Department of Physics and Astronomy, University of New Mexico

Electricity and Magnetism Preliminary Examination

Spring 2014

Instructions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed. This sheet must be attached to your answers and submitted.
- Total time: 3 hours.

Possibly Useful Formulas

- Vector identity:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A}\,.$$

- Maxwell's equations:

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0}\,,$$
$$\vec{\nabla}.\vec{B} = 0\,,$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\,,$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right)\,.$$

- Power radiated by a non-relativistically moving charge q with acceleration $\vec{a}$ (Larmor formula):

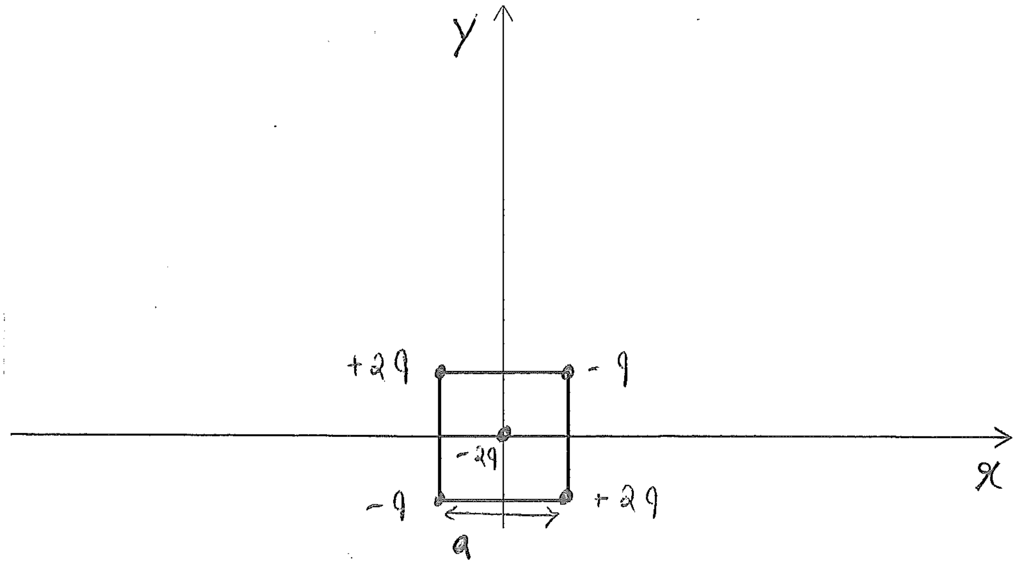
$$P = \frac{q^2 |\vec{a}|^2}{6\pi\epsilon_0 c^3}\,.$$

- Fresnel formulas for the amplitude reflection coefficient of a plane wave incident at a planar interface between two media:

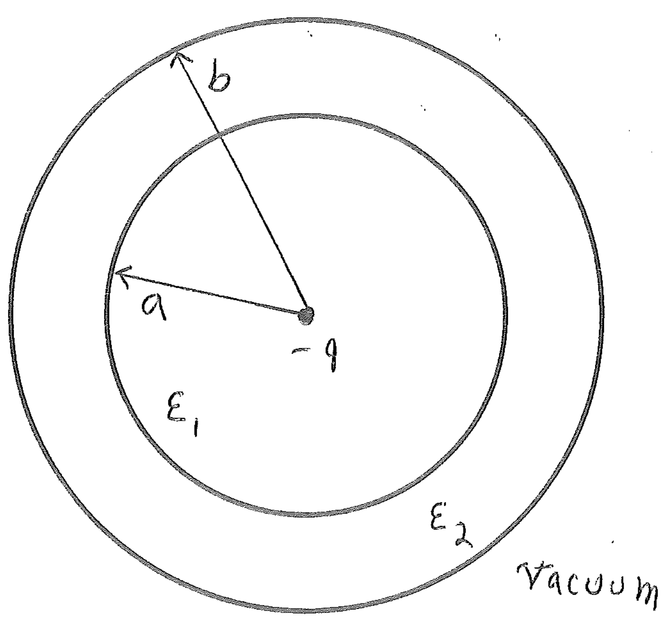
$$r_{\perp} = \frac{n \cos\theta - n' \cos\theta'}{n \cos\theta + n' \cos\theta'} \quad , \quad r_{||} = \frac{n' \cos\theta - n \cos\theta'}{n' \cos\theta + n \cos\theta'} ,$$

