Preliminary Examination: Electricity and Magnetism

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Instructions:

- This exam consists of 10 problems with 10 points each.
- Read all 10 problems before you begin to solve any problem, and solve the problems that seem easiest to you first. Spend your time wisely. If you are stuck on one problem, move on to the next one, and come back to it if you have time after you have solved all other problems.
- Show necessary intermediate steps in each solution. Partial credit will be given if merited.
- No textbook, personal notes or external help may be used other than what is provided by the proctor.
- This exam takes 3 hours.

Potentially Useful Information:

Physical constants and symbols:

ϵ	permittivity,	ϵ_0	vacuum permittivity,
μ	permeability,	μ_0	vacuum permeability,
e	electric charge of the proton,	m_e	mass of the electron,
$c = 1/\sqrt{\epsilon_0\mu_0}$	speed of light,		
ρ	electric charge volume density,	$\mathbf{j}$	electric current density.

Formulas and relations:

- Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \qquad \nabla \times \mathbf{H} = \mathbf{j}_{\text{free}} + \partial_t \mathbf{D},$$

  where $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E}$ and $\mathbf{H} = \mu_0^{-1}\mathbf{B} - \mathbf{M} = \mu^{-1}\mathbf{B}$.

- $Z\mathbf{H} = \hat{\mathbf{k}} \times \mathbf{E}$ and $k = n\omega/c$ for a monochromatic plane wave in medium, where $Z = \sqrt{\mu/\epsilon}$ is the impedance, and $n = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$ is the refractive index.

- Fresnel's relations:

$$\frac{E_2}{E_1} = \frac{2Z_2\cos\theta_1}{Z_1\cos\theta_1 + Z_2\cos\theta_2}, \qquad \frac{E_1'}{E_1} = \frac{Z_1\cos\theta_1 - Z_2\cos\theta_2}{Z_1\cos\theta_1 + Z_2\cos\theta_2}$$

for a wave with a linear polarization parallel to the incident plane, and

$$\frac{E_2}{E_1} = \frac{2Z_2\cos\theta_1}{Z_2\cos\theta_1 + Z_1\cos\theta_2}, \qquad \frac{E_1'}{E_1} = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2}$$

for a wave with a linear polarization perpendicular to the incident plane. Here the subscripts 1 and 2 refer to the media of the incident wave and the transmitted wave, respectively, and the primed quantities are for the reflected wave.

- Snell's law: $\theta_1 = \theta_1'$ and $n_1\sin\theta_1 = n_2\sin\theta_2$.

- The radiation magnetic field at position **r** due to a time-varying electric dipole **p** at the origin:

$$\mathbf{B}(t,\mathbf{r}) = -\frac{\mu_0}{4\pi rc}\hat{\mathbf{r}} \times \ddot{\mathbf{p}}(t - r/c).$$

Problem 1: A certain charge distribution produces an electrostatic potential $\phi(\mathbf{r}) = \mathbf{a} \cdot \mathbf{r}/r^3$ in space, where $\mathbf{r}$ is the position vector, and $\mathbf{a}$ is a constant vector.
(a) Verify that there is no electric charge for $r \neq 0$.
(b) What kind of charge distribution may give rise to this electrostatic potential?

Problem 2: Consider an infinitely long, thin, cylindrical shell of radius R and uniform surface charge density σ. Find the electric field outside and inside the cylindrical shell.

Problem 3: A thin spherical shell of radius R and uniform surface charge density σ rotates at a constant angular speed ω about the z axis which passes through the center of the shell. The magnetic vector potential generated by this spinning shell can be written as

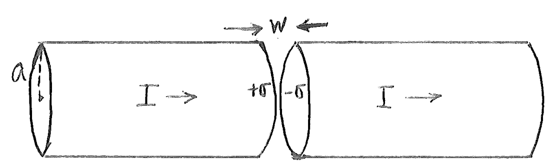
$$\mathbf{A}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0 R \sigma \omega}{3} \hat{\mathbf{z}} \times \mathbf{r} & \text{if } r \leqslant R, \\ \dfrac{\mu_0 R^4 \sigma \omega}{3r^3} \hat{\mathbf{z}} \times \mathbf{r} & \text{if } r \geqslant R, \end{cases}$$

where $\mathbf{r}$ is the position vector. Find out the magnetic field outside and inside the spherical shell.

Problem 4: A point charge q moves with constant angular speed ω along a circular orbit in the x-y plane which is of radius R.
(a) Find the instantaneous rate at which the charge loses energy by electromagnetic radiation.
(b) What is the polarization of the electromagnetic wave which is emitted in the $+z$ direction?

Problem 5: Consider a parallel-plate capacitor which has area A and distance d between the plates. Half of the gap between the plates is filled with an dielectric of permittivity ϵ and the rest is empty. (See the figure below.) Find the capacitance of the capacitor.

Problem 5



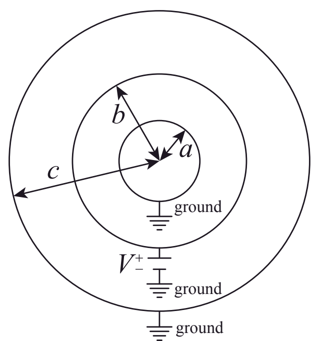
Problem 6

Problem 6: A capacitor is formed by a small gap w in a wire of radius a where $w \ll a$. (See the figure above.) The initial surface charge density σ on either side of the gap is 0. Starting at $t = 0$, a constant uniform current I flows in the wire. Find the energy density and the Poynting vector of the electromagnetic field in the gap. Ignore fringing fields.

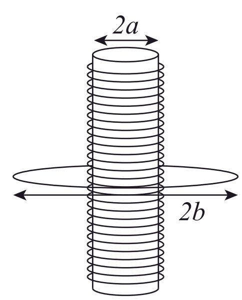
Problem 7: A circularly polarized plane electromagnetic wave of frequency ω is incident from vacuum onto a transparent slab of refractive index n. Find the incident angle for which the reflected wave is linearly polarized. Assume that the slab has permeability $\mu \approx \mu_0$.

Problem 8: An electromagnetic plane wave of (time-averaged) intensity I is incident normally from vacuum onto a thin transparent glass slab. The back surface of the slab is coated with silver so that it completely reflects the incident radiation.
(a) Find the (time-averaged) pressure on the slab due to radiation.
(b) Repeat the calculation in (a) if the silver coating reflects only a fraction $R < 1$ of the incident power and absorbs the rest.

Problem 9: Consider a system of three conducting thin spherical shells with radii a, b, and c ($a < b < c$). (See the figure below.) The inner and outer shells are connected to the ground, while the middle one is connected to a potential source V. (See the figure below.) Find the electric charge on each of the conducting shells.



Problem 9



Problem 10

Problem 10: An infinitely long solenoid of radius a and n turns of wire per unit length along the axial direction carries a current $I(t) = (1 - \alpha t)I_0$, where I_0 and α are constants. Coaxial with the solenoid is a large, circular conducting ring of radius $b \gg a$ and resistance R. (See the figure above.) Find the electric current in the ring.