

**Preliminary Examination: Statistical Mechanics**

*Department of Physics and Astronomy*

*University of New Mexico*

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**Instructions:**

- You should attempt all 10 problems (10 points each).
  - Where possible, show all work; partial credit will be given if merited.
  - Personal notes on two sides of an 8×11 page are allowed.
  - Total time: 3 hours.
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**P1.** State the partition function of a quantum mechanical gas of  $N$  noninteracting free distinguishable particles in a cube of length  $L$  at temperature  $T$ . How is it related to a ratio of  $L$  to the thermal deBroglie wavelength of each particle? Comment on the significance of the largeness or smallness of that ratio for the corresponding system of indistinguishable particles.

**P2.** Comment on the relation, if any, between the Pauli exclusion principle on the one hand and the Fermi-Dirac and Bose-Einstein distribution functions on the other. Give two examples each of particles that obey these two distribution functions.

**P3.** Consider the average magnetization  $M$  of a large number  $N$  of non-interacting spins each of magnetic moment  $\mu$  at temperature  $T$  and subjected to a magnetic field  $B$ . Present two sketches and label any important quantities on the sketches:  $M$  as a function of  $T$  at fixed  $B$ , and  $M$  as a function of  $B$  at fixed  $T$ . Now let  $B = 0$  and indicate what change you expect in the  $M - T$  plot if the spins interact among themselves as to align cooperatively.

**P4.** Draw sketches to show how the pressure  $P$  of an ideal gas varies with its volume  $V$  at constant temperature  $T$  and also with  $T$  at constant  $V$ . Combining these and additional observations if necessary, write down an equation of state for the gas. Generalize the equation of state for nonideal gases in which the constituent molecules exert repulsive forces on each other. Explain your reasoning. Finally, incorporate intermolecular interactions which are attractive if the molecules are within a short enough distance with respect to each other. This final generalization is what is known as Van der Waals equation of state. Draw  $P - V$  curves at several constant temperatures  $T$  corresponding to this last equation of state.

**P5.** By writing the energy of a 1-d harmonic oscillator of frequency  $\omega$  as  $E_n = (n+1/2)\hbar\omega$ , and performing a geometric sum (show your work explicitly), show that the partition function is given by

$$Z = \sum_n e^{-\beta(n+1/2)\hbar\omega} = \frac{1}{2 \sinh(\beta\hbar\omega/2)}.$$

From this expression calculate the temperature dependence of the heat capacity of a 1-d insulating solid. Explain what feature(s) of this dependence agrees and does not agree with experiment and indicate this on the sketch.

**P6.** Give an estimate of the following quantities in terms of a number and units where required:

- chemical potential of a collection of photons in a black body cavity of volume  $10\text{cm}^3$  at temperature  $4\text{K}$ .
- the factor by which the Fermi energy of electrons in a normal metal at room temperature would go up if the electron number density goes up by a factor of 27. You may treat the electrons as 3-d free particles with an energy density of states that is proportional to the square root of the energy.

**P7.** Calculate as a function of temperature the entropy of a collection of  $N$  noninteracting distinguishable two-level systems (for instance atoms), the energy difference between the two levels being  $\Delta$ .

**P8.** You are given no other information about a system in equilibrium at temperature  $T$  except that its energy can have any value in the continuum between 0 and  $\infty$  and that its energy density of states is a constant  $C$  (independent of the energy). Calculate the  $T$ -dependence of the free energy of this system.

**P9.** What kind of systems would exhibit heat capacities that tend to (i) zero at zero temperature, (ii) zero at very large temperatures? Explain your reasoning.

**P10.** Estimate the critical temperature for Bose-Einstein condensation in a system of  $10^{20}$  free bosons in a 2-dimensional box of size 1 cm on the side.