Department of Physics and Astronomy, University of New Mexico

Thermodynamics and Statistical Mechanics Preliminary Examination

Spring 2011

Instructions:

- The exam consists of 10 short-answer problems (10 points each).
- Where possible, show all work; partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed.
- Total time: 3 hours.

Unless otherwise noted, commonly used symbols are defined as follows:

P: Pressure

T: Absolute Temperature

k: Boltzmann's constant

R: Gas constant

$\beta = (kT)^{-1}$ (k is Boltzmann constant)

S: Entropy

V: Volume

U: Internal energy

μ: Chemical potential

$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}$ Js; Planck's constant

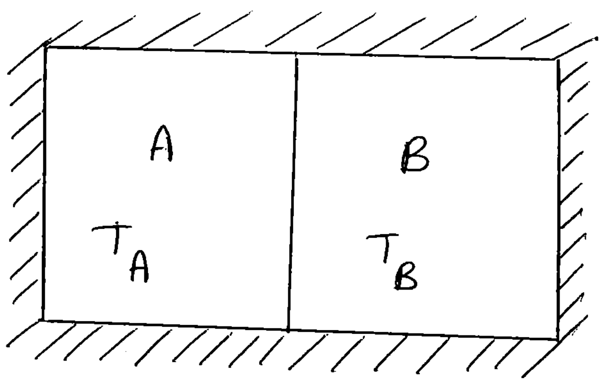
$\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}; Stefan-Boltzmann constant

1– An ideal diatomic gas occupies a volume of 2 m^3 at a pressure of 5 atm (1 atm $\approx 10^5$ Pa) and a temperature of 293 K. The gas is compressed reversibly and adiabatically to a final pressure of 10 atm.

Compute the final volume, the work done on the gas, the heat released to the surrounding, and the change in the internal energy.

2– Two objects A and B, mechanically and thermally isolated form the surrounding, have initial temperatures $T_A > T_B$. Each object has heat capacity C_V, the same for both objects, which is independent of temperature.

The objects are brought into contact until equilibrium is reached. Assuming the volumes of A and B remain constant, find the final temperatures of the objects. How much has the entropy of the system $A + B$ changed?



3– The Helmholtz free energy of N molecules of a certain gas at temperature T and volume V is given by

$$F = -NkT\,\ln(V - b) - \frac{a}{V}, \tag{1}$$

where $a < 0$ and $b > 0$ are constants.

Find the equation of state of the gas. What is the minimum volume for this gas? Give a physical interpretation for your answer.

4– The conduction electrons in a monomolecular sheet of graphene can be modeled, to first approximation, as a system of N noninteracting fermions freely moving in a two-dimensional space having area $A = L^2$. In the limit of zero temperature, obtain an expression for the chemical potential as a function of the electron density $\sigma = N/A$.

5– The temperature T_s at the surface of a star can be obtained by fitting the spectrum to a blackbody curve. For $T_s \simeq 6000$ K and a luminosity $L \cong 4 \times 10^{26}$ W, estimate the radius of the star.

6– A particle with mass m is constrained to move freely on the surface of a sphere with radius R, as shown in the figure. Its motion is described by the Hamiltonian

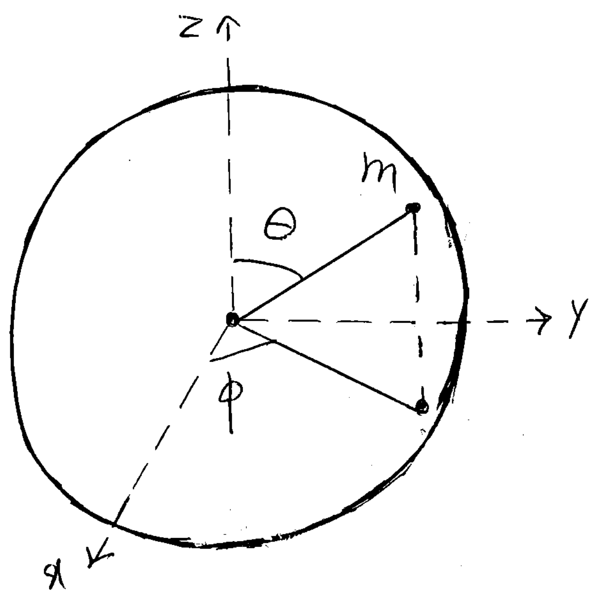
$$H = \frac{P_\theta^2}{2mR^2} + \frac{P_\phi^2}{2mR^2 \sin^2\theta},$$

where θ and ϕ are the azimuthal and polar angles, and P_θ and P_ϕ are their conjugate momenta, respectively.

