Preliminary Examination: Electricity and Magnetism
Department of Physics and Astronomy
University of New Mexico
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Instructions: ??????

Problem 1: A thin ring, of radius a, has a non-uniform, linear charge density on it, of amount $\lambda = \lambda_0 \sin\phi$. Please find the net charge of the ring, and its dipole moment. Along the direction normal to the ring, which we call z, at very large distances the electrostatic potential is approximately proportional to $1/z^n$. What is the value of n?

Problem 2: Find the capacitance of a pair of concentric, spherical metal shells, which have radii a and b, with $b > a$.

Problem 3: A particular parallel plate capacitor has an area A and a distance h separating the plates. It has been charged so that one plate has a charge Q, and the other the negative of that, but has recently been disconnected from the charging battery. Half of the area is filled, over the entire separation distance, with a dielectric material with dielectric constant ϵ. You now attempt to pull this material out from between the two plates. What is the minimum force you will have to exert in order to do this?

Problem 4: An originally-uncharged, metal sphere of radius a is placed in an otherwise uniform electric field, $\vec{E} = E_0\hat{z}$, which induces a charge distribution on the sphere. As a result of this the total electrostatic potential exterior to the sphere may be written as

$$V(r,\theta) = -E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta\ .$$

What is the induced charge distribution? What is the potential's dependence on r interior to the sphere?

Problem 5: A rectangular loop of wire hangs vertically, and supports a mass m that hangs downward from it, under the influence of gravity. The upper end of the loop finds itself in a region where there is a uniform magnetic field $\vec{B}$, which points into the page. For what current, I, in the loop, would the mass be suspended in mid-air? What direction must that current have?

Problem 6: What current density, $\vec{J}$, would produce the magnetic vector potential, $\vec{A} = k\,\hat{\phi}$, in cylindrical coordinates, where k is a constant?

Problem 7: A square loop of wire, of side length a, lies on a table, a distance s away from a very long, straight wire which carries a current, $I = I_0 \sin\omega t$. The square loop has a total resistance R. What current flows in it?

Problem 8: Using Maxwell's Equations with sources, show that the total charge inside any fixed, finite volume is conserved, i.e., constant in time.

Problem 9: The magnetic field associated with a plane wave travelling in a nonmagnetic, dielectric medium is given by

$$\vec{B}(\vec{r},t) = B_0\{\cos[\omega(2x/c - t)]\hat{z} + \sin[\omega(2x/c - t)]\hat{y}\} \ .$$

a. What is the direction of propagation of the wave?

b. What is the direction of the polarization of the wave, and what is its nature?

c. What is the dielectric constant of the material through which it is travelling?

d. What is the intensity of the radiation?

Problem 10: A particular observer, $\mathcal{O}$, measures fields in a small region in space, where he finds approximately uniform electric and magnetic fields as follows, where A is a constant:

$$\vec{E} = A\hat{x} \ , \quad \vec{B} = 3A\hat{y} \ .$$

a. Another observer, $\mathcal{O}'$, passes by at a velocity $\vec{v} = \alpha\hat{z}$. What must the value of α be in order that this observer measures no electric field?

b. Could there be yet another observer, $\mathcal{O}''$, moving at a different velocity such that she would see no magnetic field? If so, what would her velocity be?