Compton Scattering

\[ p_\parallel = p_1 \cos \theta + p \cos \varphi \]

and

\[ p_\parallel \sin \theta = p \sin \varphi \]

Squaring these equations, we obtain

\[ (p_\parallel - p_1 \cos \theta)^2 = p^2 \cos^2 \varphi \]

and

\[ p_\parallel^2 \sin^2 \theta = p^2 \sin^2 \varphi \]

Figure 2-7 Compton's interpretation. A photon of wavelength \( \lambda \) is incident on a free electron at rest. On collision, the photon is scattered at an angle \( \theta \) with increased wavelength \( \lambda' \), while the electron moves off at angle \( \varphi \).

Adding, we find

\[ p_\parallel^2 + p_1^2 - 2p_0p_1 \cos \theta = p^2 \quad (2-9) \]

Conservation of total relativistic energy requires

\[ E_0 + m_0c^2 = E_1 + K + m_0c^2 \]

Thus

\[ E_0 - E_1 = K \]

According to (2-7), this is

\[ c(p_0 - p_1) = K \quad (2-10) \]

Writing \( K + m_0c^2 \) for \( E \) in (2-6), that equation becomes

\[ (K + m_0c^2)^2 = c^2 p^2 + (m_0c^2)^2 \]

which simplifies to

\[ K^2 + 2Km_0c^2 = c^2 p^2 \]

or

\[ K^2/c^2 + 2Km_0 = p^2 \]

Evaluating \( p^2 \) from (2-9) and \( K \) from (2-10), we have

\[ (p_0 - p_1)^2 + 2m_0c(p_0 - p_1) = p_\parallel^2 + p_1^2 - 2p_0p_1 \cos \theta \]

which reduces to

\[ m_0c(p_0 - p_1) = p_0p_1(1 - \cos \theta) \]

or

\[ \frac{1}{p_1} - 1 = \frac{1}{m_0c} (1 - \cos \theta) \]

Multiplying through by \( h \), and applying (2-8), we obtain the Compton equation

\[ \Delta \lambda = \lambda_1 - \lambda_0 = \lambda_C(1 - \cos \theta) \quad (2-11) \]

where

\[ \lambda_C \equiv h/m_0c = 2.43 \times 10^{-12} \text{ m} = 0.0243 \text{ Å} \quad (2-12) \]

is the so-called Compton wavelength.