Preliminary Examination: Thermodynamics and Stat. Mech.

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Instructions:

- The exam consists two parts: 5 short answers (6 points each) and your pick of 2 out 3 long answer problems (35 points each).
- Where possible, show all work, partial credit will be given.
- Personal notes on two sides of a 8X11 page are allowed.
- Total time: 3 hours

Good luck!

Short Answers:

S1. Describe the microscopic mechanism by which evaporation cools a cup of hot coffee?

S2. A molecule of oxygen is composed of two oxygen atoms, each having a mass of 2.7×10^{-26} kg, separated from one another by a fixed distance of 1.24×10^{-10} m. Below what characteristic temperature will the rotational degrees of freedom of O_2 gas be effectively "frozen out"? Sketch a graph of the heat capacity at constant volume as a function of temperature for one mole of dilute O_2 gas, over a range which includes this characteristic temperature.

S3. The fermi level of a certain metal is 5.25 eV at room temperature. What is the probability that a state which is 0.10 eV above the fermi level is occupied by an electron?

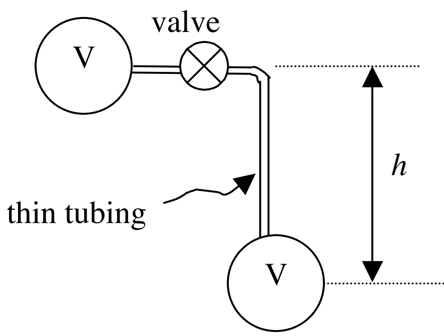
S4. To model the process by which gas is adsorbed on the surface of a metal, the metal surface can be described as a corregated muffin-tin potential, as shown in the figure. Gas atoms can lower their energy by sitting in the potential minima on the surface, which serve as a set of identical adhesion sites. Interactions between the gas atoms can be ignored, except for the fact that each site may be occupied no more than one atom.

Consider a thermodynamic system consisting of a metal surface with M adhesion sites which are occupied by N indistinguishable gas atoms ($N<M$). What is the change in the entropy of this system if one more gas atom is added to the surface?

S5. A narrow potential well consists of only two bound states, a ground state and a first excited state. The potential well is occupied by exactly two noninteracting indistinguishable particles, which are bosons. The energy to place a particle in the excited state is higher than the energy to place a particle in the ground state by Δ. What is the probability that both particles are in the excited state at a temperature T.

Long Answers: Pick two out of three problems below

L1. A glass bulb of volume V containing N atoms of ideal gas each with mass m is connected by a long thin tube to a second bulb of volume V, which is evacuated. The first bulb is located at a height h above the second bulb, as shown in the figure below, and initially the tube is closed off by a valve. The entire system is in contact with the surroundings at a temperature T. When the stopcock is opened, the gas expands to fill both bulbs. The volume of gas residing in the connecting tube is negligible.



(a) The partition function for an ensemble describing an ideal gas having N atoms which is in equilibrium at constant T and V in a vessel which is at an elevation h is given by

$$\frac{V^N}{N!}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3N/2}\exp\left[-\frac{Nmgh}{kT}\right]$$

What is the Helmholtz free energy of the gas before the stopcock is opened?
Show how to derive the equation of state from the Helmholtz free energy.

(b) After the expansion, what is the gas pressure in the upper bulb? What is the gas pressure in the lower bulb?

(c) How much heat is absorbed by the gas from the surroundings in the expansion?

L2. A crystalline solid contains N similar, immobile, statistically independent defects. Each defect has 5 possible states with energies, $\varepsilon_1 = \varepsilon_2 = 0$, $\varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \Delta$.

(a) Find the partition function of the system.

(b) Find the defect contribution to the entropy of the crystal as a function of Δ and the temperature T.

(c) Without doing a detailed calculation, find the contribution to the internal energy due to the defects, in the high temperature limit $kT >> \Delta$.

L3. A cylinder containing n moles of ideal gas is positioned vertically as shown in the figure below. The ambient pressure and temperature are P_{ext} and T respectively, and the heat capacity of the gas at constant volume is $(5/2)nR$ where R is the ideal gas constant. The cylinder is closed by a piston which has a mass M and surface area A, and slides with no friction on the walls of the cylinder. In equilibrium, the total downward force $P_{ext}A + Mg$ on the piston is equal to the upward force PA exerted by the gas. When the piston is depressed slightly and released, it oscillates with a frequency

$$\omega = \sqrt{\frac{A^2}{M}\left(-\frac{\partial P}{\partial V}\right)}.$$

(a) Assuming that the oscillation frequency is slow enough so that the gas compressions are nearly isothermal, show that

$$\omega = \sqrt{\frac{\left(P_{\text{ext}}A + Mg\right)^2}{nRTM}}$$

(b) Alternatively, obtain an expression for ω as a function of T and P_{ext} for the case that the compressions may be considered to be adiabatic. Compare this result to the result from part (a) and discuss the physical origin of the difference.