Preliminary Examination: Thermodynamics and Statistical Mechanics

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Instructions:

•The exam consists of 10 short-answer problems (10 points each).
•Where possible, show all work; partial credit will be given if merited.
•Personal notes on two sides of an 8×11 page are allowed.
•Total time: 3 hours.

P1: The application of statistical mechanics results in an expression for the average magnetic moment of an atom in the direction of a uniform applied magnetic field of strength B in certain common physical systems at temperature T, which is of the form

$$\mu = const \cdot f(B,T).$$

where the limit of $f(B,T)$ as T tends to 0 or B tends to ∞ is 1. What is the physical meaning of the factor *const*? Give the explicit form of the function $f(B,T)$ for two different physical systems (indicate which is which and explain symbols) and point out what the expected qualitative behavior of $f(B,T)$ is, in both examples you give, as T and B are varied.

P2: Calculate the energy density of states for free noninteracting quantum particles in 2 dimensions and answer from your expression whether it increases, decreases or remains constant as the energy varies.

P3: The Hamiltonian for an extreme classical anharmonic oscillator in 1-dimension is given by

$$H = \frac{p^2}{2m} + \frac{1}{10}kx^{10}.$$

The oscillator is in contact with a heat reservoir at temperature T. The equipartition law states that the average kinetic energy of a particle in thermal equilibrium is $k_B T/2$ per degree of freedom. Using that law together with the virial theorem,

$$\langle p\frac{\partial H}{\partial p}\rangle = \langle x\frac{\partial H}{\partial x}\rangle,$$

determine the average energy of the oscillator.

P4: How is the partition function of a system in equilibrium related to the energy state density of the system? How can you calculate the free energy of the system knowing the partition function? A simplified model of a gas at volume V and temperature T results in the following expression for the free energy A :

$$A + Nk_B T \ln(V - b) + \frac{a}{V} = 0$$

Work out an expression for the pressure exerted by the gas as a function of N, T, V, and comment on what physical meaning N, a, b might have in this model.

P5: Consider a *charmonic* oscillator whose energy levels are given by $E_n = n\epsilon$ where n is a non-negative integer smaller or equal to a finite number N. Calculate the specific heat of a collection of noninteracting *charmonic* oscillators and specify clearly the difference relative to a collection of *harmonic* oscillators , each with the same energy spacing ϵ but with a nonvanishing zero-point energy and infinite N. Make plots of the two specific heats and indicate on the plot the effects of the difference in N and in the zero-point energy.

P6. Consider a gas of N atoms which can be considered to be non-interacting bosons the energy spectrum of each comprising only of two states of energies E_1 and E_2. The gas is in thermal equilibrium at temperature T. Calculate the average energy as a function of N, T, E_1 and E_2 and sketch as a function of T.

P7. Briefly explain in *any three* of the following cases the term and its relevance:
(i) Gibbs mixing paradox, (ii) self-avoiding random walker, (iii) Poincare recurrence, (iv) ergodic hypothesis, (vi) Fermi energy.

P8. A system of N identical noninteracting particles is in equilibrium at temperature T. The single-particle energy, i.e., the energy that any of the particles can possess, varies from 0 to ∞. Sketch the dependence of the average number of particles in a (single-particle) state of a given energy versus that energy for the three cases when the statistics are: (i) Maxwell-Boltzmann, (ii) Fermi-Dirac, and (iii) Bose-Einstein. Explain under what physical conditions any of these may be approximated by any other(s).

P9. Explain the origin of the Boltzmann factor $\exp(-E/k_BT)$ where E is the energy and T the temperature? Is it a law of nature, a postulate of mathematics, a mysterious religious belief or something else?

P10. The specific heat of a metal has an important part that varies strongly as T^n. So does that of an insulating solid at high T and also at low T. The values of n are different in these three cases: 0, 1, and 3, NOT necessarily in that order. Explain what physical characteristics of the systems lead to these three power law dependences and why n has the particular value for each system. Justify your answer on the basis of analytic expressions and clear arguments.