Department of Physics and Astronomy, University of New Mexico

Electricity and Magnetism Preliminary Examination

Spring 2015

Instructions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- NO cheat sheets are allowed.
- Total time: 3 hours.

Useful Formulas and Relations

- Time-averaged power radiated by an oscillating electric dipole:

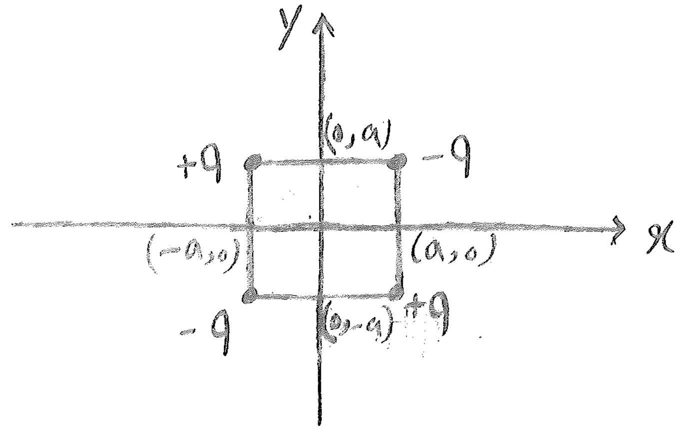
$$P = \frac{\mu_0 |p|^2 \omega^4}{12\pi c}.$$

- Fresnel formulas for the amplitude reflection coefficient of a plane wave incident at a planar interface between two media:

$$r_\perp = \frac{n\ \cos\theta - n'\ \cos\theta'}{n\ \cos\theta + n'\ \cos\theta'} \quad , \quad r_{||} = \frac{n'\ \cos\theta - n\ \cos\theta'}{n'\ \cos\theta + n\ \cos\theta'},$$

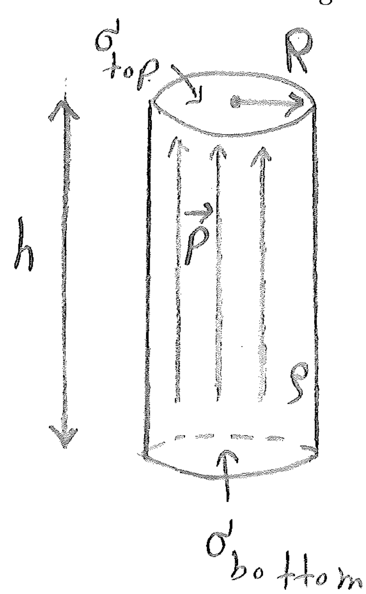
where $\perp$ and $||$ refer, respectively, to polarizations perpendicular and parallel to the plane of incidence. The angles of incidence and refraction are θ and θ', and n, n' are the refractive indices of the medium of incidence and the medium of transmission, respectively.

1– Four charges are placed in the xy plane as shown in the figure. Find the first non-zero contribution in the multipole expansion of the electric potential at points (d, d), $(d, -d)$, $(-d, d)$, $(-d, -d)$ where $d \gg a$.

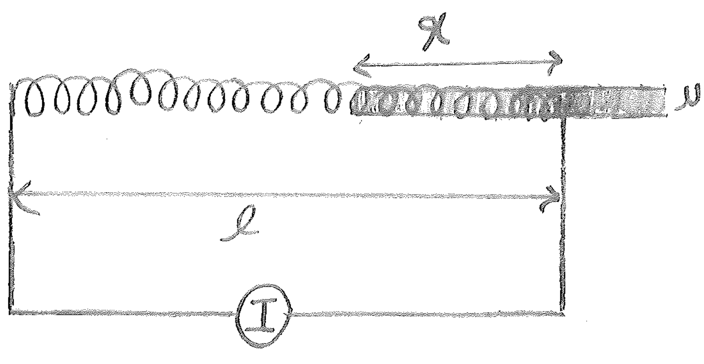


2- A solid spherical conductor with a spherical hole has a net charge of $+q$. Determine the total charge, and sketch the surface charge distribution, on each of the surfaces in each of the following situations:
(a) An additional point charge $+q$ is placed outside the conductor.
(b) An additional point charge $-q$ is placed inside the hole.

3– A solid cylinder of radius R and height h is polarized along its axis with polarization density $\vec{P}$ that decreases linearly with height from a value $P_1\hat{z}$ at the bottom face to $P_2\hat{z}$ at the top face. Find the bound surface and volume charge densities labelled as σ_{top}, σ_{bottom}, and ρ in the figure. What is the total bound charge? Comment.