  where $\perp$ and $||$ refer, respectively, to polarizations perpendicular and parallel to the plane of incidence. The angles of incidence and refraction are θ and θ', and n, n' are the refractive indices of the medium of incidence and the medium of transmission, respectively.

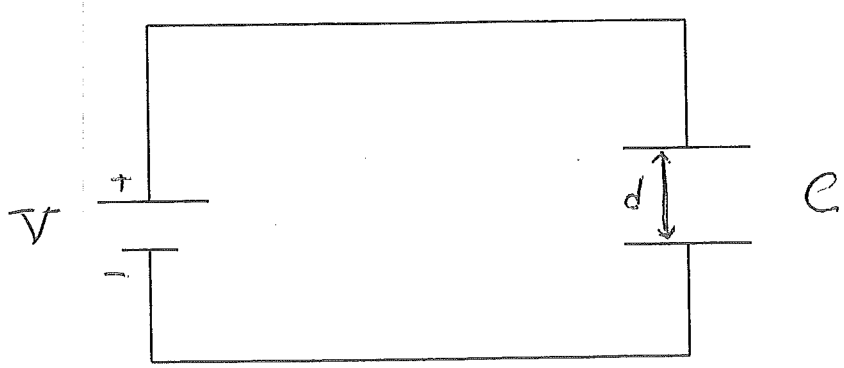
1– Five charges are placed at the corners and center of a square in the xy plane as shown in the figure. Assuming $d \gg a$, find the first non-zero contribution in the powers of a/d to the electric potential at the location (d, d).



2– A point charge $-q$ is situated at the center of a dielectric spherical shell (permittivity ϵ_1) with inner and outer radii a and b respectively. The space at $r < a$ is filled with dielectric material with permittivity ϵ_2. Determine the electric field in all regions, and sketch a graph of the magnitude of $\vec{E}$ versus r. Assume $\epsilon_1 < \epsilon_2$.



3– A parallel plate capacitor consists of two plates of area A in the vacuum separated by a variable distance d. The capacitor is connected to a battery with voltage V. What is the force of one plate on the other as a function of V and d? Show that the external work that must be performed to slowly increase the separation distance from $d = d_1$ to $d = d_2$ is given by negative of the change in $CV^2/2$, where C is the capacitance.

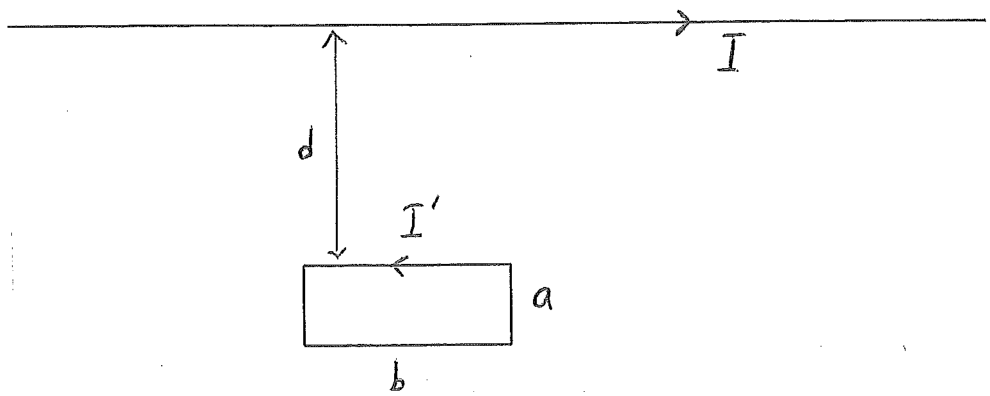


4– Find the magnetic field $\vec{B}(r, \theta, z)$ that is generated by the following current density:

$$\vec{j} = j_0 \exp(-\frac{r^2}{r_0^2})\ \hat{z}\,.$$

j_0 and r_0 are constants, and (r, θ, z) denote cylindrical coordinates.

