Department of Physics and Astronomy, University of New Mexico

Electricity & Magnetism Preliminary Examination

Spring 2010

Instructions:

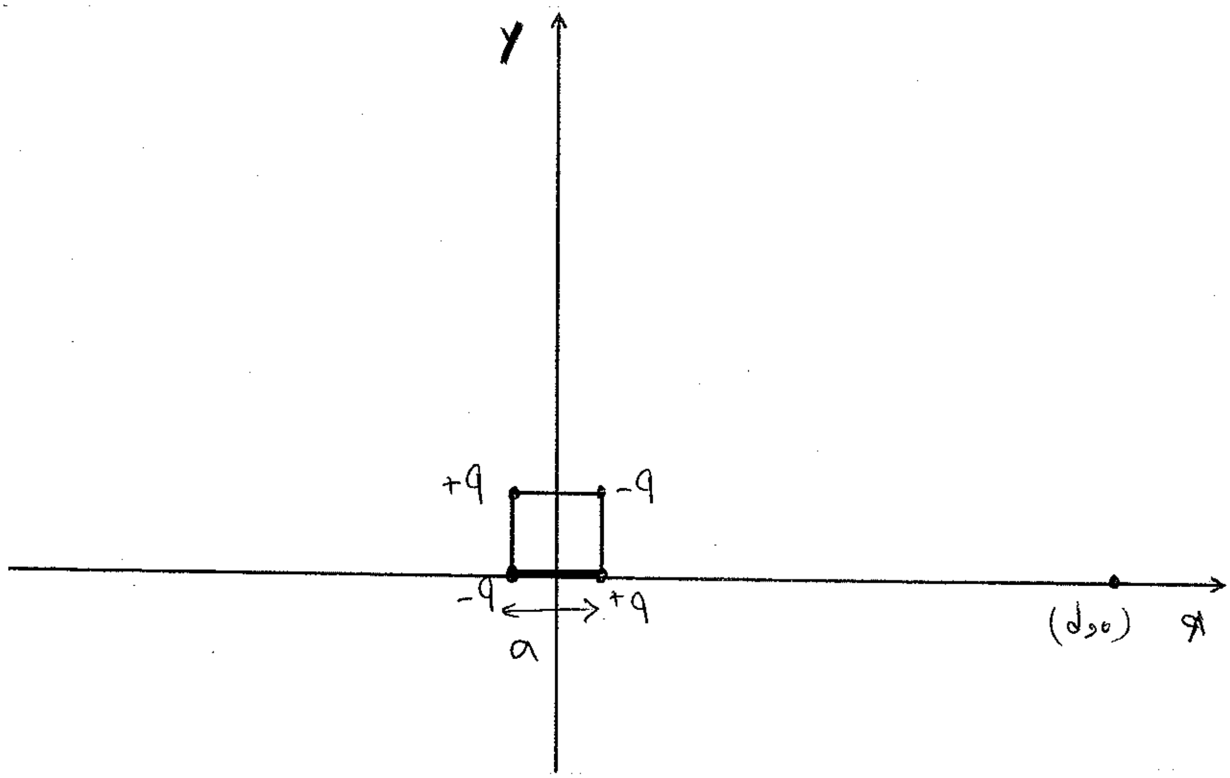
- The exam consists of 10 short-answer problems (10 points each).
- Partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed.
- Total time: 3 hours.

Some useful relations:

