - a^2} + b \right) \Rightarrow \Phi = \mu_0 I \left( b - \sqrt{b^2 - a^2} \right)
\]

Now, we get to take the derivative of this!

\[
E_{\text{ind}} = -\frac{d\Phi}{dt}, \text{ the easiest way to do this is to use the Chain Rule.}
\]
\( E_{\text{ind}} = -\frac{d\Phi}{dt} = -\frac{d\Phi}{db} \cdot \frac{db}{dt} = -\frac{d\Phi}{db} \cdot V \)

\[
\Phi = \mu_0 I \left(b - \frac{\beta^2 a^2}{b^2 - a^2}\right) \Rightarrow \frac{d\Phi}{db} = \mu_0 I \left(1 - \frac{1}{2} \left(\frac{b^2 - a^2}{b^2 - a^2}\right)^2 b\right)
\]

\[
= \mu_0 I \left(1 - \frac{b}{\beta^2 a^2}\right)
\]

\[
\therefore \quad E_{\text{ind}} = -\mu_0 I V \left(1 - \frac{b}{\beta^2 a^2}\right)
\]

b.) Checking for Following Cases:

i) Not moving \( \Rightarrow \) No change in flux \( \Rightarrow \) No induction

Putting \( V = 0 \) into the eqn for \( E_{\text{ind}} \) does indeed make \( E_{\text{ind}} = 0 \).

ii) \( a \rightarrow 0 \Rightarrow \) No Area for \( B_0 \) to go through \( \Rightarrow \) No flux

\( \Rightarrow \) No induction.

Notice: \( \left(1 - \frac{b}{\beta^2 a^2}\right) \) becomes \( 1 - \frac{b}{\beta^2 a^2} \) when \( a = 0 \Rightarrow \left|1 - 1 = 0 \Rightarrow \quad E_{\text{ind}} = 0\right|

iii) \( b \rightarrow \infty \) as \( b \rightarrow \infty \), \( B_0 \rightarrow 0 \Rightarrow \) No More Flux \( \Rightarrow E_{\text{ind}} = 0 \)

\[
\lim_{b \to \infty} \left(1 - \frac{b}{\beta^2 a^2}\right) = \lim_{b \to \infty} \left|1 - \frac{b}{\beta^2 a^2}\right| = 0
\]

too small to notice
\[ \mathbf{B} = \mathbf{B}_0 = \frac{\mu_0 I}{\pi r^2}, \circ \] Since we only have to integrate around the outside of the circle \( x = b + a \cos \theta \)

\[ \mathbf{V} \text{ to right } \Rightarrow \mathbf{B}_0 \text{ and } \mathbf{V} \text{ are } 90^\circ \text{ to each other} \]

\[ \Rightarrow |\mathbf{V} \times \mathbf{B}_0| = |\mathbf{V} \mathbf{B}_0 \sin 90^\circ = V B_0 = \frac{V \mu_0 I}{\pi r (b \cos \theta)} \]

\[ \mathbf{V} \times \mathbf{B}_0 \]

\[ \mathbf{R} + \mathbf{L} \Rightarrow \mathbf{V} \times \mathbf{B}_0 \text{ points straight up at all points on circle.} \]

We have to integrate around the circle clockwise (curling fingers clockwise makes thumb point into page \( \Rightarrow \) in same direction as \( \mathbf{B}_0 \))

\[ (\mathbf{V} \times \mathbf{B}_0) \cdot d\mathbf{r} = |\mathbf{V} \times \mathbf{B}_0| |d\mathbf{r}| \cos \phi \]

\[ \phi = 90^\circ - \theta + 90^\circ = 180^\circ - \theta \]

\[ \cos(180^\circ - \theta) = -\cos \theta \]

\[ \Rightarrow (\mathbf{V} \times \mathbf{B}_0) \cdot d\mathbf{r} = |\mathbf{V} \times \mathbf{B}_0| |d\mathbf{r}| \cos \theta \]
\[ |\mathbf{V} \times \mathbf{B}_0| = \frac{\sqrt{\mu_0 I}}{2\pi} \frac{1}{b + a \cos \theta} \quad dl = a d\theta \]

\( \Rightarrow \text{Integrate over full circle } \theta \Rightarrow 0 \leq \theta \leq 2\pi \)

\[ E_{in} = \int_0^{2\pi} -\frac{\mu_0 I V}{2\pi} \frac{a \cos \theta d\theta}{b + a \cos \theta} = -\mu_0 I V \int_0^{2\pi} \frac{a \cos \theta d\theta}{b + a \cos \theta} \]

From Hint:\[ \int_0^{2\pi} \frac{a \cos \theta d\theta}{b + a \cos \theta} = 2\pi \left( -\frac{b}{16\pi^2 \omega^2} + 1 \right) \]

\[ \Rightarrow E_{in} = -\frac{\mu_0 I V}{2\pi} 2\pi \left( 1 - \frac{b}{16\pi^2 \omega^2} \right) = -\mu_0 I V \left( 1 - \frac{b}{16\pi^2 \omega^2} \right) \]

\[ \uparrow \quad \text{Exactly the same!} \]
#6

\[ 0.65 \text{ m} = 0.65 \text{ m} \]

\[ R_{\text{circuit}} = 5 \Omega \]

\[ B = 0.25 \text{T} \]

What speed to generate \( I_{\text{no}} = 0.3 \text{ A} \)?