5– Consider a rectangular loop of wire with width b and height a as shown in the figure. The loop carries current I' and is oriented parallel to a long straight wire carrying current I. How does the force exerted on the loop depend on the separation distance d when $d \gg a$? (Both currents are held constant)



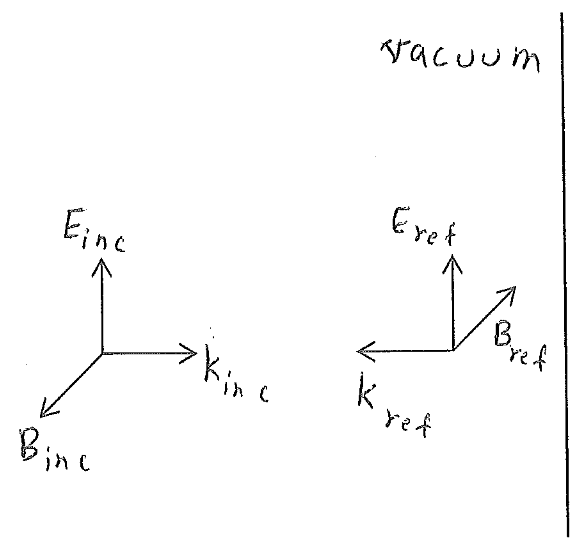
6– An electron with speed $v \ll c$ undergoes cyclotron motion (in the xy plane) in a magnetic field $B(r)$ (along the z direction) at the cyclotron radius r_0. Under a particular condition the speed of the electron can be increased without changing the orbit radius by slowly changing the B-field in time. Show that this is possible if

$$B(r_0, t) = \frac{\int_0^{r_0} B(r,t) r dr}{r_0^2} .$$

7– Consider a monochromatic electromagnetic wave traveling in the vacuum at normal incidence to a medium with permittivity ϵ and permeability μ as shown in the figure. By using the boundary conditions dictated by Maxwell's equations, show that the ratio of the reflected to incident electric field amplitude is

$$\frac{E_{\text{ref}}}{E_{\text{inc}}} = \frac{1-Z}{1+Z},$$

where $Z = \sqrt{\mu/\epsilon}$ is the relative wave impedance in the material.



8– An electron (charge: $-e$, mass: m) enters a uniform magnetic field $\vec{B} = B_0\hat{x}$ with an initial velocity $\vec{v} = 3v_0\hat{x} - 2v_0\hat{y} + v_0\hat{z}$ ($v_0 \ll c$). Describe the trajectory of the electron and find the initial rate of energy loss due to electric dipole radiation.

9– A transverse electromagnetic wave travels inside a neutral plasma, inducing a current density $\vec{j} = -n_e e\vec{v}$, where n_e is the density of free electrons in the plasma that are driven with instantaneous velocity $\vec{v}$ by the electric field. By using the Maxwell's equations, show that this wave satisfies the equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{\omega_p^2}{c^2}\vec{E},$$

where $\omega_p = (n_e e^2/\epsilon_0 m)^{1/2}$ is the plasma frequency. Use this to show that for a plane wave with angular frequency ω and wavenumber k the dispersion relation $\omega^2 = \omega_p^2 + c^2k^2$ is held.

10– A circularly polarized plane electromagnetic wave of angular frequency ω travels in the air. It comes to an interface with a transparent slab of refractive index n with incident angle θ. Show that there is a unique angle θ for which the reflected wave is linearly polarized and find this angle.