<u>Preliminary Examination: Quantum Mechanics</u>

Department of Physics and Astronomy

University of New Mexico

Spring 2006

Instructions:

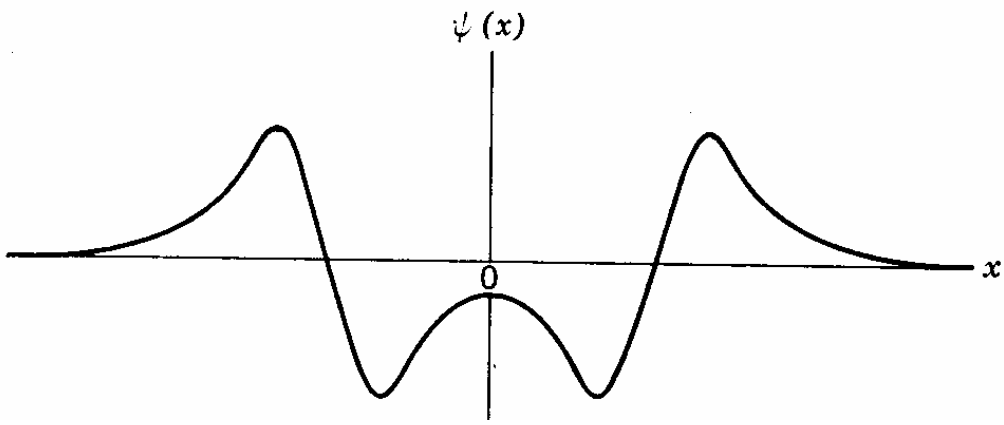
- The exam consists of 10 short answers (10 points each).
- Where possible, show all work, partial credit will be given.
- Personal notes on two sides of an 8X11 page are allowed.
- Total time: 3 hours

Good luck!

1. A particle of mass *m* moves in a potential: $V(x) = \frac{1}{2}kx^2 + cx$. Find the eigenvalue of the nth state (n= 0,1,2,…) to lowest *nonvanishing* order in perturbation theory, treating the linear term *cx* as a perturbation.

2. A particle of mass *m* is in the ground state of an infinite square well potential, $V(x) = \begin{cases} 0, 0 \le x \le a \\ \infty, \text{otherwise} \end{cases}$. Suddenly, the well expands to three times its original size – the right wall moving from *a* to *3a* – leaving the wave function momentarily undisturbed. The energy of the particle is now measured. Write an integral expression for the probability of finding the particle in the ground state of the new potential.

3. Given the eigenfunction below, make a sketch of the potential (on the same graph), indicating on the graph the energy level associated with this eigenfunction. Describe the energy of this eigenfunction relative to the lowest allowed energy.



4. Quarks carry spin ½. Three quarks bind together to make a baryon (such as the proton or neutron); two quarks (or more precisely a quark and an antiquark) bind together to make a meson (such as the pion or kaon). Assume the quarks are in the ground state (so the orbital angular momentum is zero). What magnitudes of the spin vector are possible for baryons and mesons?

5. In the representation in which S^2 and S_z are both diagonal, a spin- ½ wave function is:

$$\psi = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ 3i \end{pmatrix}.$$

What is the probability that a measurement of S_x on ψ yields $-\hbar/2$?

6. A particle with zero spin in a spherically symmetric potential is in a state represented by the wavefunction:

$\psi(x,y,z) = C(xy + 2yz)e^{-\alpha r}$, where C and α are constants.

a) What is the eigenvalue of L^2 for this particle?
b) If L_z were measured, what are the possible outcomes?

You may need the following spherical harmonics:

$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8}}\sin\theta e^{\pm i\varphi} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_2^{\pm 2} = \mp\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\varphi} \quad Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\varphi} \quad Y_2^0 = \mp\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right)$$

7. Operators $\hat{A}$ and $\hat{B}$ obey the following equations: $\hat{A}\psi_{ab} = a\psi_{ab}$ and $\hat{B}\psi_{ab} = b\psi_{ab}$, where the ψ_{ab} form a complete set in Hilbert space and *a* and *b* are real numbers. Show that the operator $\left(\hat{A}\hat{B}\right)$ is Hermitian.

8. A particle in a finite square well potential:

$$V(x) = \begin{cases} 0, & x < -a \\ -V_0, & -a \le x \le a \\ 0, & x > a \end{cases},$$

with only three bound states, initially is in a state such that the probability to measure the energy is given by: $P(E_1) = 1/3$, $P(E_2) = 1/3$ and $P(E_3) = 1/3$. The parity is then measured in such a way that the particle stays bound, and found to be -1. If some time later, the energy is measured, what value is found?

9. In a photoelectric experiment, electrons are emitted from a surface illuminated by light of wavelength 4000Å, and the stopping potential for these electrons is found to be Φ_0 = 0.5V. What is the longest wavelength of light that can illuminate this surface and still produce a photoelectric current?

10. A free electron is prepared in a Gaussian wave packet with standard deviation σ_x. Estimate the time for the uncertainty in the electron's position to increase to 2 σ_x.