

## Preliminary Examination: Thermodynamics and Statistical Mechanics

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### **Instructions:**

- No notes or cheat sheets of any kind are allowed.
- Attempt all 10 questions (for 10 points each).
- Where possible, show all work; partial credit will be given if merited.
- Total time for the test: 3 hours.

### **Information that may or may not be useful in this exam:**

- 1 joule =  $6.2 \times 10^{18}$  eV.
- $k_B T$  at room temperature is approximately equal to 0.025 eV.
- Equation of state of an ideal gas is  $PV = nRT$  in usual notation.
- $$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}.$$

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**P1:** A cubic box, each side being 1 meter long, isolated from the rest of the universe, is divided into two equal parts by the means of a partition. Initially, one half of the box contains 4 moles of an ideal gas at room temperature, while the other half is empty. Suppose that the partition is suddenly lifted. After a sufficiently long time the gas will occupy the entire box. Find the final temperature in degrees Celsius, and the change in entropy (in appropriate units) for this process. Assume any information that you feel you need but have not been given.

**P2:** The internal energy of an ideal diatomic gas is  $(5/2)nRT$  where  $T$  is the absolute temperature,  $R$  is the universal gas constant, and  $n$  is the number of moles. Calculate the specific heat capacity for this gas: at constant volume on the one hand and at constant pressure on the other .

**P3:** Consider electrons in a metal at room temperature as non-interacting fermions. (a) Write an expression for their distribution function as a function of energy, explaining the symbols you use. Sketch the distribution function for zero, intermediate and high temperature (define appropriately the words intermediate and high in this context). (b) Calculate the number density of the electrons at *room temperature* in terms of its Fermi energy by using *zero temperature* analysis. Justify your use of the zero temperature analysis for room temperature calculations.

**P4:** The vibrational motion of a solid containing  $N$  atoms is sometimes modeled as that of a collection of  $3N$  independent harmonic oscillators, each having the same natural frequency  $\omega_0$ . Obtain an expression for the heat capacity of such a system and graph it versus temperature for small and large temperatures. Specify what you mean by small and large. What physical properties of the solid are NOT explained by this model?

**P5:** The underlying equations of mechanics are reversible in time. Yet every day behavior we observe is irreversible. Briefly explain how you understand the resolution of this paradox.

**P6:** The heat capacity of certain systems tends to zero at very small temperatures. Explain carefully what physical features of the systems would lead to such behavior and what ‘very small’ temperatures could mean. Now consider, instead, certain other systems whose heat capacity tends to zero at very large temperatures. Explain what physical features would result in this behavior and explain ‘very large’. Is it possible for the same system to exhibit both behaviors? Explain your reasoning.

**P7:** According to the Debye model for the heat capacity of an insulating solid, the vibrational internal energy is proportional to

$$\int_0^{\omega_0} d\omega \frac{\omega^3}{e^{\beta \hbar \omega} - 1}$$

where the proportionality constant depends on the volume of the solid and the speed of sound in the solid,  $\beta$  being  $1/k_B T$ . State what the dependence on the volume is. Show that the vibrational contribution to the heat capacity is proportional to the cube of the temperature  $T$  at low temperature and explain what ‘low’ means here.

**P8:** Consider a system of  $N$  distinguishable, noninteracting, freely orientable (in 3-dimensions), classical magnetic dipoles, each of magnetic moment  $\mu$ , in a constant magnetic field of strength  $B$  pointing in a fixed direction in space. (a) Write down the partition function for this system. (b) Calculate the magnetic moment for this system at a temperature  $T$  and sketch how it varies with respect to  $T$  and  $B$ .

**P9:** An ideal classical gas of  $N$  particles fills a cubic vessel of volume  $V$  in equilibrium at temperature  $T$ . If the particles each have a mass  $m$ , at what rate do the particles collide with the walls of the container?

**P10:** State the Fermi-Dirac distribution, the Bose-Einstein distribution and the Maxwell-Boltzmann distribution describing the occupation number of a fixed number  $N$  of particles each with a given energy  $\epsilon$  in terms of the chemical potential  $\mu$  and the temperature  $T$ . The crucial ratio in these expressions is  $(\epsilon - \mu)/k_B T$ . Doesn’t this ratio become large as the temperature becomes very small? Shouldn’t therefore the Fermi-Dirac distribution tend to the classical distribution for *small*  $T$  rather than for *large*  $T$  as is always claimed? Explain what is wrong with the reasoning.