4– A long solenoid with n turns per unit length and cross-sectional area A carries an electric current I. Part of the solenoid is filled with a magnetic material with permeability μ as shown in the figure. Find the magnitude and direction of force required to hold the material in place as a function of x. Current I in the solenoid is held constant.



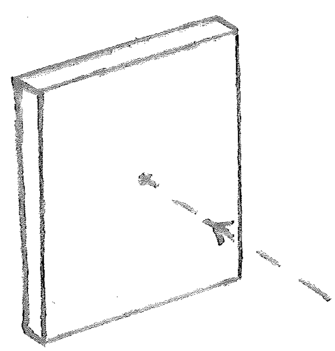
5– Find the magnetic field produced by the following current density and draw its profile:

$$\vec{J} = J_0 \exp(\frac{-r}{r_0})\ \hat{z}\,.$$

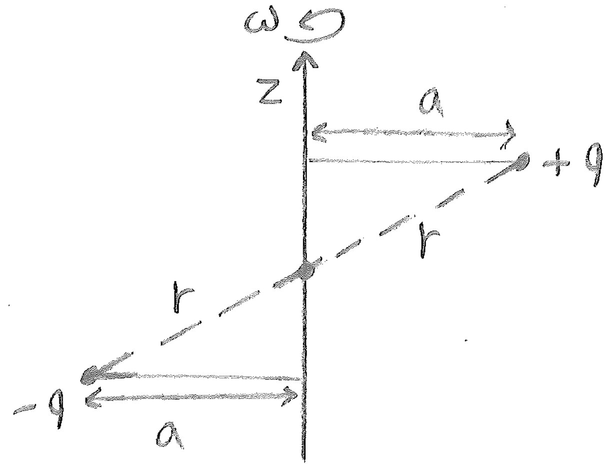
J_0, r_0 are constants and (r, θ, z) denotes the cylindrical coordinates.

6– A charge q moves through the interstellar medium in a region with a uniform magnetic field $\vec{B} = B_0\hat{z}$. Ignoring any radiation, describe the motion of the charge assuming that it is initially moving in the xy plane. Show that there exists a particular origin of coordinates such that the linear momentum of the charge obeys the relation $\vec{p} = q\vec{r} \times \vec{B}$, where $\vec{r}$ is the position of the charge. Find the distance of the charge from your chosen origin.

7– A thin transparent glass slab is oriented normally to an incident electromagnetic plane wave with time-averaged intensity I. The back surface of the slab is coated with silver so it completely reflects the incident radiation. What is the time-averaged force of radiation if the exposed front surface of the slab has area A? Repeat the problem if the silvering is removed so only a fraction $R < 1$ of the incident power is reflected.



8– Two point charges $+q$ and $-q$ are set in circular orbits with angular velocity ω about the z axis as shown in the figure. Find the instantaneous rate of energy loss due to electric dipole radiation. What is the state of polarization of radiation according to an observer on the z axis?



9- An unpolarized monochromatic electromagnetic plane wave of angular frequency ω is incident from vacuum on the plane surface of an ideal plasma. The refractive index of the plasma may be expressed as

$$n(\omega) = \left(1 - \frac{\omega_P^2}{\omega^2}\right)^{1/2},$$

where $\omega_P = \left(n_e e^2/\epsilon_0 m_e\right)^{1/2}$ is the plasma frequency (e and m_e denote electric charge and mass of the electron respectively). We assume that $\omega_P < \omega$.
(a) What fraction of the power of the wave would be reflected back at normal incidence?
(b) For what range of values of the angle of incidence would the wave be fully reflected?

10– A plasma interacting with a static magnetic field $\vec{B}_0$ along the direction of propagation of an electromagnetic wave with angular frequency ω yields two slightly different indices of refraction for the two circular polarizations of the wave:

$$n_\pm = \left(1 - \frac{\omega_P^2}{\omega^2 \mp \omega\omega_B}\right)^{1/2},$$

where $\omega_B \equiv eB_0/m_e$ and $\omega_P = \left(n_e e^2/\epsilon_0 m_e\right)^{1/2}$ is the plasma frequency (e and m_e denote electric charge and mass of the electron respectively). Find the group velocity of the wave for the left- and right-circularly polarized states (upper sign and lower sign in above respectively). For which polarization will the photon move faster?