Preliminary Examination: Thermodynamics and Statistical Mechanics

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Instructions:

•The exam consists of two parts: Complete 5 short answer problems (6 points each) and your choice of 2 of 3 long answer problems (35 points each).

•Where possible, show all work; partial credit will be given.

•Personal notes on two sides of an 8×11 page are allowed.

•Total time: 3 hours.

Short Answers:

S1. A heat pump is used to heat a house in Albuquerque in the winter. The pump can maintain an inside temperature of 21° C when the outside temperature is 0° C. Assuming optimal efficiency, how many kJ of electrical energy must be expended to operate the pump for every kJ of heat that is deposited in the house?

S2. The work to place a charge Q on a capacitor at constant temperature T is given by the change in Helmoholtz free energy,

$$\Delta A = \frac{Q^2}{2C},$$

where C is the capacitance. If the capacitance $C = \lambda/T$ is inversely proportional to temperature, where λ is a constant, how does the capacitor's internal energy U depend on the charge Q?

S3. The free energy of a system containing N identical particles at a temperature T is given by

$$A = NkT\left(\ln\frac{N}{N_0} - 1\right) + N\varepsilon_0,$$

where the number N_0 and the energy ε_0 are constants. What is the value of N in equilibrium if the system is placed in contact with a particle reservoir having a chemical potential μ?

S4. The Hamiltonian for an *anharmonic* oscillator in one dimension is given by

$$H = \frac{p^2}{2m} + \frac{1}{4}kx^4.$$

The oscillator is in contact with a heat reservoir at a temperature T which is high enough so that classical mechanics is applicable. According to the law of equipartition, the average kinetic energy of a particle in thermal equilibrium is $kT/2$ per degree of freedom. Use the law of equipartition together with the virial theorem,

$$\left\langle p\frac{\partial H}{\partial p}\right\rangle = \left\langle x\frac{\partial H}{\partial x}\right\rangle,$$

to determine the average energy of the oscillator.

S5. The vibrational motion of a solid containing N atoms is sometimes modeled as a collection of $3N$ independent harmonic oscillators, each having the same natural frequency ω_0. Obtain an expression for the heat capacity of such a system, and graph the heat capacity versus temperature for $kT \ll \hbar\omega_0$ to $kT \gg \hbar\omega_0$.

Long Answers: Choose 2 out of 3 problems below.

L1. A system in equilibrium at room temperature consists of a dilute noninteracting gas of N molecules which are confined to a vessel having a volume V. Each molecule has a permanent dipole moment $\vec{p}_0$ and a mass m. A uniform electric field of strength E is maintained inside the vessel.

(a) Enumerate the states semiclassically and calculate the partition function associated with this system. You may neglect the rotational kinetic energy.

(b) Obtain an expression for the polarizability $\vec{\pi} = \frac{N}{V} \langle \vec{p}_0 \rangle$, and deduce from this the linear susceptibility χ.

(c) Suppose that the same gas is maintained at a constant pressure $P_{\rm ext}$ and field $E = 0$ in the environment external to the vessel. If a small hole is drilled in the vessel to allow gaseous exchange with the environment, how will the pressure P in the vessel differ from $P_{\rm ext}$ in equilibrium?

L2. Many of the electronic properties of a metal may be understood though a simple model in which the mobile electrons in the conduction band are treated as a gas of noninteracting particles having spin 1/2 and mass m. For a typical metal, the conduction band electron density is $n \simeq 30 \times 10^{27}$ m^{-3}, and $m \simeq 0.51$ MeV/c^2 (9.1×10^{-31} kg). For these material parameters, carry out the following calculations in the low temperature ($T = 0$) limit:

(a) Calculate the chemical potential μ for the electron gas (in eV).

(b) Derive an expression for the distribution of particle speeds $v = |\vec{v}|$.

(c) Use the distribution of particle speeds together with arguments from kinetic theory to determine the electron gas pressure in atmospheres (1 atm $\simeq 10^5$ N/m^2). If you would prefer to find the pressure in some other manner, you may do so, but be sure to elucidate your method.

L3. A solid is composed of atoms whose nuclei have spin one. Because the nuclear charge distribution is not spherically symmetrical, the energy ε of the nucleus of each atom depends on its azimuthal quantum number m. Assume that this orientation energy is $\varepsilon = \varepsilon_0$ for $m = 1$ and $m = -1$, and $\varepsilon = 0$ for $m = 0$.

(a) Find the contribution of the nuclear orientation energy to the molar energy of the crystal as a function of temperature.

(b) Find the corresponding entropy.

(c) Sketch the temperature dependence of the nuclear contribution to the molar specific heat. Give a physical explanation for the behavior at low and high temperatures.