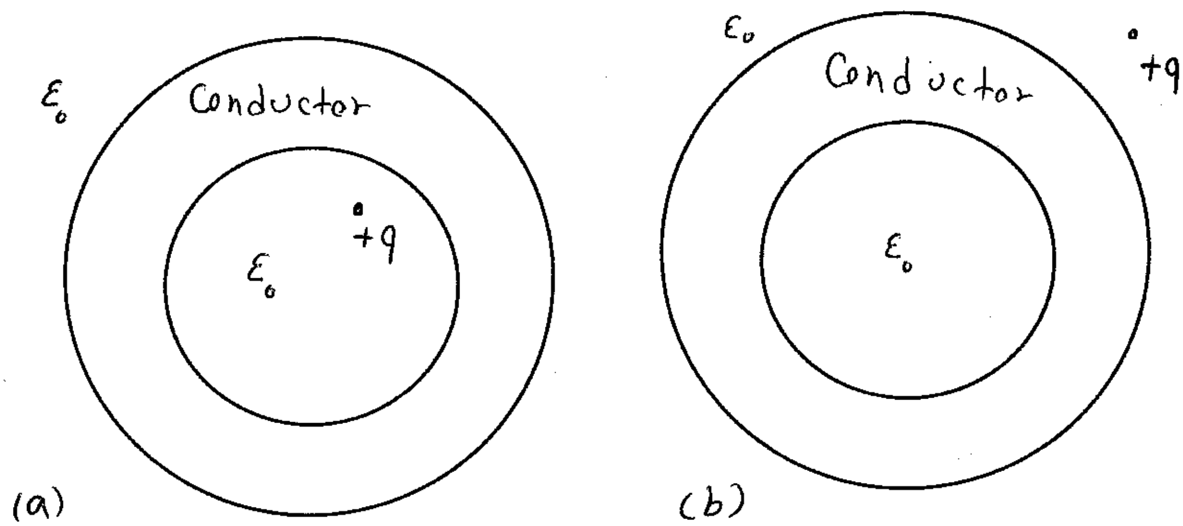
$\int_0^x x' e^{-\frac{x'}{x_0}}\, dx' = x_0^2 - x_0(x + x_0)e^{-\frac{x}{x_0}}.$

$\nabla \times (\nabla \times \vec{V}) = \nabla(\nabla \cdot \vec{V}) - \nabla^2 \vec{V}.$

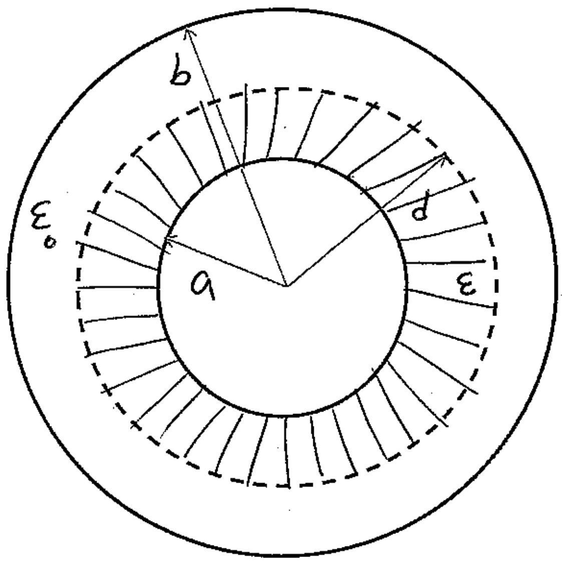
1– Four charges are placed at the corners of a square in the xy plane as shown in the figure. Assuming $d \gg a$, find the first nonzero contribution in powers of a/d to the electric potential at the location $(d, 0)$.



2– Shown is a solid spherical conductor with a concentric spherical hole in the center in two situations:
(a) with a charge $+q$ placed inside,
(b) with a charge $+q$ placed outside.
The conductor has zero net charge. In each case, determine the total charge on each of the two surfaces. Also sketch the surface charge density and electric field lines.



3– A capacitor consists of two concentric spherical surfaces of radii a, b. Part of the space between these surfaces is filled with dielectric material of permittivity ϵ as shown in the figure. What is the capacitance?

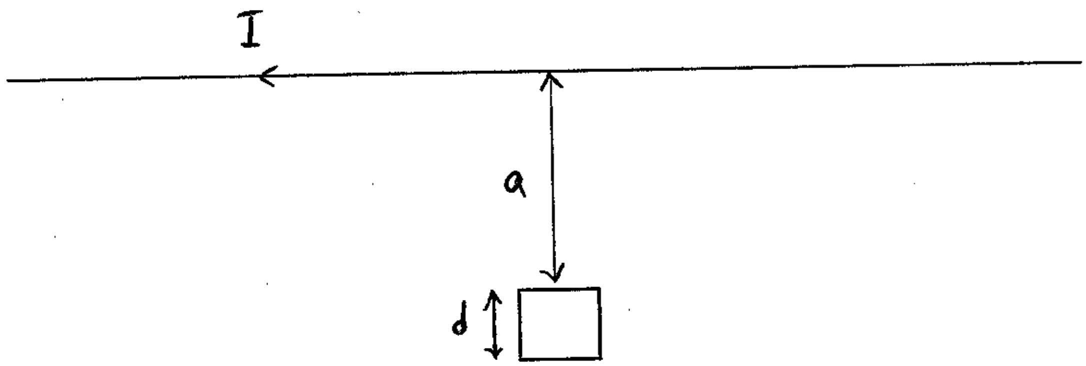


4– Find the magnetic field $\vec{B}(r,\theta,z)$ for the following current distribution:

