Preliminary Examination: Statistical Mechanics

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Instructions:

•You should attempt all 10 problems (10 points each).
•Where possible, show all work; partial credit will be given if merited.
•Personal notes on two sides of an 8×11 page are allowed.
•Total time: 3 hours.

It may help you to remember that

- if n is an integer that runs from n_1 to n_2, the geometrical sum

$$S = \sum_{n_1}^{n_2} x^n = \frac{x^{n_1} - x^{n_2+1}}{1-x};$$

- room temperature is about $0.025eV$.

P1. A system is in thermal equilibrium with a heat bath at temperature T. The ground state of the system has angular momentum 0 and linear momentum 0. A certain state ξ of the system energetically higher than the ground state by an amount E has angular momentum L units and linear momentum p units. What is the relative occupation of state ξ relative to that of the ground state? How does the principle of equal a priori probability conflict or not conflict with your answer? State in a few sentences (say 4) your reasoning, making clear how your answer may be justified.

P2. A system of N noninteracting particles is in equilibrium at temperature T. The single-particle energy, i.e., the energy that any of the particles can possess, varies from 0 to ∞. Sketch the dependence of the number of particles in a (single-particle) state of a given energy versus that energy for the three cases when the statistics are: (i) Maxwell-Boltzmann, (ii) Fermi-Dirac, and (iii) Bose-Einstein. Explain under what physical conditions any of these may be approximated by any other(s).

P3. Draw sketches to show how the pressure P of an ideal gas varies with its volume V at constant temperature T and also with T at constant V. Combining these and additional observations if necessary, write down an equation of state for the gas. Generalize the equation of state for nonideal gases in which the constituent molecules exert repulsive forces on each other. Explain your reasoning. Finally, incorporate intermolecular interactions which are attractive if the molecules are within a short enough distance with respect to each other. This final generalization is what is known as Van der Waals equation of state. Draw $P-V$ curves at several constant temperatures T corresponding to this last equation of state.

P4. Consider a system of noninteracting particles each of which has energy $E_n = n\Delta$ where n varies through integer values from 0 to a finite number N. Calculate the heat capacity of this system and SKETCH it (important!) indicating characteristic values if possible.

P5. An electron in in thermal equilibrium in a metal at room temperature. Calculate the probability that it occupies a state $0.01eV$ higher than the Fermi energy. State also whether you expect the chemical potential of the electron to vary (increase or decrease?) appreciably or remain essentially constant as the temperature is varied by (i) 10 deg. C (ii) 1000 deg. C.

P6. The magnetic moment of a system of N noninteracting classical spins subjected to a magnetic field B at temperature T increases with increasing field and saturates at large fields. Derive an expression for the moment considering the system is 1-dimensional (which means that the spins are either aligned or anti-aligned with B). What qualitative and/or quantitative differences would appear for a 2-d and 3-d counterpart of the system?

P7. Would the measurement of the heat capacity of a substance have different values if performed under constant pressure versus constant volume conditions? If your answer is yes, express quantitatively their difference in case the system is a perfect gas. Derive the expression you show.

P8. The specific heat of an insulating solid plunges to zero at low temperatures and saturates to a constant value at high temperatures. Does the dependence of the specific heat on temperature at either of these ends follow an approximate power law? If so, does the exponent in the power law depend on any universal characteristic or on the specifics of the solid? Justify your answer on the basis of an analytic expression.

P9. The mixing paradox of Gibbs appears to represent a serious failure of statistical mechanics. Briefly explain the paradox and mention how it may be 'resolved'. Comment on the 'resolution' and express your opinion on whether this has anything to do with Quantum Mechanics.

P10. The Gibbs free energy of N molecules of a given gas at pressure P and temperature T is $NkTlnP + f(P)$ where $f(P) = a + bP + cP^2 + dP^3$, a, b, c and d being constants. Determine the equation of state of the gas.