Preliminary Examination: Quantum Mechanics

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Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours.*

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1. A polished silicon surface is seen by a free neutron as an (infinitely sharp) impenetrable barrier. Under the influence of gravity, unlike a classical particle that rests on a hard horizontal surface, the neutron will hover above the surface.
   (a) Sketch the wave function of the ground state of such a neutron.
   (b) Derive an approximate expression for the expectation value of the height of the neutron above the silicon using the uncertainty principle (in terms of the acceleration of gravity g, the mass of the neutron m_n, and other constants).
2. Consider a particle of mass m in a one-dimensional potential well $V(x) = -V_0$ for $|x| < a$ and equal to zero otherwise. Show that at least one bound state will always exist.
3. Consider a Hamiltonian
   $$\hat{H} = \hat{H}_1 + \hat{H}_2.$$
   (a) When can the energy eigenvalue of the total Hamiltonian $\hat{H}$ be written as the sum of eigen-energies E_i for the $\hat{H}_i$ ($i = 1, 2$),
   $$E = E_1 + E_2?$$
   (b) For a diatomic molecule, what is the rigid rotor approximation and show explicitly that E can be written as the sum of vibrational and rotational energies.
4. Find the reflection coefficient for 1 dimensional scattering of a particle of mass m off of a delta function potential $V = -a\delta(x)$.
5. Consider a complex potential $V = V_0 - i\Gamma$ where V_0 and Γ are real constants with dimensions of energy. Use the one dimensional Schrödinger equation to obtain an equation for the probability density and show that the total probability $P = \int \psi^\star \psi dx$ is not conserved; Find dP/dt.
6. An electron is in the spin-up state with respect to a uniform magnetic field. The field direction is suddenly rotated at t=0 by 60°. What is the probability to measure the electron in the spin-up state with respect to the new field direction immediately after the field is rotated?

7. The spin of an electron interacts with a uniform, static magnetic field with

$$\hat{H}_0 = \omega_0 \hat{S}_z.$$

At $t = 0$ the spin state is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)),$$

with the notation $|\uparrow\rangle = |\frac{1}{2}, +\frac{1}{2}\rangle, |\downarrow\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$.

Find $\langle \hat{S}_x \rangle$ as a function of time t.

8. For the two-dimensional harmonic oscillator, the unperturbed Hamiltonian is given by

$$\hat{H}_0 = \frac{1}{2m}\left(\hat{p}_x^2 + \hat{p}_y^2\right) + \frac{1}{2} m\omega^2 \left(\hat{x}^2 + \hat{y}^2\right).$$

Use perturbation theory to determine the first-order energy shifts to the ground state and the (degenerate) first-excited states due to the perturbation

$$\hat{H}_1 = 2b\hat{x}\hat{y}.$$

9. Consider a particle of mass m in an **infinite** spherical well of radius a. Determine the energy eigenstates and eigenvalues for $\ell = 0$. The energy eigenstates should be written in terms of a normalization constant (A) that you leave undetermined.

10. Consider the two-electron states of Helium in perturbation theory, where we take the e-e Coulomb interaction to be the perturbation.
a) What is the unperturbed energy and degeneracy of the multiplet of first excited states?

Explicitly construct these states in terms of single electron hydrogen atom (spatial) energy eigenstates $|n_i, \ell_i, m_i\rangle_i$ and spinors $|\pm\rangle_i \equiv |\frac{1}{2}, \pm\frac{1}{2}\rangle_i$ (where $i = 1, 2$ labels the electron) for states with **total** orbital angular momentum quantum number ℓ and **total** spin quantum number s for the cases:
b) $\ell = 0$, $s = 1$; and
c) $\ell = 1$, $s = 0$.