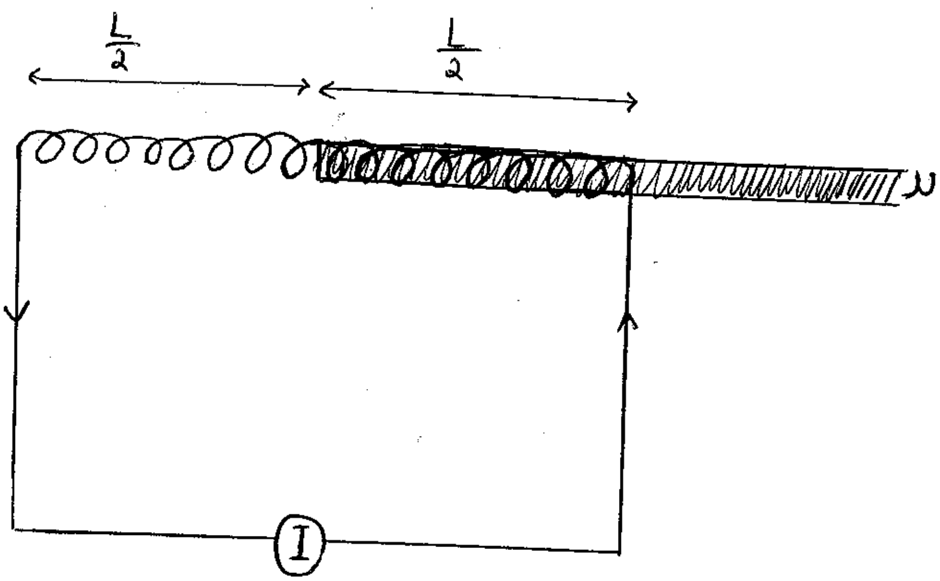
$$\vec{j} = j_0 \exp(\frac{-r}{r_0})\hat{z}\,.$$

Here j_0, r_0 are constants and (r,θ,z) denotes cylindrical coordinates.

5– Consider a square loop of wire with side length d that is oriented parallel to a long straight wire carrying current I. The loop has mass m. Determine the magnitude and direction (clockwise or counter-clockwise) of the current I' in the loop such that it will be suspended in mid-air.



6– A long solenoid with n turns per unit length and cross-sectional area A carries an electric current I. Half of the solenoid is filled with a magnetic material with permeability μ. Find the magnitude and direction of force required to hold the material in place. (Current I is held constant.)



7– An electron with speed v undergoes cyclotron motion (in the xy plane) in a magnetic field $B(r)$ (along the z direction) at the cyclotron radius $r_0 = mv/eB(r_0)$. The speed of the electron can be increased by slowly increasing the B-field in time. Show that if the field at r_0 is half of the average across the orbit

$$B(r_0, t) = \frac{1}{2}\frac{\int B(r,t)da}{\pi r_0^2}$$

the radius of orbit is constant in time. (Assume non-relativistic speeds)

8– A transverse electromagnetic wave travels inside a neutral plasma, inducing a current density $\vec{j} = -n_e e \vec{v}$, where n_e is the density of electrons driven with instantaneous velocity $\vec{v}$ by the electric field. Use Maxwell's equations to show that the electric field for these waves satisfy the equation

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{E} = \frac{\omega_p^2}{c^2}\vec{E}\,,$$

where $\omega_p = \left(n_e e^2/\epsilon_0 m\right)^{1/2}$ is the plasma frequency. Ignore any electron damping.

9– Given that the magnetic field associated with a plane wave traveling in a nonmagnetic, dielectric medium is given by

$$\vec{B}(\vec{r},t) = B_0 \sin[\omega(3y/2c - t)]\hat{x} + B_0 \cos[\omega(3y/2c - t)]\hat{z} ,$$

find the direction of the propagation of the wave, its polarization, the dielectric constant of the material, and the intensity of the radiation?

10– An electron enters a uniform magnetic field $\vec{B} = B_0\hat{z}$ with an initial velocity $\vec{v} = v_0\hat{x} + 2v_0\hat{y} + 3v_0\hat{z}$ (with $v_0 \ll c$). It then loses its energy as a result of electromagnetic radiation. What is the initial rate of energy loss due to electric dipole radiation?