\[ \text{Motional emf } \quad E_{\text{ind}} = VBL \quad \text{since } B \text{ uniform, and } \vec{V} \text{ is } 90^\circ \text{ to } \vec{B}. \]

\[ V = ? \quad B = 0.25 \text{T} \quad L = 0.65 \text{ m} \quad \text{use Ohm's law to find } E_{\text{ind}} \]

\[ E_{\text{ind}} = I_{\text{no}} R = (0.3 \text{ A})(5 \Omega) = 1.5 \text{ V} \]

\[ \therefore V = \frac{E_{\text{ind}}}{BL} = \frac{1.5 \text{ V}}{(0.25 \text{T})(0.65 \text{ m})} = \frac{9.23 \text{ m/s}}{} \]

b) What direction is \( I_{\text{ind}} \)? Decreasing area \( \Rightarrow \) decreasing flux \( \Rightarrow \) \( B_{\text{ind}} \) also \( \Rightarrow \) counterclockwise.

b) As rod slides to left the amount of railing is decreasing

\[ \frac{\text{less railing}}{\rightarrow \text{ less wire}} \rightarrow \text{ for current to flow through} \]

\[ \text{decreasing resistance } \& E_{\text{ind}} = I_{\text{no}} R \Rightarrow E_{\text{ind}} \text{ must get smaller too to keep } I_{\text{no}} \text{ constant, } E_{\text{ind}} = VBL \Rightarrow \text{go slower}. \]
a) Estimate Amount of Displacement Current.

Displacement Current with $V_{KM} \Rightarrow i_d = \varepsilon_0 \frac{d\Phi_e}{dt}$

As usual, we'll assume that the electric field is **uniform at all locations in the wire.** (It's obviously not uniform in time, or we would get no displacement current.)

\[ \vec{A} \rightarrow \Phi_e = EA, \text{ Constant Cross-Sectional Area} \]

\[ \Rightarrow i_d = \varepsilon_0 A \frac{d\varepsilon}{dt} \]

Current Review: In a wire, $E = \int J$ where $J = \frac{I}{A}$

\[ \Rightarrow E = \int \frac{I}{A} \text{ both } \varepsilon \text{ and } A \text{ are constant} \Rightarrow \frac{d\varepsilon}{dt} = \frac{\varepsilon}{A} \frac{dI}{dt} \]

\[ \therefore i_d = \varepsilon_0 A \left( \frac{\varepsilon}{A} \right) \frac{dI}{dt} = \varepsilon_0 \int \frac{dI}{dt} \]

Estimate \( \Rightarrow \frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} \Rightarrow \Delta I = 0 - I_0 = -I_0 = -1.5A \)

\[ \Delta t = 0.6 \text{ ns} = 0.6 \times 10^{-9} \]
To find amount of displacement current just take absolute value: 

\[ |i_d| = \frac{\partial}{} \left| \frac{\Delta E}{\partial t} \right| \]

\[ |i_d| = \frac{(8.85 \times 10^{-12} \text{C}^2)}{\text{N.m}^2} \left(2 \times 10^{-8} \text{m} \right) (1.5 \text{A}) \frac{0.6 \times 10^{-8}}{0.6 \times 10^{-8}} = 4.425 \times 10^{-10} \text{A} \]

Unit: \( \frac{\text{C}^2}{\text{N.m}^2} \cdot \frac{\text{m}}{\text{s}} \cdot \text{A} = \text{A} \)

\[ \frac{}{\text{N.m}^2} \cdot \frac{\text{m}}{\text{s}} \]

These should cancel \( \Rightarrow \) \( \Delta = \frac{\chi}{\varepsilon_0} = \frac{\mu}{\varepsilon_0} = \frac{\varepsilon_0}{c^2} \)

\[ \frac{}{\text{N.m}^2} \cdot \frac{\text{J}}{\text{C}^2} \cdot \text{m}^2 = \frac{\text{N.m} \cdot \text{m}}{\text{N.m}^2} = 1 \text{, they do} \]

b) \( 4.425 \times 10^{-10} \text{A} \) is very small compared to \( 1.5 \text{A} \) of conduction current that we start with, so the field it creates will be negligible. By the time the conduction current gets this small, the whole process is basically over!
#8 The Sun!

Density, \( \rho = 3 \times 10^{-6} \text{kg/m}^3 \)

Mass of CME, \( M = 1 \times 10^{15} \text{kg} \)

Needs \( 1 \times 10^5 \text{m/s} \) speed to escape Sun's gravity

So the CME material needs to be given a kinetic energy of

\[ K = \frac{1}{2}mv^2 = \frac{1}{2} (1 \times 10^{15} \text{kg})(1 \times 10^5 \text{m/s})^2 = 1.8 \times 10^{39} \text{J} \]

Assume this energy comes from the magnetic field energy. We have

\[ u = \frac{B^2}{2 \mu_0} \quad \Rightarrow \quad B = \sqrt{\frac{2u \mu_0}{B^2}} \]

Potential energy

\[ u = \frac{U}{\text{Vol}} \quad \Rightarrow \quad 100\% \text{ conversion} \quad \Rightarrow \quad u = \frac{K}{m/\text{density}} = \frac{K}{M/\text{density}} \]

\[ = \frac{1.8 \times 10^{39} \text{J}}{1 \times 10^{15} \text{kg}} \]

\[ = 5.4 \times 10^7 \text{J/m}^3 \]

\[ B = \sqrt{\frac{2(4 \pi \times 10^{-7} \text{Tm}\text{A}^{-1}) (5 \times 10^7 \text{J/m}^3)}{1.8 \times 10^{39} \text{J}}} \]

\[ = 11.6 \text{T} \]

A little too big to be realistic but a few calculations...