Preliminary Examination: Electricity and Magnetism

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Instructions:

- This exam consists of 10 problems with 10 points each.
- Read all 10 problems before you begin to solve any problem, and solve the problems that seem easiest to you first. Spend your time wisely. If you are stuck on one problem, move on to the next one, and come back to it if you have time after you have solved all other problems.
- Show necessary intermediate steps in each solution. Partial credit will be given if merited.
- No textbook, personal notes or external help may be used other than what is provided by the proctor.
- This exam takes 3 hours.

Potentially Useful Information:

Physical constants and symbols:

ϵ	permittivity,	ϵ_0	vacuum permittivity,
μ	permeability,	μ_0	vacuum permeability,
e	electric charge of the proton,	m_e	mass of the electron,
$c = 1/\sqrt{\epsilon_0 \mu_0}$	speed of light,		
ρ	electric charge volume density,	$\mathbf{j}$	electric current density.

Formulas and relations:

- Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \qquad \nabla \times \mathbf{H} = \mathbf{j}_{\text{free}} + \partial_t \mathbf{D},$$

  where $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E}$ and $\mathbf{H} = \mu_0^{-1} \mathbf{B} - \mathbf{M} = \mu^{-1} \mathbf{B}$.

- Lorentz force law:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$

- Bio-Savart's law for the magnetic field at position $\mathbf{r}$ due to a steady current element $I\mathrm{d}\boldsymbol{\ell}'$ located at position $\mathbf{r}'$:

$$\mathrm{d}\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{I\mathrm{d}\boldsymbol{\ell}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}.$$

- Poynting vector: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$.

- $Z\mathbf{H} = \hat{\mathbf{k}} \times \mathbf{E}$ and $k = n\omega/c$ for a monochromatic plane wave in medium, where $Z = \sqrt{\mu/\epsilon}$ is the impedance, and $n = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$ is the refractive index.

- The radiation magnetic field at position $\mathbf{r}$ due to a time-varying electric dipole $\mathbf{p}$ at the origin:

$$\mathbf{B}(t, \mathbf{r}) = -\frac{\mu_0}{4\pi rc} \hat{\mathbf{r}} \times \ddot{\mathbf{p}}(t - r/c).$$

Problem 1: A charge distribution $\rho(x, y, z)$ results in an electrostatic potential $V(x, y, z) = a|y - y_0|$ in space, where a and y_0 are two constants. Find the form of $\rho(x, y, z)$.

Problem 2: A square loop of wire with side length d and electric resistance R is placed in the x-y plane with two sides parallel to the y axis. It is pulled with a constant velocity $v\hat{\mathbf{x}}$ through space which is filled with the magnetic field $\mathbf{B} = \alpha x\hat{\mathbf{z}}$, where α is a positive constant, and $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ are unit vectors in the x and z directions, respectively. Find the magnitude and direction of the induced current I in the loop.

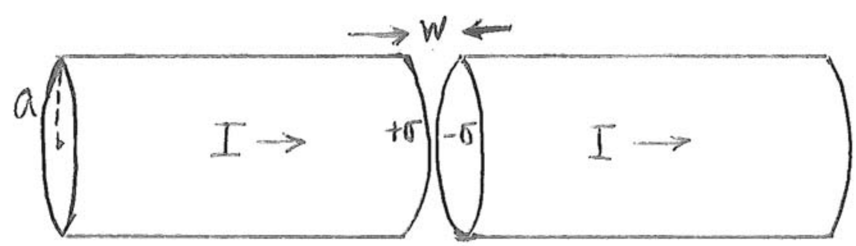
Problem 3: A point particle of charge q and mass m is initially at rest in a region filled with uniform magnetic field B in the y direction. Taking into account the gravitation force in the $-z$ direction, write down the equation of motion for the particle and solve it.

Problem 4: A uniformly charged ring of radius R and charge Q is centered at the origin and rotates in the x-y plane about the z axis with an angular velocity ω. Determine the electric and magnetic fields at the center of the ring.

Problem 5: A solid cylinder of radius R and height h is polarized along its axis (chosen to be the z axis) with the polarization density $\mathbf{P}$ increasing linearly with the height from a value $P_1\hat{\mathbf{z}}$ at the bottom face to $P_2\hat{\mathbf{z}}$ at the top face. Calculate the bound surface charge density and the volume charge density. Show by integrating the total bound charge densities that there is no net charge in the cylinder.

Problem 6: A point charge q is situated at the center of a dielectric spherical shell of permittivity ϵ_1 with inner and outer radii a and b, respectively. The space at $r > b$ is vacuum, and the space at $r < a$ is filled with another dielectric material of permittivity ϵ_2. Determine the electric field $\mathbf{E}$ in all regions.

Problem 7: A capacitor is formed by a small gap w in a wire of radius a where $w \ll a$. The initial surface charge density σ on either side of the gap is 0. Starting at $t = 0$, a constant, uniform current I flows in the wire. Find the electric and magnetic fields in the gap. Ignore the fringing fields.



Problem 8: A circularly polarized electromagnetic plane wave of frequency ω is incident from vacuum onto a transparent slab of refractive index n. Find the incident angle for which the reflected wave is linearly polarized. Assume that the slab has a permeability $\mu \approx \mu_0$.

Problem 9: A thin spherical shell of radius R and uniform surface charge density σ rotates at a constant angular speed ω about the z axis which passes through the center of the shell. The magnetic vector potential generated by this spinning shell can be written as

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0 R \sigma \omega}{3} \hat{\mathbf{z}} \times \mathbf{r} & \text{if } r \leqslant R, \\ \dfrac{\mu_0 R^4 \sigma \omega}{3r^3} \hat{\mathbf{z}} \times \mathbf{r} & \text{if } r \geqslant R, \end{cases}$$

where $\mathbf{r}$ is the position vector. Find out the magnetic field outside and inside the spherical shell.

Problem 10: A point charge q moves with constant angular speed ω along a circular orbit in the x-y plane which is of radius R.
(a) Find the instantaneous rate at which the charge loses its energy by electromagnetic radiation.
(b) What is the polarization of the electromagnetic wave emitted in the $+z$ direction?