Preliminary Examination: Thermodynamics and Statistical Mechanics

Department of Physics and Astronomy
University of New Mexico

Spring 2013

Instructions:
•The exam consists of 10 short-answer problems (10 points each).
•Where possible, show all work; partial credit will be given if merited.
•Personal notes on two sides of an 8×11 page are allowed.
•Total time: 3 hours.

P1: Consider a collection of a large non-interacting number N of a certain kind of anharmonic oscillators (CAO) whose energy spectrum is similar to that of the harmonic oscillator but starts out with a gap near the ground state. Specifically, the energy E_n of the CAO equals 0 for $n = 0$ and $\Delta + (n-1)\epsilon$ for $n > 0$, where n is an integer. Calculate the partition function and compare with that of the harmonic oscillator. With or without calculation, sketch the temperature dependence of the specific heat of the CAO system showing differences, if any, from that for the harmonic oscillator. Comment on three cases: $\epsilon/\Delta = 1, >> 1, << 1$.

P2. Einstein's theory of the specific heat of an insulating solid assumes the solid to be composed of noninteracting harmonic oscillators. What observed feature of the temperature dependence of the specific heat is incompatible with Einstein's prediction? How would you extend Einstein's theory to address this incompatibility? How would your extension show (present clear analytic arguments with expressions) that the temperature dependence of the specific heat depends on the dimensionality d of the solid (d=1,2,3)?

P3: What are micro-canonical, canonical and grand canonical ensembles and how would you choose which of them to use, in a given situation, for calculations in equilibrium statistical mechanics? Explain if there are any conditions under which they give the same results for thermodynamic quantities and explain why.

P4: Consider an ideal 3-dimensional classical gas of a large number N of free noninteracting particles in equilibrium at temperature T. Each particle has energy proportional to the magnitude of its momentum. Find the free energy of the gas and comment on the difference between its temperature dependence and that of its normal counterpart in which the particle energy is proportional to the square of the momentum.

P5. A random walker has its probability density $P(x, y, z, t)$ of being at position (x,y,z) at time t governed by the equation

$$\frac{\partial P(x, y, z, t)}{\partial t} = D\nabla^2 P(x, y, z, t)$$

where D is a constant that describes how fast the walker walks. Place the walker at the origin at the initial time. Multiply the probability density by the square of the distance of the walker from the origin and integrate over all space. Call the square root of the value of the integral the average distance of the walker from the origin at time t. By making some reasonable assumptions about the behavior of P at infinite distances, show that the average distance varies as $K\sqrt{t}$ where K is a constant. What is the precise dependence of K on D? How would K differ if the random walker were to move on a flat surface instead of in 3-dimensional space?

P6. Explain the origin of the Boltzmann factor $\exp(-E/k_B T)$ where E is the energy and T the temperature. Is it a law of nature, a postulate of mathematics, a mysterious religious belief or something else?

P7. Recall that the classical partition function of a single particle in a box of volume V is $z = V/\lambda^3$ with λ as the thermal de Broglie wavelength of the particle. Surely then, the partition function of two identical counterparts of the particle noninteracting with each other should be z^2 and that of N noninteracting particles all together in a system should be z^N. Show that this argument leads to the violation of the expectation that thermodynamic quantities such as the free energy of the gas are extensive.

P8. Explain a "correction" argument based on the supposed indistinguishability of the N particles that might be used to fix the problem discussed in P7 above. Comment on whether, and why, you find the argument acceptable or unacceptable. If you do not find it acceptable, explain how you reconcile yourself to this unsatisfactory situation in statistical mechanics.

P9. Calculate the dependence (at zero temperature) of the energy of a d-dimensional gas of noninteracting free fermions on the density of the gas for d=1,2,3.

P10. Sketch without calculation the following. Show as much detail indicating characteristic values of relevant quantities as you can.

a. The Maxwell-Boltzmann distribution as a function of velocities.

b. The energy density of states of a free particle in a box as a function of the energy.

c. The allowed frequencies in the Debye theory of the specific heat of an insulating solid as a function of the wavenumber.