Calculate the canonical partition function for the single particle at a temperature T. Using this partition function, find the average energy and show that it has the value you expect. Assume that m is large enough when integrating over phase space. (Useful information: the number of states in a differential element of phase space is given by $dP_\theta dP_\phi \sin\theta d\theta d\phi / h^2$. Do the momentum integrals first. You may use the following expression for a Gaussian integral

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \qquad a > 0,$$

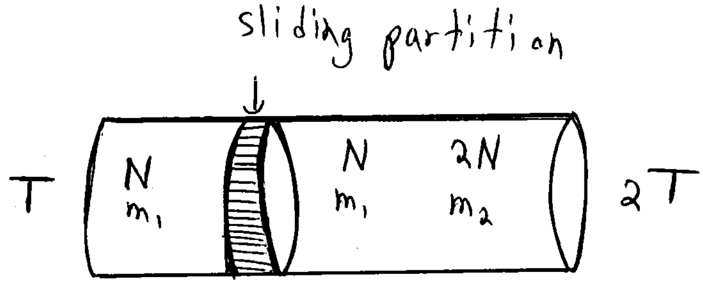
without proof.)



7– Estimate the critical temperature below which a Bose-Einstein condensate starts to form for ^{87}Rb atoms with a density $n \approx 10^{12}$ cm^{-3}. (Mass of ^{87}Rb atom is $m \approx 1.46 \times 10^{-25}$ Kg.)

8– A cylinder with total volume V is partitioned into two parts as shown in the figure. The left-hand part is filled with an ideal gas consisting of N particles of mass m_1. The right-hand part contains an ideal gas that is an admixture of N particles of mass m_1 and $2N$ particles of mass m_2. The left- and right-hand parts of the cylinder are in contact with reservoirs at temperature T and $2T$ respectively. There is no heat transfer between the two parts through the partition.

Assuming that the partition can slide freely, where will it be located when equilibrium is reached?



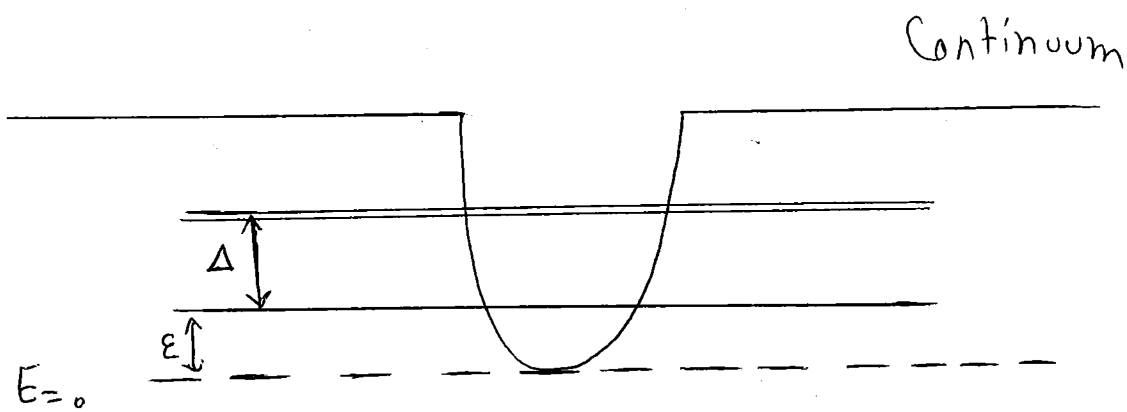
9– Consider a system of N indistinguishable and non-interacting bosons confined to a potential in three dimensions. The potential allows two bound states for a single particle: the ground state with energy ϵ and the first excited state, which is doubly degenerate, and has energy $\epsilon + \Delta$. The system is in equilibrium at a temperature T. Show that in the limit $N\beta\Delta \gg 1$ the partition function for this system is:

$$Z = \frac{e^{-\beta N \epsilon}}{(1 - e^{\beta\Delta})^2}\,.$$

(Hint: You may use the following identity

$$\sum_{n=0}^{+\infty} (n+1)a^n = \frac{1}{(1-a)^2} \qquad 0 < a < 1\,,$$

without proof.)



10– A vessel with volume V in contact with the surroundings at a temperature T is initially filled with N atoms of helium gas. After some time, it is found that N_1 of the atoms remain in the gas phase, and $N_2 = N - N_1$ atoms adhere to the surface of the vessel. The partition function for the gas phase is

$$Z_1 = \frac{1}{N_1!}\left(\frac{V}{\lambda^3}\right)^{N_1},$$

and the partition function for the surface atoms

$$Z_2 = \frac{e^{\beta N_2 \Delta}}{N_2!}\left(\frac{A}{\lambda^2}\right)^{N_2}.$$

Here $\lambda = (2\pi\hbar^2/mkT)^{1/2}$ is the thermal de Broglie wavelength for an atom having mass m, A is the surface area of the vessel and $\Delta > 0$ is the surface binding energy.

What are the chemical potentials μ_1 and μ_2 for the two subsystems, respectively? When equilibrium is reached, what will be the ratio N_2/N_1?