Preliminary Examination: Electricity and Magnetism

Department of Physics & Astronomy
University of New Mexico

January 2017

Instructions:

- This exam consists of 10 problems with 10 points each.
- Read all 10 problems before you begin to solve any problem, and solve the problems that seem easiest to you first. Spend your time wisely. If you are stuck on one problem, move on to the next one, and come back to it if you have time after you have solved all other problems.
- Show necessary intermediate steps in each solution. Partial credit will be given if merited.
- No textbook, personal notes or external help may be used other than what is provided by the proctor.
- This exam takes 3 hours.

Potentially Useful Information:

Physical constants and symbols:

ϵ_0	vacuum permittivity
ϵ_r	relative permittivity
μ_0	vacuum permeability
μ_r	relative permeability
$c = 1/\sqrt{\epsilon_0\mu_0}$	speed of light
e	electric charge of the proton
m_e	mass of the electron

Formulas and relations:

- Maxwell's equations:

$$\boldsymbol{\nabla}\cdot\mathbf{D} = \rho_\mathrm{free}, \qquad \boldsymbol{\nabla}\cdot\mathbf{B} = 0,$$
$$\boldsymbol{\nabla}\times\mathbf{E} = -\partial_t\mathbf{B}, \qquad \boldsymbol{\nabla}\times\mathbf{H} = \mathbf{j}_\mathrm{free} + \partial_t\mathbf{D},$$

  where $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon_0\epsilon_\mathrm{r}\mathbf{E}$ and $\mathbf{H} = \mu_0^{-1}\mathbf{B} - \mathbf{M} = (\mu_0\mu_\mathrm{r})^{-1}\mathbf{B}$.

- Poynting vector: $\mathbf{S} = \mathbf{E}\times\mathbf{H}$.
- $Z\mathbf{H} = \hat{\mathbf{k}}\times\mathbf{E}$ and $k = n\omega/c$ for a monochromatic plane wave in medium, where $Z = \sqrt{\mu_0\mu_\mathrm{r}/\epsilon_0\epsilon_\mathrm{r}}$ is the impedance, and $n = \sqrt{\epsilon_\mathrm{r}\mu_\mathrm{r}}$ is the refractive index.

Problem 1: A charge distribution $\rho(x,y,z)$ results in an electrostatic potential $V(x,y,z) = a|z|$ in space, where a is a constant. Find the form of $\rho(x,y,z)$.

Problem 2: An electromagnetic plane wave of intensity I is incident from vacuum onto a large planar metallic foil of area A at angle θ from the normal. The foil is slightly blackened so that only a fraction R of the electromagnetic energy is reflected and the rest is absorbed. Calculate the radiation force on the foil.

Problem 3: The positive terminal of a battery (ground taken at infinity) is attached to a perfectly conducting sphere of radius R in vacuum by a long, thin wire and brings it to potential V. How much work does the battery do in bringing the initially uncharged sphere to the same potential? Assume that no energy is lost in radiation or heat.

Problem 4: A transverse electromagnetic wave travels inside a neutral plasma, inducing a current density $\mathbf{j} = -en_e\mathbf{v}$, where n_e is the number density of electrons driven with instantaneous velocity $\mathbf{v}$ by the electric field. Protons, being much heavier have a negligible contribution to $\mathbf{j}$ at frequencies of interest, and collisional damping can also be ignored. One can show that such a wave satisfies equation

$$\left(\boldsymbol{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\omega_{\mathrm{P}}^2}{c^2}\right)\mathbf{E} = 0,$$

where $\omega_{\mathrm{P}} = \sqrt{n_e e^2/\epsilon_0 m_e}$ is the plasma frequency. Find the index of refraction $n(\omega)$ of the plasma as a function of the (angular) frequency ω of the wave. In what frequency range is the plasma transparent to the wave?

Problem 5: A monochromatic electromagnetic wave traveling in vacuum incident normally onto a non-dispersive medium with real relative permittivity ϵ_{r} and permeability μ_{r}. By using the boundary conditions dictated by Maxwell's equations, show that a fraction

$$R = \left(\frac{1 - Z_{\mathrm{r}}}{1 + Z_{\mathrm{r}}}\right)^2$$

of the incident electromagnetic energy is reflected, where $Z_{\mathrm{r}} = \sqrt{\mu_{\mathrm{r}}/\epsilon_{\mathrm{r}}}$ is the relative wave impedance in the medium.

Problem 6: An infinitely extended plane with a uniform charge density σ coincides with the x-z plane and moves in the z direction with a uniform velocity v. Find the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ at a distance d on either side of the plane.

Problem 7: Consider four point charges $+q$, $+q$, $-q$ and $-q$ placed on the x-y plane with (x, y) coordinates $(0, a)$, $(0, -a)$, $(-a, 0)$ and $(2a, 0)$, respectively, where $a > 0$. Another point charge Q is placed on the x-axis at a distance $r \gg a$ from the origin. Find the direction and magnitude of the force on Q up to the lowest non-vanishing order in a/r.

Problem 8: A square loop of wire with side length d and electric resistance R is placed in the x-y plane with two sides parallel to the y axis. It is pulled with constant velocity $v\hat{\mathbf{x}}$ through the space with magnetic field $\mathbf{B} = \alpha x \hat{\mathbf{z}}$, where α is a positive constant, and $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ are unit vectors in the x and z directions, respectively. Find the magnitude and direction of the induced current I in the loop.

Problem 9: Two capacitors A and B are identical except that A is filled with a dielectric of relative permittivity $\epsilon_{\mathrm{r}} = 2$ and B is filled with nothing. Initially A has a charge Q. Find the final charges in A and B when the two capacitors are connected.

Problem 10: A point particle of charge q and mass m is initially at rest in a region filled with uniform magnetic field B in the y direction. Taking into account the gravitation force in the $-z$ direction, write down the equation of motion for the particle and solve it.