Preliminary Examination: Quantum Mechanics

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Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours.*

Table of Constants and Conversion factors

Quantity	Symbol	$\approx$ value
speed of light	c	3.00×10^8 m/s
Planck	h	6.63×10^{-34} J·s
electron charge	e	1.60×10^{-19} C
e mass	m_e	0.511 Mev$/c^2 = 9.11 \times 10^{-31}$ kg
p mass	m_p	938 Mev$/c^2 = 1.67 \times 10^{-27}$ kg
fine structure	α	1/137
Boltzmann	k_B	1.38×10^{-23} J/K
Bohr radius	$a_0 = \hbar/(m_e c\alpha)$	0.53×10^{-10} m
conversion constant	$\hbar c$	200 eV·nm
conversion constant	k_BT@ 300K	1/40 eV
conversion constant	1eV	1.60×10^{-19} J

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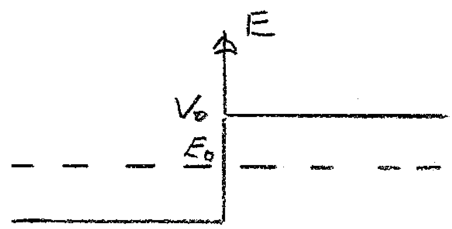
1. At what speed is the deBroglie wavelength of an alpha particle equal to that of a 10 KeV photon.
2. Argue using the deBroglie wavelength whether or not QM is needed to describe the physical system:
   a) atomic electron (note that for the ground state of hydrogen $\langle v \rangle = c\alpha$)
   b) hydrogen gas with number density $10^{21}/\text{cm}^3$ at T=300K
3. Three essential features of the non-relativistic Schroedinger equation are that it is linear, it depends on the first time derivative and it explicitly contains the complex number i in the time derivative term. State the physical consequences of each of these features.
4. A particle is confined to a 1D box $0 < x < a$ and at $t = 0$ has the wave function

$$\psi = Ax(a - x).$$

   (a) Determine the normalization constant A.
   (b) Find the expectation value of the energy for this state.
   (c) How does your result in (b) compare qualitatively $(>, =, <)$ with the ground state energy and explain your reasoning.

5. A particle moving in 1D with kinetic energy E_0 is incident on an infinitely wide potential barrier with $V_0 > E_0$. Find the penetration depth of the particle into the barrier.



6. A photon is in the state

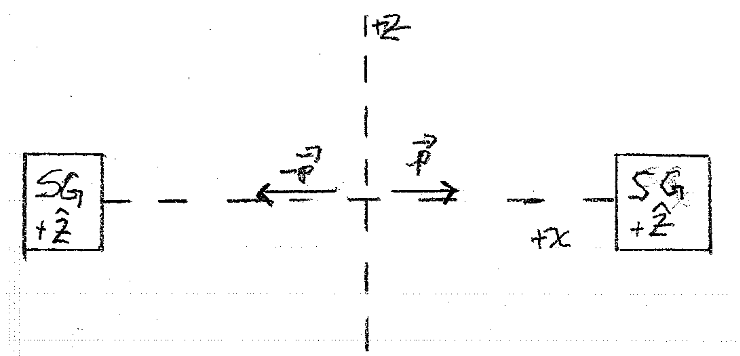
$$|\psi\rangle = \frac{1}{5}(3|\underline{x}\rangle + i4|\underline{y}\rangle$$

where $|\underline{x}\rangle$, $|\underline{y}\rangle$ refer to photon states polarized along the x and y axes respectively.
(a) What is the probability of the photon being right circularly polarized?
(b) Suppose the photon passes through a plate that introduces a relative phase shift of π between the states $|\underline{x}\rangle$, $|\underline{y}\rangle$. What is the probability of the photon being right-circularly polarized now?

7. Consider two electrons with equal and opposite momenta directed along the x-axis and produced in the entangled spin state

$$\frac{1}{\sqrt{2}}(|\Uparrow_z, \Downarrow_z\rangle - |\Downarrow_z, \Uparrow_z\rangle).$$

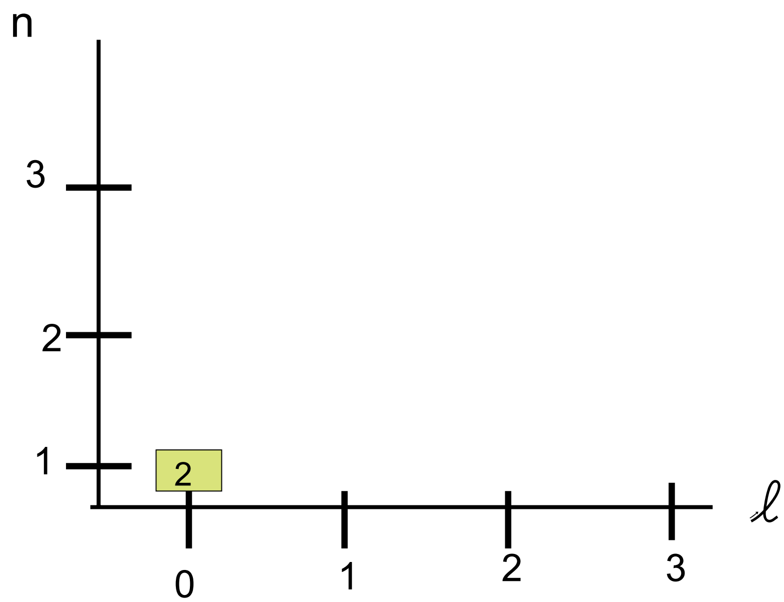
The electron with momentum $\vec{p}$ is the first in the product ket, and the $-\vec{p}$ is the second. This is sketched in the figure below, where the boxes labeled "SG $+\hat{z}$" represent Stern-Gerlach devices that deflect electrons in the $|\Uparrow_z\rangle$ state in the $+\hat{z}$ direction and electrons in the $|\Downarrow_z\rangle$ state in the $-\hat{z}$ direction thereby measuring the electron spin components along the z direction.



a) What are the possible outcomes for the two SG measurements? What are the probabilities for the outcomes given this entangled state.
b) What is the total spin of this state? (Think of the symmetry of this state and how you would write down the other possible two-electron product states.)
c) The experiment is repeated with SG devices oriented in the y direction thereby measuring the spin components along the y axis. What is the probability for each of the possible outcomes now? (No calculations needed if you use the symmetry property of this state.)

8. Calculate the first order correction to the energy of a simple 1D harmonic oscillator with displacement in the x direction due to the perturbation bx^4.

9. a) For the $1/r$ potential (simple hydrogen atom) the states are labeled by quantum numbers n,ℓ,m_ℓ,s,m_s. The states with magnetic quantum number m_ℓ but the same ℓ are degenerate. Why? Same question for the spin quantum numbers s,m_s.
   b) A typical energy level (not to scale) diagram is shown below, where each box represents states of the same n,ℓ and is labeled by the degeneracy including spin (number of states in the box). Complete the diagram up to n=3.



10. Add the spin-orbit coupling to the simple hydrogen atom. For the $n = 2$ level, label **all** the states according to the "good" quantum numbers. Show that the total number of states sums to $2n^2 = 8$ (2 for spin).