\[ T = \frac{I_t}{I_i} \frac{\cos \theta_t}{\cos \theta_i} \bigg|_{\theta_i = \theta_t = 0} = \frac{I_t}{I_i} \]

\[ I_t = T \cdot I_i \]

But \[ T = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} \bigg|_{\theta_i = \theta_t = 0} = \frac{n_t}{n_i} \left( \frac{2n_i^2}{n_i + n_t} \right) \]

So \[ I_t = I_i \frac{4n_t}{(n_i+n_t)^2} = 400 \text{ W/m}^2 \cdot \frac{4(1.33)(1.376)}{(1.33 + 1.376)^2} \]

Thus \[ I_t = 399.9 \text{ W/m}^2 \]

4.57 The fish can only see objects above the surface of the water in a cone defined by the critical angle.

\[ \theta_c = \arcsin \left( \frac{1}{n_{\text{water}}} \right) = \arcsin \left( \frac{1}{1.33} \right) \]

\[ \theta_c = 48.6^\circ \]

The cone is surrounded by darkness if there is no light coming from under water. Otherwise the fish would see outside the cone and it would be light from underwater objects reflected at the water-air interface.
\[ (-r_1) = \frac{\sin \theta_i - \theta_i}{\sin \theta_i + \theta_i} \] and expand

\[ (-r_1) = \frac{\sin \theta_i \cos \theta_t - \cos \theta_i \sin \theta_t}{\sin \theta_i \cos \theta_t + \cos \theta_i \sin \theta_t} \]

Use Snell's law \( \sin \theta_t = \frac{\sin \theta_i}{n} \) and small angle expansion:

\[ \sin \theta_i = \theta_i - \theta_i^3/6 = \theta_i (1 - \theta_i^2/6) \]
\[ \cos \theta_i = 1 - \theta_i^2/2 \]

Also \( \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = \sqrt{1 - \frac{\sin^2 \theta_i}{n^2}} = \sqrt{1 - \frac{\theta_i^2(1 - \theta_i^2/6)^2}{n^2}} \approx 1 - \frac{\theta_i^2}{2n^2} \) \( \text{(to 1st order in } \theta_i^2) \)

Keep only 1st order and let \( \Theta_i = \Theta \)

\[ (-r_1) = \frac{\Theta (1 - \Theta^2/6) (1 - \frac{\Theta^2}{2n^2}) - (1 - \frac{\Theta^2}{2}) \Theta (1 - \Theta^2/6)}{\Theta (1 - \frac{\Theta^2}{2n^2}) + (1 - \frac{\Theta^2}{2}) \Theta (1 - \Theta^2/6)} \]

\[ = \frac{(1 - \frac{1}{n}) - \Theta^2 (\frac{1}{6} + \frac{1}{2n^2} - \frac{2}{3n})}{(1 + \frac{1}{n}) - \Theta^2 (\frac{1}{6} + \frac{1}{2n^2} + \frac{2}{3n})} \]

\[ \approx \frac{(n-1) - n \Theta^2 (\frac{1}{6} + \frac{1}{2n^2} - \frac{2}{3n})}{(n+1) - n \Theta^2 (\frac{1}{6} + \frac{1}{2n^2} + \frac{2}{3n})} \]

Now factor \((n+1)\) out of the denominator and expand \([1 + e]^{-1}\) keeping only first order \((e \ll \theta^2)\)

\[ (-r_1) = \left( \frac{1}{n+1} \right) [(n-1) - n \Theta^2 (\frac{1}{6} + \frac{1}{2n^2} - \frac{2}{3n})] [1 - \frac{n}{n+1} \Theta^2 (\frac{1}{6} + \frac{1}{2n^2} + \frac{2}{3n})] \]

Multiply it out keeping only 1st order in \( \theta^2 \)
\[ (-r_\parallel) = \left( \frac{1}{n+1} \right) \left[ (n-1) - n \Theta^2 \left[ \frac{1}{6} + \frac{1}{2n^2} - \frac{2}{3n} - \left( \frac{n-1}{n+1} \right) \left( \frac{1}{6} + \frac{1}{2n^2} + \frac{2}{3n} \right) \right] \right] \]

\[ = \left( \frac{1}{n+1} \right) \left[ (n-1) - n \Theta^2 \left( -1 + \frac{1}{n^2} \right) \right] \]

\[ = \left( \frac{1}{n+1} \right) \left[ (n-1) + \frac{\Theta^2}{n} (n-1) \right] \]

Note: Many steps were left out in these expansions, but it does work.

(4.47) We know that

\[ r_\parallel = \frac{\tan (\Theta_i - \Theta_t)}{\tan (\Theta_i + \Theta_t)} = \frac{\sin (\Theta_i - \Theta_t)}{\sin (\Theta_i + \Theta_t)} \cdot \frac{\cos (\Theta_i + \Theta_t)}{\cos (\Theta_i - \Theta_t)} \]

\[ = (-r_\perp) \cdot \frac{\cos (\Theta_i + \Theta_t)}{\cos (\Theta_i - \Theta_t)} = (-r_\perp) B \]

Expand B,

\[ B = \frac{\cos \Theta_i \cos \Theta_t - \sin \Theta_i \sin \Theta_t}{\cos \Theta_i \cos \Theta_t + \sin \Theta_i \sin \Theta_t} \]

Using the same expansion techniques as we found in (4.46), and again keeping only terms to 1st order in \( \Theta_i = \Theta \), we get

\[ B = 1 - \frac{2\Theta^2}{n} \]

Now use results of (4.46) for \(-r_\perp\)

\[ r_\parallel = \left( \frac{n-1}{n+1} \right) \left( 1 - \frac{\Theta^2}{n} \right) \left( 1 - \frac{2\Theta^2}{n} \right) \]

or

\[ r_\parallel = \left( \frac{n-1}{n+1} \right) \left( 1 - \frac{\Theta^2}{n} \right) \]

to 1st order in \( \Theta^2 \)
\[ \theta_c = \sin^{-1} \frac{1.33}{1.55} \]

\[ \theta_c = 59.0^\circ \]

4.63 From Eq 4.43:

\[ r_{II} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \]

When \( \theta_i = \theta_p = \frac{\pi}{2} - \theta_t \Rightarrow \theta_i + \theta_t = \frac{\pi}{2} \)

So \( r_{II} = \lim_{\theta_i + \theta_t \to \frac{\pi}{2}} \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = \frac{\text{finite}}{\infty} = 0 \)

But \( \tan(\theta_i - \theta_t) \to \tan(\frac{\pi}{2} - 2\theta_t) \) which does not blow up for \( \theta_i < \frac{\pi}{2} \).

Also \( r_\perp = \lim_{\theta_i + \theta_t \to \frac{\pi}{2}} \left[ \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \right] \)

\[ = \lim_{\theta_i + \theta_t \to \frac{\pi}{2}} \left[ \frac{\sin(\theta_i + \theta_t) + \sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \right] \]

\[ = \lim_{\theta_i + \theta_t \to \frac{\pi}{2}} \sin(\theta_i - \theta_t). \text{ This is not zero, so the parallel component is not reflected while the perpendicular component is reflected 100\% polarized light which is 1 to incident plane.} \]